

VOLUME 36, NUMBER 1, MARCH 2019-2020

ALGEBRAS, GROUPS AND GEOMETRIES Volume 36, Number 1, March 2019-2020

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ALGEBRAS GROUPS AND GEOMETRIES

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PROCEEDINGS OF THE INTERNATIONAL TELECONFERENCE

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Albert Einstein, Boris Podolsky and Nathan Rosen in the 1930s.

**PROCEEDINGS OF
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EINSTEIN-PODOLSKY-ROSEN ARGUMENT THAT
"QUANTUM MECHANICS IS NOT A COMPLETE THEORY"**

September 1 to 5, 2020

Also published by

Curran Associates Conference Proceedings, New York, U.S.A. (2021)

<http://www.proceedings.com/60007.html>

<http://www.proceedings.com/59404.html>

Dedicated to the memory of

Prof. Zbigniew Oziewicz

Poland 1941 - Mexico 2020

Universidad Nacional Autónoma de México

Facultad de Estudios Superiores Cuautitlán

for his independence and depth of scientific thought.

Biographical Notes

<http://eprdebates.org/docs/Zbigniew-Oziewicz-biography.pdf>

PROCEEDINGS OF THE INTERNATIONAL TELECONFERENCE

ALGEBRAS GROUPS AND GEOMETRIES
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Albert Einstein, Boris Podolsky and Nathan Rosen in the 1930s.

**OVERVIEW
OF HISTORICAL AND RECENT VERIFICATIONS OF
THE EPR ARGUMENT AND THEIR APPLICATIONS
IN PHYSICS, CHEMISTRY AND BIOLOGY**

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Reprinted from

APAV - Accademia Piceno Aprutina dei Velati, Pescara, Italy

2021

PREFACE OF THE OVERVIEW

Almost from the moment it was first published in 1935, the famous - some might say infamous - article by Einstein, Podolsky and Rosen, claiming quantum mechanics to be an incomplete theory, has courted controversy. Following the initial furore, things calmed down with many accepting Bohr's refutation of the Einstein, Podolsky, Rosen argument. However, the topic has resurfaced periodically over the intervening years with no completely clear resolution emerging as far as some are concerned. In 2018 the entire issue resurfaced with the publication of some experimental results from a laboratory in Basle, results which served to support the view of that 1935 paper by Einstein, Podolsky and Rosen. This was followed by the international conference to be held in Florida in 2020 but which had to be turned into an online conference because of the covid-19 problem occurring in most, if not all, of the World. The conference ended up being a success at publicizing so many views, from so many people, which again supported the Einstein-Podolsky-Rosen line of thought. In particular, the conference brought to the fore the enormous, unheralded contribution to the debate by Sir Ruggero Maria Santilli (www.i-b-r.org/Ruggero-Maria-Santilli.htm - "Ruggero" hereon), with his first major contribution having come in 1998 after many years devoted to developing the new mathematics needed to cope adequately with the problems surrounding the Einstein, Podolsky, Rosen issue.

This book is intended to outline a collection of primary works from the mid 1960's to date by Ruggero and his collaborators on the verification and application of the Einstein-Podolsky-Rosen (EPR) argument that *Quantum mechanics* [and, therefore, quantum chemistry] *is not a complete theory*, in the expected recovering of classical determinism at least under limit conditions [1] (references indicated with square brackets refer hereon to those of Ruggero's Overview, while references indicated with upper numbers refer to additional works quoted in this preface).

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This Preface is intended to provide a guide through a rather voluminous collection of works in various fields, as well as indicate important references following Ruggero's writing of the Overview.

During his Ph. D. studies at the University of Torino, Italy, in the mid 1960's, Ruggero discovered that the most advanced mathematics available at that time was insufficient for the representation of systems more complex than atomic structures, such as nuclear fusions, extended particles in deep mutual overlapping, combustion, biological structures, anti-matter and other complex systems in the universe.

Extensive research carried out in 1964 at European mathematics libraries and repeated in 1978 at Cantabridgean mathematics libraries, convinced Ruggero that the mathematics needed for the effective representation of the indicated complex systems did not exist but had to be built and he had the courage to do it. In fact, Ruggero first recognized the need for new mathematics, constructed it himself and then proceeded to use it to examine complex systems not only in physics but in chemistry and biology as well.

In fact, throughout his research life, Ruggero first constructed the needed new mathematics and then passed to the treatment of complex systems in physics, chemistry or biology.

It then follows that no serious understanding of Ruggero's works is possible without a prior knowledge of the underlying new mathematics that, for the reader's convenience, are merely listed below with their primary references.

Quite notable is Ruggero's quote that "*There cannot exist a truly new physical theory without a new mathematics, and there cannot exist a new mathematics without new numbers.*"

I. NEW MATHEMATICS.

I-1. Lie-admissible mathematics.

As part of his Ph. D. curriculum, Ruggero studied the works by Giuseppe

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Luigi Lagrange who lived in Torino and wrote a number of papers in Italian. In this way, Ruggero learned that Lagrange represented physical reality via his celebrated analytic equations containing potentials represented with Lagrangians L , plus *external terms* F representing non-conservative systems that, as such, are irreversible over time.

A comparison of Lagrange's original equations with the theories he was studying in Ph. D. courses soon revealed that Lagrange's external terms were not present in any of the available theories and that said external terms could not be represented with the available mathematics for various technical reasons.

A study of the works by Sir William Rowan Hamilton revealed their full parallelism with Lagrange's works, only formulated in phase space. In fact, the celebrated Hamilton's equations comprise a Hamiltonian H plus external terms F representing non-conservative and irreversible effects which were absent in the scientific literature of the mid 1960's.

Ruggero also learned that quantum mechanics cannot represent time irreversible systems because its main dynamical equations, Heisenberg's equation for an observable A , $idA/dt = [A, H] = AH - HA$ (where AH is the conventional associative product) can only represent the conservation of the energy, since $idH/dt = [H, H] = 0$.

To build the new mathematics needed for the representation of energy releasing, irreversible processes, Ruggero identified the main mathematical structure of quantum mechanics, which is given by Lie's theory with brackets $[A, B] = AB - BA$ and decided to generalize Lie's algebras into an algebra with brackets $(A, B) = A < B - B > A = ARB - BSA$ (where R, S are different, positive-definite operators) that turned out to be Lie-admissible according to the American mathematician, A. A. Albert [9]. The identification of the foundations of the new Lie-admissible mathematics immediately allowed Ruggero to generalize Heisenberg's equation into the form, today called *Heisenberg-Santilli Lie-admissible equations*, $idA/dt = (A, H) = A < H - H > A = ARH - HSA$ which repre-

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sents the *time rate of variation* (rather than the conservation) of the energy $idH/dt = (H, H) = H(R-S)H \neq 0$ (see papers [6] [7] [8] of Ruggero's Ph. D. thesis although the unpublished version of the thesis better illustrates the originating thoughts).

The above studies in irreversibility attracted NASA attention and Ruggero moved to the U.S.A. in 1967 with his wife Carla and their newborn daughter Luisa for a one year appointment at the Center for Theoretical Physics of the University of Miami, Florida, with NASA support. He then accepted a faculty position at the Department of Physics of Boston University where he remained from 1968 to 1974 with partial support from the U. S. Air Force by teaching mathematics and physics at all levels and writing various papers with his associates and graduate students listed in his curriculum. From 1974 to 1977, Ruggero was a visiting scientist at the MIT Center for Theoretical Physics following an invitation from his Director Francis Low.

During his three year stay at the MIT-CTP, Ruggero wrote works¹⁻¹¹ which are the *analytic foundation of verifications [210]-[214] of the EPR argument*, and comprise:

A. MIT-CTP preprints¹⁻⁵ on the necessary and sufficient conditions for the existence of a Lagrangian in field theory, technically known as the *conditions of variational selfadjointness* (SA), and their use for the representation of systems that are variationally non-selfadjoint (NSA) that are *analytic* in the sense of being derivable from a generalized action principle.

B. MIT-CTP memoirs^{6,7,8} published at Annals of Physics which summarize the content of preprints¹⁻⁵.

C. MIT-CTP preprints^{9,10} presenting the Newtonian particularization of the preceding field theoretical works, that Ruggero submitted in early 1977 to Springer-Verlag, Heidelberg, Germany, for publication as mono-

graphs.

D. MIT-CTP preprint¹⁰ intended to be a third field theoretical volume of the Newtonian references^{9,10}, which preprint has remained unpublished, yet available as Annals of physics papers^{6,7,8}.

As we shall see in Section III, the above field theoretical works are important for the EPR completion of quantum electrodynamics into a form representing deviations of the theory from recent measures.

In September 1977, Ruggero joined Harvard University with a joint appointment as a visiting scientist at the Lyman Laboratory of Physics and at the Department of mathematics. In view of his works at MIT, Ruggero received on arrival an invitation from the U. S. Department of Energy for a grant intended to search for possible new clean nuclear energies. Soon thereafter, Ruggero also received the acceptance from Springer-Verlag for the publication of MIT preprints^{9,10}.

Encouraged by these openings, Ruggero plunged himself, firstly, in the construction of the Lie-admissible mathematics and secondly, in its application for the treatment of energy releasing irreversible systems.

The resulting main studies at Harvard University in the Lie-admissible mathematics, also called *genomathematics*^{6,7,8}, are given by: the 200 page memoir [19] of 1978 setting up the foundations of the new mathematics; Springer-Verlag monographs [21] [22] of 1978 proposing for the first time the completion of quantum mechanics into *hadronic mechanics* (see page 112) via the Lie-admissible generalization of Lie's theory and Heisenberg's equations and monographs [39] [40] on the Lie-admissible formulation of various aspects of 20th century applied mathematics.

In 1982, Ruggero accepted the position of President and Professor of Physics at the Institute for Basic Research (IBR) located at the Prescott House within the compound of Harvard University, which was moved to Florida in 1989 (www.i-b-r.org).

Among a rather large scientific production at the IBR reviewed in the

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Overview, we here merely mention the systematic presentations of the various branches of hadronic mechanics in monographs [23]-[25] of 1996, and the five volumes [74] of 2006, all works having extensive references on independent contributions.

Various lectures on Lie-admissible mathematics are available on the website www.world-lecture-series.org. A tutoring lecture specifically intended to the verification of the EPR argument is available in Ref. [145].

Contributions in Lie-admissible mathematics that are important for the verification of the EPR argument are the following:

1. Paper [37] of 1979 established the bimodular structure of Lie-admissible mathematics, in the sense that the product $(A, B) = A < B - B < A = ARB - BSA$ can be reduced to two nodular actions, one to the right (representing motion forward in time) $H > |\psi \rangle = HS|\psi \rangle = E|\psi \rangle$, and one to the left (representing motion backward in time) $\langle \psi | < H = \langle \psi |RH = \langle \psi |E'$, $E' \neq E$, whose in-equivalence assures the axiomatic representation of irreversibility. Said bimodular structure also assured the preservation of quantum mechanical axioms by the Lie-admissible branch of hadronic mechanics, since in a bimodular structure, quantum axioms are merely formulated per each selected time ordering.

2. In early 1980, it became known that physical applications of Lie-admissible methods are inconsistent when formulated over conventional numeric fields. This impasse was resolved with the discovery in the 1993 paper [33] of the genotopic numbers with multiplicative genounit to the right $I^> = 1/s$ and to the left $I^< = 1/R$.

3. In the mid 1990's it became also known that the representation of extended particles via Lie-admissible methods were inconsistent when elaborated via Newton-Leibnitz differential calculus due to its strict local character. In particular, hadronic mechanics was still missing in the mid 1990's a consistent generalized formulation of the angular momentum

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due to the lack of a consistent generalization of the quantum mechanical linear momentum $p|\psi(r) \rangle = -i\hbar\partial_r|\psi(r) \rangle$. These additional impasses were resolved via the generalization-completion in Ref. [34] of 1996 of the conventional differential calculus into a form defined on *volumes*, rather than points, with *genomomentum* $p^> \rangle |\psi^>(r^>) \rangle = -i\partial_{r^>}^> |\psi^>(r^>) \rangle = -iI^>\partial_{r^>} |\psi^>(r^>) \rangle$ and to the left $\langle \psi^<(<r) | \langle^< p = -i\partial_{<r}^< \langle \psi^<(<r) | = -i^<I\partial_{<r} |\psi^<(<r) \rangle$ where volumes are represented by the *genounits* $I^> = 1/S$ and $^<I = 1/R$, respectively.

4. Paper [35] of 1989 identified a very simple method for the completion of 20th century mathematics into the Lie-admissible covering via the following two nonunitary transformations $UW^\dagger = I^>$, $WU^\dagger = ^<I$, $UU^\dagger \neq I$, $WW^\dagger \neq I$ and the proof of the invariance over time of the genounits with consequential invariance of the shape and density of the represented particles.

5. Memoir [41] of 2006 proved the universality of the Heisenberg-Santilli geno-equation for the representation of all possible (regular) non-linear, non-local, non-conservative, NSA systems via realizations of the type $R = 1$, $S = 1 - = F/H$ with dynamical equation $idA/dt = (A, H) = A < H - H > A = AH - HA - AF$ where F is a suitably normalized operator form of Lagrange's and Hamilton's external terms.

In closing, we should indicate that there exists considerable literature on Lie-admissible algebras within the context of non-associative algebras in pure mathematics (see, e.g., Ref.¹³ and Vol. I of Refs. [74]). However, these studies are formulated over conventional numeric fields, and even though mathematically correct, they cannot be used for the verification of the EPR argument because of a number of insufficiencies identified in Ruggero's Overview.

I-2. Lie-isotopic mathematics.

All objections against the EPR argument (see, e.g., Refs. [2]-[4]) are for-

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mulated for quantum mechanical, thus *conservative* systems for which Lie-admissible mathematics is inapplicable.

Additionally, in order to search for possible new clean nuclear energies under his DOE grant, Ruggero had to study nuclear structures, that when stable and isolated, verify all conventional total conservation laws, yet admit a more general formulation of internal strong interactions.

These requirements mandated the construction of a new mathematics for the representation of isolated, thus conservative systems with *extended* constituents in deep mutual penetration-entanglement, under non-linear, non-local and NSA interactions, yet such to verify conventional total conservation laws.

The needed new mathematics was identified in paper [19] of 1978 as the particular case of the Lie-admissible mathematics for $R = S = T > 0$ with basic brackets $[A, B]^* = A \star B - B \star A = ATB - BTA$, where T is called the isotopic element, which verify Lie's axioms, for which reason Ruggero called the new mathematics *Lie-isotopic* or *isomathematics* for short. Following the original proposal [19], isomathematics was studied in detail in monograph [22] of 1981 via the identification of its universal enveloping isoassociative algebra ξ with isoproduct $A \star B = ATB$, the initiation of the isofunctional analysis and the isotopic completion of the various branches of Lie's theory.

Isomathematics achieved maturity with the discovery of isonumbers [33], the isodifferential calculus [34] and the simple method for its construction in Ref. [35]. Systematic presentations of isomathematics were then provided in monographs [23]-[25] and [74].

Various lectures in isomathematics are available from www.world-lecture-series.org. A tutoring lecture in isomathematics specifically intended for the verification of the EPR argument is available from Ref. [143].

We should note the completion of the quantum entanglement of particles into the covering *EPR entanglement* (introduced by Ruggero in his

Overview and first released in paper¹⁸), which represents the non-linear, non-local and NSA interactions due to the overlapping of the wavepackets of particles via realizations of the isotopic element T , by jointly providing an explicit and concrete realizations of Bohm's *hidden variables* [17] (see Figure 2 and Section 7.3.2 of the Overview).

There is little doubt that Ruggero's EPR entanglement will have applications in all quantitative sciences. To see it, it is sufficient to note that the conception of a nucleus, or a molecule or a virus, as being composed of extended constituents under EPR entanglement, implies the inapplicability of all objections against the EPR argument [2]-[4], thus opening the door for a new physics as well as chemistry and biology.

We should also indicate that isomathematics has attracted considerable interest in mathematical circles and has seen a number of important contributions by pure mathematicians identified in the Overview, with complete listing in Vol. I of monographs⁷⁴.

I-3. Hypermathematics.

In line with his belief that mathematics will never admit final formulations, Ruggero has stated various times, that despite their vast representational capabilities, Lie-admissible and Lie-isotopic formulations cannot describe "all elements of reality" [1] because they are single-valued (in the sense that the multiplication of two quantities yields one single result, e.g., $2 \times 3 = 6$). This is an excessive limitation for the representation of complex systems, such as biological structures, which suggested Ruggero to turn Lie-admissible and Lie-isotopic mathematics into *multi-valued* (rather than multi-dimensional) forms via genotopic R , S and isotopic T elements representing an ordered set of values. As an example, the assumption that the isotopic element has three values $T = \{2, 3, 1/2\}$ implies that $2 \times 3 = \{12, 18, 1/3\}$ [27].

Even though sufficient for "simple" biological structures such as seashells, the above formulation of hypermathematics turned out to be in-

sufficient for the representation of a living organism such as a cell.

Thanks to the participation of the Greek mathematician Thomas Vougiouklis, the above multi-valued formulations were generalized-completed into the most general and complex mathematics conceivable nowadays by the human mind, that of the *Lie-admissible and Lie-isotopic hyperstructures defined on hyperspaces over hyperfields* (see the Tutoring Lecture [146]).

I-4. Isodual mathematics.

Despite the advances indicated above, Ruggero still considered Lie-admissible and Lie-isotopic mathematics as being unable to represent *all* elements of reality [1].

During his graduate studies in the mid 1960's, Ruggero wanted to study whether a far away galaxy is made up of matter or of antimatter, but was prohibited from doing such a study because the most advanced mathematics and physics available at that time identified no difference between matter and other complex systems in the universe.

Additionally, Ruggero has been a supporter of Dirac's view that *anti-matter has negative energy* [11], as a pre-requisite for the representation of matter-antimatter annihilation.

Recall that, as discovered by Dirac himself, negative energies violate causality in the sense that the effect generally precedes the cause in the solution of quantum mechanical equations with negative energy).

For the intent of resolving Dirac's causality problem, while being at the Department of mathematics of Harvard University in the early 1980's, Ruggero decided to build the foundations of yet another new mathematics, this time based on the *negative unit* " -1 " under the name of *isodual mathematics* where the word "isodual" stands to indicate an axiom-preserving duality of 20th century mathematics.

This lead to the construction of the isodual images of the conventional, Lie-isotopic and Lie-admissible mathematics [29].

In view of the construction of the above new mathematics and their application in physics, chemistry and biology, Ruggero was listed in 1990 by the Estonia Academy of Sciences among the most illustrious applied mathematicians of all times (see <http://santilli-foundation.org/santilli-nobel-nominations.htm>).

II. VERIFICATIONS OF THE EPR ARGUMENT.

Thanks to the new mathematics indicated in the preceding section, Ruggero and his associates have constructed the axiom-preserving completion of quantum mechanics into *hadronic mechanics* comprising the *Lie-isotopic*, *Lie-admissible*, *hyperstructural* and *isodual* branches [23]-[25] [74], with corresponding completions for *hadronic chemistry* [30] and *hadronic biology* [27] (see the outline in Section 6 of the Overview). Following these preparatory studies, as well as the teaching from historical completions of quantum mechanics by W. K. Heisenberg [16], Prince L. V. P. R. de Broglie [17], D. J. Bohm [18] and others reviewed in Section 5 of the Overview, Ruggero achieved the following verifications of the EPR argument (see also the presentation at the 2020 EPR conference by E.T. D. Boney on Gödel's incompleteness theorems [217] and by A.A. Nassikas on the minimum contradiction theory [218], as well as their recorded talks available from Ref. [0]):

II-1. Verification of the EPR argument for irreversible processes.

We are here referring to Ruggero's 1967 Ph. D. thesis [6]-[8] (see also memoir [41]) in which he proved the lack of completion of quantum mechanics for energy releasing, thus time irreversible processes and constructed the foundations of the Lie-admissible completion of quantum mechanics, also called *genomechanics*.

Besides scientific and industrial applications, the irreversible character of the Lie-admissible mechanics has stimulated studies to achieve a

connection between mechanics and thermodynamics, of course, via the intermediate step of irreversible statistical mechanics, see P. Roman et al¹⁴, A. A. Bhalekar¹⁵, J. Fronteau et al [42], J. Dunning Davies [52] and other contributions.

II-2. Verification of the EPR argument for classical counterparts.

In a paper of 1964, J. S. Bell [3] proved a theorem essentially stating that a system of quantum mechanical particles with spin $1/2$ does not admit a classical counterpart.

Thanks to the prior development of the Lie-isotopic mathematics, Ruggero proved in his 1998 paper [210] that Bell's theorem is inapplicable (rather than being violated) for a system of extended particles with spin $1/2$ under deep EPR entanglement and said system does indeed admit a fully defined classical counterpart.

The proof was essentially based on the completion of Bell's theorem via the isoproducts $A \star B = A\hat{t}B$, $\hat{T} > 0$, which allows an explicit and concrete realization of Bohm's *hidden variables* [17].

In the same paper [210], Ruggero illustrates the validity of hadronic mechanics via a numerically exact representation of nuclear magnetic moments that escaped a quantum mechanical representation for close to one century.

II-3. Verification of the EPR argument for classical determinism.

In a paper of 1981 [47], Ruggero introduced a generalization-completion of Heisenberg's uncertainties for strong interactions among extended hadrons in deep mutual entanglement (as occurring in a nuclear structure) when they are represented via hadronic mechanics.

In the 2019 paper [211], Ruggero proved the progressive recovering of Einstein's determinism in the interior of hadrons, nuclei and stars and its full recovering at the limit of gravitational collapse.

The proof is based on the realization of the isolinear momentum via the isodifferential calculus [34], $\hat{p} \star |\hat{\psi}(\hat{r})\rangle = -i\hat{\partial}_{\hat{r}}|\hat{\psi}(\hat{r})\rangle = -i\hat{I}\partial_{\hat{r}}|\hat{\psi}(\hat{r})\rangle$, $\hat{I} = 1/\hat{T} > 0$ (where "hat" denotes definition in hadronic mechanics).

Let Heisenberg's uncertainties be given by $\Delta p \Delta r = (1/2) \langle \psi(r)|[r, p]|\psi(r)\rangle \leq (1/2)\hbar$ under the normalization $\langle \psi|\psi \rangle = \hbar$. The isotopies lead uniquely and unambiguously to the *isouncertainties* $\Delta \hat{p} \Delta \hat{r} = (1/2) \langle \hat{\psi}(\hat{r})| \star [\hat{r}, \hat{p}] \star |\hat{\psi}(\hat{r})\rangle \leq (1/2)\hat{T}$ under the isonormalization $\langle \hat{\psi} | \star |\hat{\psi} \rangle = \langle \hat{\psi} | \hat{T} | \hat{\psi} \rangle = \hat{T}$. The results of paper [210] then follow from the fact that according to all fits of experimental data in hadron and nuclear physics, the isotopic element $\hat{T} = 1 - F/H$ has very small values and represents Schwartzchild's horizon at the limit of gravitational collapse.

II-4. Verification of the EPR argument for electron valence bonds.

In the final statement of their historic paper [1], Einstein, Podolsky and Rosen state that the *wavefunction of quantum mechanics* [and, therefore, of quantum chemistry] *cannot represent all elements of reality*.

In the 2001 monograph [30] on hadronic chemistry (see Chapter 4 on), Ruggero proved the above statement by showing that the wavefunction $\psi(r)$ of the Schrödinger equation of quantum chemistry cannot represent the *attraction* between the *identical* electrons in valence bonds since they experience at 10^{-13} cm the extremely big *repulsive* force of 230 Newton.

By subjecting the Schrödinger equation for electron valence bonds to a non-unitary transformation of the type $UU^\dagger = \hat{I} = \exp\{-\bar{\psi}/\psi \int \psi^\dagger \psi d^3r\}$, and by using an appropriate selection of $\bar{\psi}$, the repulsive Coulomb potential is transformed into a strongly attractive Hulthén potential, with corresponding completion of the wavefunction into a form representing the *element of reality* given by electron valence bonds.

The resulting bound state was called *isoelectronium* and allowed the numerically exact representation of the experimental data for the hydrogen [31] and water [32] molecules that have escaped quantum chemistry

for half a century.

II-5. Verification of the EPR argument for antimatter.

Ruggero did not accept the conventional charge conjugation $\psi(t, r) \rightarrow \psi^c(t, r) = -\psi^\dagger(t, r)$ as being final because it provides no conjugation of matter into antimatter for neutral particles and prevents a representation of matter-antimatter annihilation (because antiparticles have the same positive energy of particles).

Hence, Ruggero developed the isodual mathematics for antimatter [29] outlined in the preceding section which is characterized by the isodual map (indicated with an upper letter "d") applied to the totality of quantities and their operations of quantum mechanics, resulting in the new *isodual charge conjugation* $\psi(t, r) \rightarrow \psi^d(t^d, r^d) = -\psi^\dagger(-t^\dagger, -r^\dagger)$, under which antimatter has negative energy as predicted by Dirac [11], evolves backward in time and all its characteristics are opposite those of matter, thus allowing a representation of matter-antimatter annihilation.

The violation of causality for particles with negative energies is resolved by the isodual mathematics because *particles with negative energy or negative time referred to negative units are as causal as positive energies or positive time referred to positive units.*

III. NEW APPLICATIONS.

Section 8 of Ruggero's Overview and the vast literature quoted therein, illustrate quite clearly that the verifications of the EPR argument, the new notion of EPR entanglement and their embodiment in the completion of quantum into hadronic mechanics, chemistry and biology, have important *new* implications (that is, applications not permitted by quantum mechanics) in all quantitative sciences.

We here merely note that what appears to be a central implication of the EPR argument, the existence of superluminal speeds under strong interactions (Section 8.4.4-VI), was first voiced by Ruggero in 1982¹⁶.

The exact character of quantum electrodynamics (QED) has been recently disproved by accurate measurements^{12,17} establishing a deviation of the measured muon g-factor from the QED prediction. Following the release of his Overview, Ruggero has provided a numerical representation of the anomalous muon g-factor via a branch of hadronic mechanics called *isoelectrodynamics* (IED)¹⁸ whose analytic counterpart is given by the MIT-CTP papers^{6,7,8}.

We should finally mention that, following the release of the Overview, an independent review of the Einstein, Podolsky, Rosen argument has been published¹⁹ and this is recommended for reading before embarking on a study of the technical issues as outlined in some detail in this book. In this article, inspired partly by the above-mentioned 2020 Florida Conference, much of the latest information, both experimental and theoretical, pertaining to this topic - a topic whose resolution is so important for the future of all science, not just physics - is provided. Also, reference is made to some earlier work supporting the Einstein-Podolsky-Rosen stance, which as far as many are concerned, has remained conveniently almost hidden ever since its ideas were advanced in the mid-1980's.

Further, the article contains some purely reflective thoughts on the position of probability theory in physics and other sciences, thoughts which may have relevance in other disciplines as well. The article also gives an independent view of some of the possible consequences of Ruggero's work - consequences which could affect each and every one of us, with the possibility of a new method for the quick and safe disposal of nuclear waste being probably the most important for many.

Even this, though, is merely one side product of his work which could lead to many more benefits for mankind.

In many ways, as with so many issues, the main problem encountered in discussions of the Einstein, Podolsky, Rosen issue has been an unwillingness to think 'outside the box', as the saying goes. However, the final resolution of this and other outstanding questions facing modern day

science will surely rely on unorthodox thinking and in this respect, all should remember that final paragraph in the 1958 edition of Dirac's well-known text on quantum mechanics:

It would seem that we have followed as far as possible the path of logical development of the ideas of quantum mechanics as they are at present understood. The difficulties, being of a profound character, can be removed only by some drastic change in the foundations of the theory, probably a change as drastic as the passage from Bohr's orbit theory to the present quantum mechanics.

This powerful statement from such an eminent theoretical physicist surely deserves careful contemplation and certainly cannot be dismissed easily. It is the contention here that Ruggero has achieved, at least in part, that drastic change envisaged as necessary by Dirac. Not that Ruggero would claim this work constituted a complete theory, because as he has said on so many occasions, there cannot be a truly complete theory which successfully encompasses every possible situation. Physics, and indeed all of science, will always be continuously evolving subjects but it is felt the work discussed in this book represents a significant step forward in helping man gain a better understanding of the universe in which we all live.

Acknowledgments. The author would like to thank Ruggero Maria Santilli and the R. M. Santilli Foundation for continuous contacts and communications without which this Preface would not have been possible. Thanks are also due to all participants of the 2020 International Teleconference on the EPR Argument for comments and suggestions.

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May 31, 2021

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Abstract of the Overview

In the 2020 Teleconference Ref. [0]: 1) We studied the 1935 objections against the quantum entanglement moved by Einstein, Podolsky and Rosen (EPR argument) [1]; 2) We pointed out that the interactions caused by wave-overlapping are of contact, zero-range, non-linear, non-local and non-Hamiltonian type; 3) We studied the new type of particle entanglement, here called **EPR entanglement**, consisting of particles in continuous and instantaneous communication via the overlapping of their wavepackets, thus without any need for superluminal speeds [1], whose non-Hamiltonian interactions are represented by the isotopic element $\hat{T}$ in the axiom-preserving product $A \hat{\times} B = A \hat{T} B$ of isomathematics and related hadronic mechanics (Tutoring Lectures [143] [144]); 4) We showed that the isotopic element $\hat{T}$ provides an explicit and concrete realization of Bohm's hidden variables; 5) We reviewed the recent verifications of the EPR argument by R. M. Santilli [210]-[214] showing the inapplicability of Bell's inequalities and other objections against the EPR argument for extended particles under non-potential interactions, with ensuing progressive recovering of classical images and Einstein's determinism in the structure of hadrons, nuclei and stars, and its full recovering at the limit of Schwartzchild's horizon **under the full preservation of quantum axioms merely subjected to a broader realization**; 6) We studied otherwise impossible advances in physics, chemistry and biology, including the exact representation of nuclear data, the achievement of an attractive force in valence electron bonds with ensuing exact representation of molecular data, and a new conception of life consisting of extended constituents under continuous EPR entanglement represented via hyperstructures [231]; 7) We showed the impossibility for the Copenhagen interpretation of quantum mechanics to solve our increasingly alarming environmental problems, such as recycling nuclear waste, achieving controlled nuclear fusions, and reaching the full combustion of fossil fuels; and pointed out their possible resolution under the EPR argument according to which "quantum mechanics is not a complete theory."

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Acknowledgements

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1. FOREWORD.

As it is well known, Nazism reached in 1935 the peak of its military, political as well as, lesser known, scientific power, the latter being due to the conception and construction of quantum mechanics by German scientists, such as M. Planck, I. Schroedinger and W. Heisenberg and others.

The R. M. Santilli Foundation (<http://www.santilli-foundation.org>) and the Family of Israel Foundation (<http://www.i-b-r.org/translational-medicine.htm>) organized, conducted and recorded the International Teleconference from September 1 to 5, 2020 (see Ref. [0] and 8 proceedings papers [210]-[233]) for the study of old and new verifications of the historical view by Albert Einstein, expressed jointly with his graduate students B. Podolsky and N. Rosen, that "*quantum mechanics is not a complete theory*," with ensuing expectation that a suitable completion of quantum mechanics would recover classical determinism at least under limit conditions (EPR argument) [1].

This Overview is intended to provide an outline of studies conducted over one century in old and new verifications of the EPR argument and their applications in physics, chemistry and biology, with particular reference to the search for possible resolutions of our increasing alarming environmental problems via the new sciences permitted by the EPR argument, which resolutions appear to be impossible via quantum mechanics.

In Section 2, we present a brief outline of the EPR argument and its historical objections, including Bell's inequalities; in Section 3, we review known insufficiencies of quantum mechanics in various fields; in Section 4, we review the experimental verifications of the validity of Bell's inequality for point-like particles under linear, local and potential interactions as well its inapplicability (rather than violation) for extended particles under non-linear, non-local and non-potential interactions; in Section 5, we review the first historical completions of quantum mechanics essentially along the EPR argument by W. Heisenberg, L. de Broglie, D. Bohm et al.; in Section 5, we outline the completion of quantum mechanics,

chemistry and biology into the various fields; in Section 7, we review five different verifications of the EPR Argument by R. M. Santilli [210]-[214]; in Section 8 we review applications and predictions in physics, chemistry and biology, as well as the implications of the EPR argument for high energy scattering experiments; and in Section 9, we present what appears to be the ultimate implications of the EPR argument.

To properly present and document a century of research in the field, we have made an effort to provide free pdf downloads of:

i) Reprints of Santilli's 1998 and 2019 verifications of the EPR argument [210]-[214];

ii) Papers [6]-[188] by numerous authors that resulted in time to be significant for the proofs the EPR argument, including:

ii-1) Refs. [32]-[36] that are important for the axiomatic structure and elaboration of Lie-isotopic theories;

ii-2) Refs. [49]-[52] that contain original important contributions by various authors for the construction of hadronic mechanics;

ii-4) Refs. [53]-[67] that provide a step-by-step Lie-isotopic completion of conventional space-time symmetries, their implications for physical laws and the proof of their isomorphism with conventional symmetries, which references play an important role in the construction of explicit and concrete realizations of Bohm's hidden variables with ensuing verification of the EPR argument.

iii) Monographs by various authors [154] [201]-[207] that review the process leading to verifications [210]-[214] of the EPR argument;

iv) References in the first representation of nuclear magnetic moments, spin, stability and other data with the consequential prediction and initial verification of new clean nuclear energies (Sections 8.1 and 8.2);

v) Internet debates for anonymous expressions of personal views in cosmology [76], neutrino [91], gravitation [180], applications of the EPR argument in physics [208], and applications of the EPR argument in chemistry [209];

vi) Tutorial lectures in isomathematics [143], verifications of the EPR argument [144], Lie-admissible formulations [145], and hyperstructures [146]; and

vii) Proceedings of international meetings in the field [188]-[200], including: five *Workshops on on Lie-Admissible Formulations* conducted at Harvard University from 1978 to 1982; twenty five *Workshops on Hadronic Mechanics* conducted from 1983 on at various locations in the U.S.A., Europe and China; and three *International Conferences on non-Potential interactions and their Lie-admissible treatments*, the first conducted in 1981 at the Université d'Orleans, France, the second conducted in 1995 at the Castle Prince Pignatelli, Molise, Italy, and the third conducted in 2011 at Katmandu University, Nepal. Out of all these meetings, the author could identify only twelve proceedings available with links for free pdf download which illustrate the number of scientists who contributed in the construction of hadronic mechanics, chemistry and biology.

It should be stressed that, by no means, the content of this Overview is expected to be accepted by all participants of our 2020 Teleconference in the EPR Argument [0] because, particularly when dealing with fundamental open aspects, debates on qualified dissident views are essential for a serious scientific process.

2. THE EPR ARGUMENT.

A most mysterious experimental evidence in nature is the capability of particles to influence each other instantly at a distance. In view of the apparent influence in the 1930's by the Nazism, the scientific community of the time assumed that such an effect is predicted by quantum mechanics, for which reason the effect continues to be called to this day *quantum entanglement*.

By contrast, Albert Einstein noted that the Schrödinger equation of

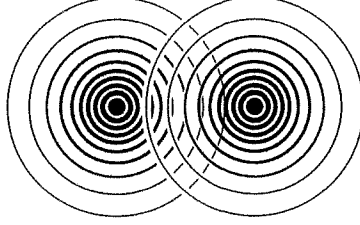


Figure 2: *A view of the new "EPR entanglement" introduced during the 2020 International Teleconference [0] (see Section 7.2.3 for a technical treatment), which consists of particles under continuous and instantaneous communication via the overlapping of their wavepackets, thus without any need for superluminal communications[1], with ensuing contact, zero-range, non-linear, non-local and non-potential interactions represented via isomathematics [54] and isomechanics [24].*

quantum mechanics (for $\hbar = 1$)

$$\left[-\frac{1}{2m}\Delta_r + V(r)\right]\psi(r) = E\psi(r), \quad (1)$$

can only represent point-like particles at a distance in vacuum and, therefore, cannot predict the entanglement of particles, in which case the sole possible representation of the entanglement is that via superluminal communications that would violate special relativity.

To avoid such a violation, Einstein Podolsky and Rosen argued that "quantum mechanics is not a complete theory" [1].

The EPR argument was quickly criticized by N. Bohr [2] a few months following its appearance although, under a sufficiently deep scrutiny, Bohr did not truly address the EPR argument, namely, the possible existence of "elements of reality" in the universe beyond those of the atomic structure which would require a suitable completion of quantum me-

chanics.

Despite such a visible insufficiency, Bohr's opposition to the EPR argument was supported quite widely by numerous scientists apparently because of the Nazism scientific power of the time.

In 1964, J. S. Bell [3] proved an inequality which essentially established that *quantum mechanical systems of particles with spin 1/2 do not admit classical counterparts*. Bell's inequality was considered by the mainstream scientific community, with due exceptions, to be the final dismissal of Einstein's dream of recovering classical determinism, and set the current widely accepted view that quantum mechanics is valid for whatever conditions exist in the universe (see review [4] and its comprehensive literature).

3. CONCEPTUAL FOUNDATIONS

3.1. Insufficiencies of quantum mechanics in particle physics.

The Copenhagen interpretation of quantum mechanics can only represent particles as being point-like at a distance in vacuum, because Schrödinger's equation (1) is characterized by wavefunctions $\psi(r)$, potentials $V(r)$, Laplacians Δ_r and other quantities that can only be defined at a finite set of isolated points r . Consequently, contrary to a rather popular belief, the Copenhagen interpretation of quantum mechanics cannot consistently predict or represent particle entanglements without superluminal communications, as correctly stated in the EPR argument [1].

The novel quantitative representation of particle entanglements studied at the 2020 teleconference [0] and called *EPR entanglement*, can be conceptually outlined as follows (see Figure 2 and Section 7.2.3 for a technical definition). Recall that the wavepacket of a particle fills up the entire universe with an intensity inversely proportional to the square of the distance. Hence, the entanglement of particles at a distance is characterized by the mutual penetration/overlapping of their wavepackets, under

which conditions particles are in continuous instantaneous communication through the overlapping of their wavepackets without any need for superluminal speeds (Figure 2).

The technically challenging problem is that the interactions caused by the overlapping of the wavepackets of particles are: 1) Non-linear in the wavefunctions; 2) Non-local because defined in a volume, and 3) Not derivable from a potential because they are of contact, thus zero-range type, hereon referred to as *non-Hamiltonian interactions*.

Also recall that quantum mechanics is strictly Hamiltonian in the sense that interactions can only be represented via the Hamiltonian H . Consequently, the vast historical literature based on quantum mechanics (not listed in this Overview for brevity because easily identifiable with an internet search) represents particle entanglements with a Hamiltonian. By contrast, the 2020 Teleconference studied the alternative approach according to which the interactions occurring in particle entanglement should be represented with an operator *other* than the Hamiltonian, by therefore mandating a completion of quantum mechanics according to the EPR argument [1].

An important objective of the 2020 Teleconference has been that of reviewing half a century of research by R. M. Santilli as well as by various scholars in the completion of 20th century applied mathematics, from its sole validity at isolated points, into a covering mathematics providing a consistent representation of interactions 1), 2), 3) above (see the recorded Tutoring Lectures of Ref.[0]). Physical and chemical completions and applications of the new notion of particle entanglement were considered only thereafter.

Note that, from the basic axioms of the $SU(2)$ -spin algebra and of Lie's theory at large, the validity of Bell's inequality [3] crucially depends for point-like particles under linear, local and potential interactions. Therefore, the study of systems of extended particles under non-linear, non-local and non-potential interactions automatically assures the *inapplica-*

bility (rather than the violation of Bell's theorem), by therefore establishing rigorous grounds for the verification of the EPR argument.

A study of Goedel's incompleteness theorems presented at the 2020 Teleconference is available in Ref. [217] and in the recorded lecture by E. T. D. Boney [0].

The connection between causality and quantum mechanics was studied at the 2020 Teleconference by S. E. Johansen, see Ref. [224], and his recorded lecture [0].

The connection between the EPR argument and the minimum contradiction theory was studied at the 2020 Teleconference by A. A. Nassikas, see contributed paper [228] and his recorded lecture [0].

3.2. Insufficiencies of quantum mechanics in nuclear physics. On serious scientific grounds, a theory can be claimed to be *exactly valid* for given systems ("elements of reality" [1]) if and only if the theory provides an exact representation of *all* experimental data of the systems considered from first axiomatic principles without the adulterations appearing in the contemporary physics literature via manipulated form factors, venturing of particles and/or entities not directly testable, and the like.

Under the above serious scientific conditions, quantum mechanics can indeed be considered to be exactly valid for the structure of the hydrogen atom. However, with the understanding that the *approximate* validity of quantum mechanics in nuclear physics is out of question, the assumption of quantum mechanics as being exactly valid for the nuclear structure implies the exiting from the boundaries of serious science due to the well known failure by quantum mechanics to achieve an exact representation of nuclear experimental data, such as nuclear magnetic moments, nuclear spins, nuclear stability (recall that the neutron is unstable.....), and other data, despite the use of billions of dollars of public funds in about one century of research. This insufficiency begins with the

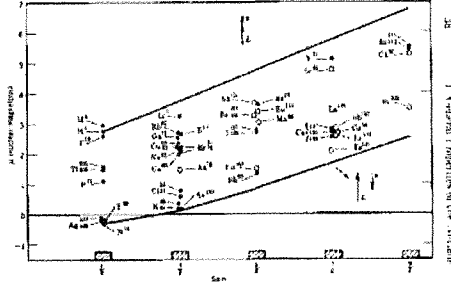


Figure 3: *A reproduction of the Schmidt limits providing a documentation of the deviations of the predictions of quantum mechanics from experimentally measured nuclear magnetic moments beginning with that of the smallest nucleus, the Deuteron. Similar insufficiencies exist for the representation of nuclear spins, nuclear stability and other data.*

lack of representation of experimental data on the smallest nucleus, the Deuteron, and becomes embarrassing for large nuclei such as the Zirconium (Figure 3) [6].

An understanding of the origin of the indicated insufficiency can be reached via the comparison of atomic and nuclear structures. In the hydrogen atom, the proton and the electron are at such a large mutual distance to allow their effective point-like approximation, with the ensuing exact validity of the theory. By contrast, experimental data on nuclear volumes and on the volume of individual nucleons, establishes that *nuclei are composed by a collection of extended and hyperdense nucleons in conditions of partial mutual penetration, thus EPR entanglement, of their charge distributions* (Figure 4) with ensuing non-Hamiltonian interactions (Section 3.1), with consequential need for a suitable completion of quantum mechanics.

3.3. Insufficiencies of quantum mechanics for irreversible

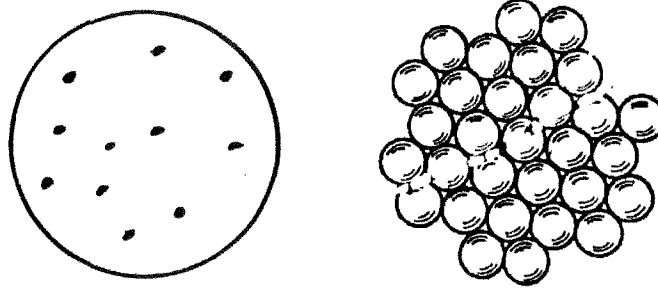


Figure 4: *An illustration on the left of the sole possible representation of nuclear structures by quantum mechanics due to its locality and linearity. An illustration on the right of the experimental reality establishing that nuclei are composed by extended and hyperdense nucleons in conditions of partial mutual overlapping with ensuing non-Hamiltonian interactions mandating a suitable completion of quantum mechanics [1].*

processes.

Beginning with his Ph. D. studies at the University of Torino, Italy, in the mid 1960's, Santilli dedicated his research life to the proof of the EPR argument [1] because quantum mechanics cannot achieve a consistent representation of energy releasing processes, such as combustion, nuclear fusions and others. This is due to the fact that all energy releasing processes are irreversible over time, while quantum mechanics can only represent systems whose time reversal image does not violate causality (Figure 5, left view) due to the invariance under anti-Hermiticity of Heisenberg's equation for Hermitean operators A

$$i \frac{dA}{dt} = [A, H] = AH - HA = -[A, H]^\dagger. \quad (2)$$

The century old objection against the verification of the EPR argument via irreversible processes, which is still widely accepted nowadays, is



Figure 5: *A view on the left of the irreversibility of combustion and its lack of representation via quantum mechanics due to its reversibility. A view on the right of the clear irreversibility of high energy particle collisions, with ensuing need for an irreversible completion of quantum mechanics (Section 8.7).*

that irreversibility is 'illusory' (sic) because, when irreversible processes are reduced to their quantum mechanical elementary constituents, reversibility is fully regained. As part of his 1967 Ph. D. thesis [6]-[8], Santilli proved a number of theorems essentially stating that *a macroscopic irreversible system cannot be consistently decomposed into a finite number of quantum mechanical particles all in reversible conditions and, vice-versa, a finite number of quantum mechanical particles cannot recover a macroscopic irreversible system under the correspondence or other principles.*

An important discovery made during the 2020 Teleconference [0] is that *macroscopic irreversibility originates at the ultimate level of elementary particles, as established by the mere visual inspection of high energy scattering experiments at CERN, FERMILAB and other particle physics laboratories (Figure 5, right view).*

In view of the above insufficiency, Santilli proposed in 1967 (*loc. cit.*) the first known completion of quantum mechanics based on the embedding of Lie algebras of quantum mechanics with brackets $[A, B] = AB - BA$ into covering algebras with brackets (A, B) that are *Lie-admissible*

(*Jordan-admissible*) according to the American mathematician A. A. Albert [8] when the attached anti-symmetric brackets $[A, B]^* = (A, B) - (B, A)$ verify the Lie algebra axioms (attached symmetric brackets $\{A, B\}^* = (A, B) + (B, A)$ verify the Jordan algebra axioms), with realization of the type

$$(A, B) = ARB - BSA = (ATB - BTA) + (AJB + BJA), \quad (3)$$

where $R = T + J$ and $S = -T + J$ are non-singular operators representing non-Hamiltonian interactions. The representation of irreversibility from first axiomatic principles is evidently assured by the violation of the invariance under anti-Hermiticity, $(A, B) \neq -(A, B)^\dagger$ which occurs whenever $R \neq S$. In particular, the Lie-isotopic operator T represents the non-Hamiltonian interactions of particle entanglement, while the Jordan-isotopic operator J represents the irreversibility of energy-releasing processes via a representation of the external terms in Lagrange's and Hamilton's equations that are completely absent in the Copenhagen interpretation of quantum mechanics (see Sections 6.2 and 7.6 for an overview).

3.4. Insufficiencies of quantum mechanics for antimatter.

Another majestic event in nature is given by the annihilation of particle-antiparticle pairs into light. The mechanism of this event cannot be represented via quantum mechanics because charge conjugation

$$\psi(r) \rightarrow \psi^c(r) = -\psi^\dagger(r), \quad (4)$$

characterizes antiparticles in the same Hilbert space of the original particles, as a result of which antiparticles have the same positive energy of particles. Consequently, a particle-antiparticle pair both having positive energy cannot possibly annihilate into light, besides being in conflict with P. A. M. Dirac historical hypothesis that antiparticles have *negative energy* [11].

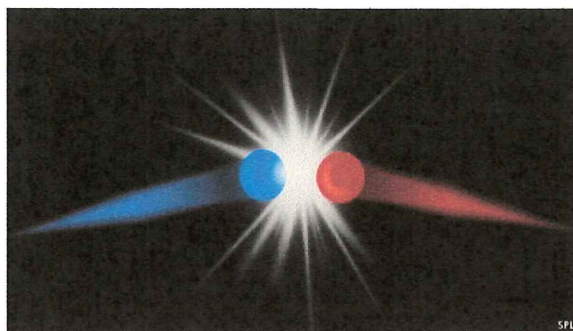


Figure 6: *Another topic studied at the 2020 Teleconference [0] has been the inability by quantum mechanics to represent the mechanism of particle-antiparticle annihilation into light because charge conjugation characterizes antiparticles with the same positive energy of particles against Dirac's view [11] that antiparticles should have negative energy.*

This occurrence clearly suggests the need for an additional completion of quantum mechanics, this time, for a representation of antiparticles in a way compatible with the experimental evidence on annihilation that can only be achieved when all characteristics of antiparticles are opposite those of particles (see Section 7.7 for an overview).

3.5. Insufficiencies of quantum mechanics in chemistry.

With the full admission of the historical advances achieved by quantum chemistry in the past century, the advancement of basic scientific knowledge requires the indication of the basic insufficiency of quantum chemistry given by the absence of a quantitative model of valence bonds due to the inability to represent the *attractive force* between *identical* valence electrons (Figure 7), with consequential lack of a quantitative model of molecular structures that carries evident environmental implications [12].

In fact, the Schrödinger equation of valence electron pairs is given by

$$\left[-\frac{1}{m}\Delta_r + \frac{e^2}{r}\right]\psi(r) = E\psi(r), \quad (5)$$

where m is the reduced mass, thus solely allowing a repulsion caused by the equal electron charge $-e$ which is represented by $+e^2/r$. Simple calculations show that the *repulsive force* between two valence electrons at $10^{-13}cm = 10^{-15}fm$ mutual distance is given by

$$F = k\frac{e^2}{r^2} = (8.99 \times 10^9) \frac{(1.60 \times 10^{-19})^2}{(10^{-15})^2} = 230 \text{ N}, \quad (6)$$

thus being so enormous for particle standards to prevent any realistic hope of being overcome by current models of valence bonds, thus confirming the need for a suitable completion of quantum chemistry. according to the EPR argument [1].

The lack of a sufficiently strong attractive force between identical valence electrons has rather serious implications in chemistry, such as:

a) The lack of an exact representation of molecular binding energies with deviations of the order of 2% that, rather than being ignorable, corresponds to about 950 *kcal/mole*.

b) The prediction that all substances are ferromagnetic in view of the lack of strongly bonded valence electron pairs.

c) The inability to achieve a quantitative representation of molecular electric and magnetic moments, with consequential inability to identify the attractive force between molecules in their liquid state, and other insufficiencies [*loc. cit.*].

4. EXPERIMENTAL FOUNDATIONS.

An important lecture delivered at the 2020 Teleconference has been that by Gerald Eigen [13] (see also Ref. [235]) who presented various experiments that essentially provide a *clear experimental verification of Bell's*

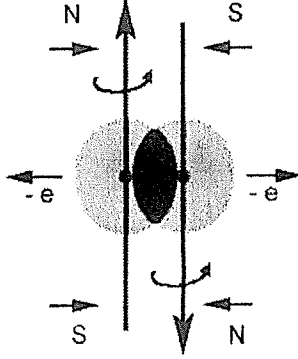


Figure 7: A schematic view of the lack of a quantitative representation of molecular structures by quantum chemistry due to the large Coulomb repulsion between identical valence electrons according to Schrödinger equation (5) which is explicitly computed in equation (6).

inequality for point-like particles in vacuum, thus including electromagnetic interactions.

Independently from the above tests, Matteo Fadel *et al* [14] conducted experiments similar to those of Ref. [13], but this time for the study of spin-correlations between space separated *atoms* of a Bose-Einstein condensate, by measuring uncertainties below the bound of Heisenberg's uncertainty principle.

Additionally, the ALICE experiments at CERN on *heavy ions* [15] have shown deviations from quantum mechanics bigger than those measured by experiments [14].

A view expressed at the 2020 teleconference is that tests [13]-[15] provide a conceptual and experimental verification of the EPR argument [1] because tests [13] confirm the acceptance by Einstein, Podolsky and Rosen of the validity of quantum mechanics for *point-like particles* in vacuum, while tests [14] [15] establish the existence of deviations from quan-

tum mechanics for *extended particles*.

The main difference between tests [13] and [14] [15]] deal with the underlying interactions. In fact, for tests [13], the sole possible interactions are those at a distance, since the particles are dimensionless. By contrast, for the case of tests [14] [15], since the constituents are given by extended atoms and ions under the known compression caused by the Bose-Einstein condensate, the interactions are of the non-linear, non-local and non-potential type causing the *inapplicability* (rather than the violation) of Bell's inequality.

As we shall see, besides the above direct tests, there exist additional experimental evidence in support of the EPR argument in particle physics, nuclear physics, chemistry and other fields.

5. HISTORICAL EPR COMPLETIONS.

In general, the limitations of basic physical laws have been best identified by their originators. For example, following the discovery of the matrix realization of quantum mechanics on a Hilbert space $\mathcal{H}$ over the field of complex numbers $\mathbb{C}$, with historical time evolution (2), Werner K. Heisenberg identified the limitations of his own theory caused by its linearity, and studied what can be well called the *first non-linear completion of quantum mechanics* [16].

Quantum mechanics is a strictly Hamiltonian theory and, therefore, can only represent non-linear interactions with the Hamiltonian, resulting in eigenvalues equations of the type

$$H(r, p, \psi)\psi(r) = E\psi(r). \quad (7)$$

As part of his decades of studies on completions according to the EPR argument, Santilli pointed out at the 2020 Teleconference [0] that, despite its clear historical value, Heisenberg's non-linear completion does not allow the characterization of the constituents $\psi_k(r)$, $k = 1, 2, \dots, N$ of a bound state with non-linear internal interactions because non-linear equations

(7) generally violate the superposition principle,

$$\psi(r) \neq \sum_{k=1,2,\dots,N} \psi_k(r). \quad (8)$$

The lack of a superposition principle then implies serious limitations on the representation of experimental data, e.g., on how the neutron, which is naturally unstable, becomes stable when bonded to a proton in a stable nucleus.

Therefore, in order to preserve the superposition principle under completion, Santilli suggested that non-linear interactions should be represented with a suitably selected operator *other than the Hamiltonian*. In any case, as indicated earlier, non-linear interactions are of contact, zero-range type thus carrying no potential energy. Their representation with the Hamiltonian would then cause consistency problems in the axiomatic structure of the completed theory.

Despite the indicated limitation, Heisenberg's non-linear completion of quantum mechanics played an important role in Santilli's studies. In fact, there is a mention in the Comments of Teleconference [0] that Santilli had various mail contacts with Heisenberg in the early 1970's precisely on the superposition principle when Santilli was in the faculty of the Department of Physics of Boston University.

As it is well known, Louis V. P. R. de Broglie was one of the primary contributors to the development of the wave structure of particles in quantum mechanics. Jointly, de Broglie was the first to identify the limitations of such a description caused by the locality of the wavefunction $\psi(r)$. The theory developed by de Broglie was semiclassical as a condition to by-pass Heisenberg's uncertainty principle and, as such, it can be considered to be one of the first attempts at recovering Einstein's determinism.

Unfortunately for science, de Broglie was dissuaded by mainstream scientists of the time to continue his studies on possible broadening of quantum mechanics. Nevertheless, de Broglie's ideas were resumed and

further developed by David J. Bohm and the theory can be well called resulting the *de Broglie-Bohm non-local completion of quantum mechanics* [17].

This theory was discussed at the 2020 Teleconference [0] because of its important influence in the development of new completions of quantum mechanics. In particular, Santilli pointed out that, in his view, no consistent representation of the entanglement of particles can be achieved without a non-local theory since the overlapping of wavepackets occurs in a volume. Yet, to be effective, the representation of said non-locality should be done via an operator other than the Hamiltonian since said wave-overlapping carries no potential energy.

The third important historical completion of quantum mechanics studied at the 2020 Teleconference [0] is that by D. J. Bohm with his theory of *hidden variables* [18]. As it is well known, quantum mechanics is a probabilistic theory, with ensuing uncertainties in the position and momentum of particles. Bohm conjectured the possible existence of more fundamental physical laws hidden in the mathematics of quantum mechanics that would allow the theory to recover Einstein's determinism. Despite a large literature in the field, no concrete formulation of hidden variables, and related recovering of Einstein's determinism, was achieved up to the early 2000's. Nevertheless, Bohm's intuition on the lack of final character of the Copenhagen interpretation of quantum mechanics remained fundamental.

Santilli's view on Bohr's hidden variables presented at the 2020 Teleconference [0] is essentially the following:

- 1) Bell's inequality on the lack of existence of classical images should be considered valid for the infinite family of unitary equivalence of quantum mechanics. Consequently, the sole known possibility of bypassing Bell's inequality and achieving concrete realizations of hidden variables is the construction of a *non-unitary completion of quantum mechanics*.

- 2) Bell's inequality has been experimentally verified by G. Eigen and

others [13] for electromagnetic interactions of point-like particles in vacuum. Consequently, the most promising applications of non-unitary completions of quantum mechanics are those for strong nuclear interactions for which quantum mechanics is known to be incomplete (Section 2).

3) As it is the case for Heisenberg's non-linear theory, and de Broglie-Bohm on-local theory, Bohm hidden variables should be realized via an *operator*— other than the Hamiltonian which is hidden in the axioms of quantum mechanics.

A geometric interpretation of hidden variables was presented at the 2020 Teleconference by O. A. Olkhov, see contribution [239] and his recorded lecture [0].

6. HADRONIC MECHANICS, CHEMISTRY AND BIOLOGY.

6.1. Hadronic mechanics.

It is evident that the completion of quantum mechanics according to Einstein, Podolsky and Rosen [1] has implications for all quantitative sciences, including physics, chemistry and biology. In order to initiate an expectedly long process, in his Ph. D. thesis at the University of Torino, Italy, in the mid 1960's, Santilli proposed the completion of quantum mechanics for irreversible processes via Lie-admissible completion of the quantum mechanical Lie algebras [6]-[8] according to A. A. Albert [9], and continued the studies at Harvard University under DOE support in the late 1970's with further developments of the Lie-admissible formulations as well as their Lie-isotopic particularization [19]-[22].

Because of important contributions by numerous mathematicians, physicists and chemists, the above studies achieved in the late 1990's a completion/covering of quantum mechanics known as *hadronic mechanics* [23]-[28] for a consistent representation of extended particles under non-linear, non-local and non-Hamiltonian interactions due to wave-overlapping, thus EPR entanglement.

An important feature of the studies herein considered is that *the Copenhagen interpretation of quantum mechanics resulted as being the simplest possible realization of quantum axioms, such as associativity, distributivity, linearity, locality, potentiality, etc., while hadronic mechanics is characterized by the progressive more general possible realizations of said axioms depending on the complexity of the considered "elements of reality" [1], according to the following:*

Classification of hadronic mechanics:

6-1-I) Quantum mechanics, with the familiar *Heisenberg's time evolution* (2) of an observable A in the infinitesimal form

$$i \frac{dA}{dt} = [A, H] = AH - HA = -[A, H]^\dagger, \quad A = A^\dagger, \quad H = H^\dagger, \quad (9)$$

and finite form

$$A(t) = UA(0)U^\dagger = e^{Ht}A(0)e^{-itH}, \quad UU^\dagger = U^\dagger U = I, \quad (10)$$

characterized by the Lie's theory with brackets $[A, H]$ for the representation of stable, thus time-reversal invariant systems of point-like particles in vacuum via the Hamiltonian H .

6-1-II) Isomechanics (see Refs. [23] [24], reviews in Section 2 of Ref. [25] and Tutoring Lecture I [143][144]), with the *iso-Heisenberg time evolution* in its infinitesimal form (first introduced in Eqs. (18), page 153, Ref. [22])

$$i \frac{dA}{dt} = [A, \hat{H}] = A\hat{T}H - H\hat{T}A = -[A, \hat{H}]^\dagger, \quad \hat{T} > 0, \quad (11)$$

and finite form

$$A(t) = WA(0)W^\dagger = e^{H\hat{T}t}A(0)e^{-it\hat{T}H}, \quad WW^\dagger \neq I, \quad (12)$$

characterized by the axiom-preserving Lie-Santilli isothory with brackets $[A, B] = A\hat{T}B - B\hat{T}A$, for the representation of stable (thus time-reversal invariant) hadrons, nuclei and stars composed by extended, therefore deformable constituents in conditions of deep mutual entanglement with conventional interactions represented with the Hamiltonian H , and non-linear, non-local and non-potential interactions represented with the isotopic element $\hat{T}$.

6.1-III) Genomechanics (see Refs. [23] [24], reviews in Section 3 of Ref. [25] and Tutoring Lecture IV [145]), with the *geno-Heisenberg equation* in their infinitesimal form (first introduced in Eqs. (19), page 153, Ref. [22])

$$\begin{aligned} i\frac{dA}{dt} &= (A, H) = A\hat{R}H - H\hat{S}A = \\ &= (A\hat{T}H - H\hat{T}A) + (A\hat{J}H + H\hat{J}A) \neq -(A, B)^\dagger, \\ \hat{R} &= \hat{T} + \hat{J} \neq \hat{S} = -\hat{T} + \hat{J}, \quad \hat{T} > 0, \quad \hat{J} > 0, \end{aligned} \quad (13)$$

and finite form

$$A(t) = W A(0) W^\dagger = e^{H\hat{R}ti} A(0) e^{-it\hat{S}H}, \quad WW^\dagger \neq I, \quad (14)$$

characterized by the Lie-admissible/Jordan-admissible theory with product (3), for the representation of time-irreversible processes between extended constituents via: A) The conventional Hamiltonian H for the representation of conventional potential interactions; B) The Lie-isotopic operator $\hat{T}$ for the representation of non-Hamiltonian interactions due to mutual EPR entanglement as in Case II; and C) The Jordan-isotopic operator $\hat{J}$ for the representation of the external terms in the Lagrange's and Hamilton's equations that are completely missing in the Copenhagen interpretation of quantum axioms.

6.1-IV) Hypermechanics (see Ref. [27] [28] and Tutoring Lecture V [146]) characterized by the formulation of genomechanics in terms of

T. Vougiouklis H_v hyperstructures [233] for the representation of irreversible systems with a large number of extended particles in deep EPR entanglement.

6.1-V) Isodual quantum, iso-, geno- and hyper-mechanics [29] for the representation of antiparticles in condition of increasing complexity via an anti-Hermitean map (called *isoduality* and denoted with the upper index d) of the preceding time evolutions, such as

$$i^d \times^d \frac{d^d A^d}{d^d t^d} = [A^d, H^d]^d = A^d \times^d H^d - H^d \times^d A^d, \quad (15)$$

providing a causal representation of antimatter with *negative energy* according to P. A. M. Dirac [11] because necessary to represent particle-antiparticle annihilation (see Section 3.4 and Figure 6, contribution [227] to the 2020 Teleconference and the recorded lecture by A. S. Muktibodh [0]).

6.2. Hadronic chemistry.

Following the achievement of mathematical [23] and physical [24] maturity, as well as a number of experimental verifications [25] [26], hadronic mechanics was applied to chemistry, resulting in a completion-covering of hadronic chemistry for the representation of molecules composed by extended constituents in condition of EPR entanglement, called **hadronic chemistry** [30] which is also axiom-preserving, thus implying that *quantum and hadronic chemistry coincide at their abstract realization-free level*.

The most important function of hadronic chemistry is to show that the basic open problems in chemistry listed in Section 3.5 cannot be solved via quantum chemistry due to its linearity, locality and potentiality. By contrast, the achievement of an attractive force between identical valence electrons in molecular bonds has been possible [30] following the admission of the "element of reality" [1] given by the non-linearity, non-locality and non-potentiality of valence electron bonds (Figure 7). This

new strong valence bond has permitted the achievement of exact representations of experimental data for the Hydrogen [31], water [32] and other molecules.

6.3. Hadronic biology.

Studies in the field were initiated by C. R. Illert [176] who proved that a conventional three-dimensional Euclidean space $E(r, \delta, I)$, $r = (x, y, z)$, $\delta = I = \text{Dag.}(1, 1, 1)$ can indeed represent all possible *shapes* of seashells, but the *representation of their growth in time requires a three-dimensional two-valued space* here denoted $\hat{E}(\hat{r}, \delta, I)$, $\hat{r} = (\hat{r}_k)$, $\hat{r}_k = (r_k^1, r_k^2)$, $k = 1, 2, 3$.

In order to initiate the expectedly long process to represent the extreme complexity of the DNA code, and by recalling that all living organisms are irreversible over time, thus requiring genomathematics, Santilli introduced the representation of biological structures via three-dimensional multi-valued geno-Euclidean genospaces $\hat{E}(\hat{r}, \hat{\delta}, \hat{I})$ over genofields with genounits having an arbitrary number N of ordered elements, $\hat{I} = \text{Diag.}(I_1, I_2, \dots, I_N)$ [34], with classical operations, e.g., the genoproduct of two observables $A > B$ yields N ordered results [27].

T. Vougiouklis formulated the preceding results via the reformulation of genomathematics in terms of his H_v -hyperstructures on a hyperfield formulated in terms of hyperoperations called *hopes* (see Tutoring Lecture V [146] and Ref. [233]).

Vougiouklis' hyperformulation allowed the introduction at the 2020 Teleconference [0] of a new conception of living organisms presented in Ref. [231] as being composed by a very large number of molecules all in EPR entanglement, thus being in continuous and instantaneous communication, resulting in such a complex structure that can only be quantitatively represented with the most complex mathematics developed by the human mind.

It is significative that the 2020 Teleconference [0] ended up with the view: *There is hope that 'hope' can represent life.*

7. VERIFICATIONS OF THE EPR ARGUMENT.

7.1. Foreword.

A main topic of the 2020 Teleconference [0] has been the study of *isotopic verifications* [210]-[214] of the EPR argument [1], namely, verifications based on the preservation of the basic axioms of quantum mechanics and their most general possible realization which verifications are the outcome of the life-long research by R. M. Santilli with contributions by numerous scholars.

In this section, we review the indicated isotopic verifications in a language as simple as possible, including a rudimentary review of the needed basic formalism for minimal self-sufficiency of this Overview, as well as for the specialization of the formalism to the EPR problem.

A central concept of this section is that *extended* particles immersed within a hyperdense medium, as it is the case for a proton in the core of a star, experience a *pressure in all radial directions* proportional to the density of the medium (Figure 8) which evidently restricts Heisenberg's uncertainties in favor of Einstein's determinism [1]. Note that such a notion of pressure is completely absent in the 20th century physics due to the point-like approximation of particles.

With the understanding that the participants to the 2020 Teleconference are available for consultations, including this author, it should be indicated that a full understanding of the isotopic verifications reviewed in this section requires a technical knowledge of hadronic mechanics [24].

7.2. Recovering of classical images.

7.2.1. Rudiments of isomathematics. As recalled earlier, Bell [3] proved the lack of existence of classical images for systems of quantum mechanical point-like particles with spin $1/2$.

Since the systems studied by Bell are described by the time-reversible Heisenberg law (9)-(10) (Sections 3 and 6), Santilli studied in the 1998 paper [210] systems characterized by isomathematics (Tutoring Lecture

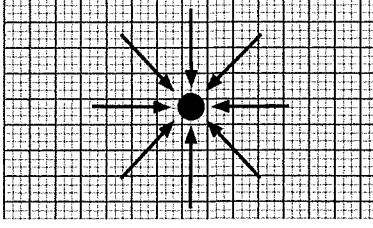


Figure 8: *This figure provides a conceptual rendering of the central notion used for the verifications of the EPR argument, namely, the inward 'pressure' experienced by 'extended' particles in 'all radial directions' when immersed in hyperdense media, such as a proton in the core of a star, which pressure evidently restricts uncertainties in favor of Einstein's determinism first identified in the 1981 paper [47] and then studied in detail in Refs.[210]-[214]. Note that the above notion of pressure did not exist in 20th century physics due to the approximation of particles as being point-like.*

I [143]) and related isomechanics [23] [24] with iso-Heisenberg's time evolution (11)-(12) for which the represented systems are equally time-reversible, yet they admit internal non-linear, non-local and non-potential interactions represented with the isotopic element $\hat{T}$.

Additionally, Bell [3] used the conventional $SU(2)$ -spin symmetry for the derivation of his result. Consequently, Santilli constructed the Lie-isotopic completion of the various branches of Lie's theory, including the isotopies of universal enveloping associative algebras, Lie algebras, and Lie groups [22], resulting in a theory nowadays known as the *Lie-Santilli isothory* [21]-[25] [50] [55] [201] (see also contribution [226] to the 2020 Teleconference and the recorded lecture by A. S. Muktibodh [0]).

The foundation of the Lie-Santilli isothory is given by the *universal isoenvolving isoassociative isoalgebra with isoproduct* between arbitrary

quantities A, B (first introduced in Eq.(5), page 71, Ref. [22])

$$A \hat{\times} B = A \hat{T} B, \hat{T} > 0, \quad (16)$$

where $\hat{T}$, called the *isotopic element*, is solely restricted by the condition of being positive-definite, but can otherwise possess an arbitrary (non-singular) dependence on all needed local variables of interior dynamical problems, such as a dependence on time t , coordinates r , momentum p , energy E , density μ , temperature τ , pressure γ , wavefunctions ψ , their derivatives $\partial_r \psi$, etc. $\hat{T} = \hat{T}(t, r, p, E, \mu, \tau, \gamma, \psi, \partial \psi, \dots)$, which dependence is hereon omitted for brevity.

It is evident that product (16) verifies the axioms of associativity and distributivity, although in the following more general form

$$\begin{aligned} (A \hat{\times} B) \hat{\times} C &= A \hat{\times} (B \hat{\times} C), \\ A \hat{\times} (B + C) &= A \hat{\times} B + A \hat{\times} C, \end{aligned} \quad (17)$$

for which reason it is called the isoproduct.

It should be indicated upfront that *the primary function of isoproduct (16) is that of providing novel explicit and concrete realizations of Bohm's hidden variables [18] via the isotopic element $\hat{T}$ which is "hidden" in the associative and distributive laws as clearly indicated by Eqs. (17).*

Santilli realized in the early 1978 that, despite its simplicity, isoproduct (16) requires for consistency the isotopic completions of *all* aspects of 20th century applied mathematics with no known exception. As an illustration of the Lie-Santilli isoalgebra, we indicate the isotopic completion $\hat{L}$ of an N -dimensional Lie algebra L with Hermitean generators $X_k, k = 1, 2, \dots, N$, characterized by the *isocommutator rules*

$$\begin{aligned} [\hat{X}_i, \hat{X}_j] &= \hat{X}_i \hat{\times} \hat{X}_j - \hat{X}_j \hat{\times} \hat{X}_i = \\ &= \hat{C}_{ij}^k(t, r, p, E, \mu, \tau, \psi, \partial \psi, \dots) \hat{\times} \hat{X}_k, \end{aligned} \quad (18)$$

whose verification of Lie algebra axioms is evident. Nowadays, Lie-Santilli isoalgebras are classified into *regular* when the structure quantities $\hat{C}$ are constant, and *irregular* when they are function of the local variables [55].

It should be noted, as illustrated below, that regular Lie-Santilli isoalgebras can be obtained via non-unitary transformations of the original Lie algebras, while irregular isoalgebras cannot, thus constituting new realizations of Lie's axioms.

The elaboration of isocommutation rules (18) with the conventional mathematics used for Lie's theory soon proved to lead to serious inconsistencies. This occurrence mandated the construction of isomathematics with the following main features:

1) The multiplicative unit of quantum mechanics $\hbar = 1$ is no longer invariant for isomechanics due to its non-unitary structure, Eq. (12). The consistent multiplicative unit under isoproduct (16) is given by the *isounit*

$$\hat{I} = 1/\hat{T}, > 0 \quad \hat{I} \hat{\times} A = A \hat{\times} \hat{I} = A; \quad (19)$$

2) The isotopy of the unit $1 \rightarrow \hat{I}$ mandates, for consistency, the corresponding isotopic completion of numeric fields $F(n, \times, I)$ into *isofields* $\hat{F}(\hat{n}, \hat{\times}, \hat{I})$, first introduced in Ref. [33] of 1993, with *isonumbers* $\hat{n} = n\hat{I}$ where $n \in F$ equipped with isoproduct (16) and isounit (19);

3) The elaboration of the Lie-Santilli isothory needs to be done, also for consistency, via the *isofunctional isoanalysis* initiated in Refs. [47] [48] of 1992 (see Refs. [23] [24] for a general treatment) with expressions, for instance, for the *isoexponent*

$$\hat{e}^X = (e^{X\hat{T}})\hat{I} = \hat{I}(e^{\hat{T}X}); \quad (20)$$

4) Recall that non-unitary transformations on conventional spaces over conventional fields generally violate causality[24] [212]. To regain causal-

ity, all non-unitary transformations can be reformulated into the *isounitary isotransforms*

$$U = \hat{U}\hat{T}^{1/2}, \quad UU^\dagger \neq I \rightarrow \hat{U} \hat{\times} \hat{U}^\dagger = \hat{U}^\dagger \hat{\times} \hat{U} = \hat{I}, \quad (21)$$

for which the non-unitary time evolution(12) is identically reformulated in the correct isounitary form

$$A(t) = \hat{W} \hat{\times} A(0) \hat{\times} \hat{W}^\dagger = \hat{e}^{Ht} \hat{\times} A(0) \hat{\times} \hat{e}^{-itH},$$

$$\hat{W} \hat{\times} \hat{W}^\dagger = \hat{W}^\dagger \hat{\times} \hat{W} = \hat{I}; \quad (22)$$

5) For consistency, scalar quantities must be elements of isofields, with ensuing expression for *isocoordinates* $\hat{r} = r\hat{I}$ and generic expression for *isofunction*

$$\hat{f}(\hat{r}) = [f(r\hat{I})]\hat{I}; \quad (23)$$

6) Also for consistency, Lie-Santilli isoalgebras must be formulated on *isospaces over isofields*, (first formulated in Ref. [34] with reference, as an illustration, for non-relativistic formulations to the *iso-Euclidean isospace* $\hat{E}(\hat{r}, \hat{\delta}, \hat{I})$ with *isocoordinates* $\hat{r} = r\hat{I}$, $r = (x, y, z)$, *isometric* $\hat{\delta} = (\hat{T}\delta)\hat{I}$ where $\delta = \text{Diag.}(1, 1, 1)$ and *isoinvariant* as an element of the isofield of isoreal isonumbers $\hat{\mathcal{R}}$

$$\hat{r}^{\hat{2}} = \hat{r}^i \hat{\times} \hat{\delta}_{i,j} \hat{\times} \hat{r}^j = \left(\frac{x^2}{n_1^2} + \frac{y^2}{n_2^2} + \frac{z^2}{n_3^2} \right) \hat{I}, \quad (24)$$

and the *iso-Minkowski isospace* for relativistic treatments $\hat{M}(\hat{x}, \hat{\eta}, \hat{I})$, with *space-time isocoordinates* $\hat{x} = x\hat{I} = (x, y, z, t)\hat{I}$, *space-time isometric* $\hat{\eta} = \hat{T}\eta$, $\eta = \text{Diag.}(1, 1, 1, -1)$ and *space-time isoinvariant*

$$\hat{x}^{\hat{2}} = \hat{x}^\mu \hat{\times} \hat{\eta}_{\mu\nu} \hat{\times} \hat{x}^\nu = \left(\frac{x^2}{n_1^2} + \frac{y^2}{n_2^2} + \frac{z^2}{n_3^2} - t^2 \frac{c^2}{n_4^2} \right) \hat{I}; \quad (25)$$

7) Since the multiplicative isounit is generally dependent on local iso-coordinates, $\hat{I} = \hat{I}(\hat{r}, \dots)$, the elaboration of Lie-Santilli isoalgebras via the conventional Newton-Leibnitz calculus leads to axiomatic inconsistencies (see also Section 7.3), and must be lifted into the *isodifferential isocalculus*, first introduced by Santilli in Ref. [34] (see also monographs. [207], contributions [222] [223] to the 2020 Teleconference, and the recorded lectures by S. Georgiev [0]), with basic *isodifferential*

$$\hat{d}\hat{r} = \hat{T}d[r\hat{I}(r, \dots)] = dr + r\hat{T}d\hat{I}' \quad (26)$$

and *isoderivatives*

$$\frac{\hat{\partial}\hat{f}(\hat{r})}{\hat{\partial}\hat{r}} = \hat{I}\frac{\partial\hat{f}(\hat{r})}{\partial\hat{r}}; \quad (27)$$

8) Contrary to a popular belief that isomathematics is excessively complex for physicists, *all aspects of (regular) isomathematics can be very easily constructed via a non-unitary transform of the corresponding aspect of 20th century applied mathematics*. For instance, the trivial unit 1 can be transformed into the isounit $\hat{I}$, the conventional associative product AB can be transformed into the isoproduct $A\hat{T}B$, etc. [35]

$$\begin{aligned} 1 &\rightarrow U1U^\dagger = \hat{I} = 1/\hat{T} \neq I, \quad \hat{T} = (UU^\dagger)^{-1}, \\ AB &\rightarrow U(AB)U^\dagger = (UAU^\dagger)(UU^\dagger)^{-1}(UBU^\dagger) = A'\hat{T}B', \\ E^X &\rightarrow \hat{e}^X = (e^{X\hat{T}})\hat{I} = \hat{I}(e^{\hat{T}X}), \text{ etc.}; \end{aligned} \quad (28)$$

9) The crucial invariance of the numeric values of the isounit and isotopic element representing non-Hamiltonian interactions is verified by the appropriate time evolution, the isounitary form (21)-(22) [35]

$$\begin{aligned} \hat{I} &\rightarrow \hat{U}\hat{\times}\hat{I}\hat{\times}\hat{U}^\dagger = \hat{I}' \equiv \hat{I}, \\ A\hat{\times}B = A\hat{T}B &\rightarrow \hat{U}\hat{\times}(A\hat{\times}B)\hat{\times}\hat{U}^\dagger = A'\hat{\times}'B' = A'\hat{T}'B', \quad \hat{T}' \equiv \hat{T}. \end{aligned} \quad (29)$$

Following the isotopic completion of Lie's theory in the period 1978-1982 [19]-[22], the step-by-step isotopies of conventional space-time mathematics, Santilli constructed in Refs. [60]-[70] the isosymmetries leaving invariant isospacetime (26).

These studies established that, rather than being violated for line element (25), the Lorentz symmetry remains valid because its isotopic image $\hat{SO}(3.1)$ is isomorphic to the original symmetry $SO(3.1)$ [60], and the same holds for all continuous as well as discrete space-time symmetries [23] [24].

We here merely recall for future comments: the *Lorentz-Santilli isotransformations* introduced in the 1983 paper [60], here indicated in the (3, 4)-plane (see Ref. [24] for their full formulation)

$$\begin{aligned} x^{1'} &= x^1, \quad x^{2'} = x^2, \\ x^{3'} &= \hat{\gamma}(x^3 - \hat{\beta} \frac{n_3}{n_4} x^4), \quad x^{4'} = \hat{\gamma}(x^4 - \hat{\beta} \frac{n_4}{n_3} x^3), \end{aligned} \quad (30)$$

where

$$\hat{\beta} = \frac{v_3/n_3}{c/n_4}, \quad \hat{\gamma} = \frac{1}{\sqrt{1 - \hat{\beta}^2}}; \quad (31)$$

the second order iso-Casimir invariant of the *Lorentz-Poincaré-Santilli isosymmetry* [66]-[69]

$$\begin{aligned} \hat{C}_2 &= (\hat{\eta}^{\mu\nu} P_\mu P_\nu) \hat{I}^c = \\ &= (n_1^2 P_1^2 + n_2^2 P_2^2 + n_3^2 P_3^2 - n_4^2 P_4^2) \hat{I}^c = m^2 C^2, \end{aligned} \quad (32)$$

where, from line element (25), C is the local speed of light left invariant by isosymmetry (30),

$$C = \frac{c}{n_4}, \quad (33)$$

and $\hat{I}^c$ is now the *contravariant isounit*, thus being equal to $\hat{T}$,

$$\hat{I}^c = \hat{T} = 1/\hat{T}^c = \text{Diag.}(\frac{1}{n_1^2}, \frac{1}{n_2^2}, \frac{1}{n_3^2}, \frac{1}{n_4^2}); \quad (34)$$

and the *iso-Klein-Gordon isoequation* [24] derivable from iso-Casimir invariant (32) which, for the simplest possible assumptions $n_k = 1$, $k = 1, 2, 3$, $\hbar = 1$ and m_4 is a positive quantity given by

$$\begin{aligned} & \left(-\frac{1}{c} \frac{\partial^2}{\partial t^2} + \frac{1}{n_4^2} \nabla\right) \hat{T}^c \hat{\psi}(\hat{x}) = \\ & = \left(\frac{m^2}{n_4^2} \frac{c^2}{n_4^2}\right) \hat{T}^c \hat{\psi}(\hat{x}) = (\bar{m}^2 c^2) \hat{T}^c \hat{\psi}(\hat{x}), \end{aligned} \quad (35)$$

where

$$\bar{m} = \frac{m}{n_4^2} \quad (36)$$

is the *isorenormalized energy* of an extended particle within a physical medium with $n_1^2 = n_2^2 = n_3^2 = 1$ holding for the particular case in which the particle is perfectly spherical and the medium is homogeneous and isotropic [24] (see the formal treatment in Section 8.4.4, Eq. (133) of Isoaxiom IV).

Finally, the isotopies $\hat{SU}(2)$ of the $SU(2)$ -spin symmetry were constructed in Refs. [64] [65] (see the review in Section 3, Ref.[213]).

7.2.2. Recovery of classical images. Thanks to the above advances, Santilli introduced in paper [210] the following simplest possible realization of Bohm's hidden variables

$$\hat{U}U^\dagger = \text{Diag}(\lambda, 1/\lambda) \neq I, \quad \text{Det}\hat{I} = 1, \quad \lambda > 0, \quad (37)$$

applied to Pauli's matrices

$$\sigma_k \rightarrow \hat{\sigma}_k = U\sigma_k U^\dagger, \quad (38)$$

with explicit form

$$\begin{aligned}\hat{\sigma}_1 &= \begin{pmatrix} 0 & \lambda^{-1} \\ \lambda & 0 \end{pmatrix}, \quad \hat{\sigma}_2 = \begin{pmatrix} 0 & -i\lambda^{-1} \\ i\lambda & 0 \end{pmatrix}, \\ \hat{\sigma}_3 &= \begin{pmatrix} \lambda & 0 \\ 0 & -\lambda^{-1} \end{pmatrix},\end{aligned}\tag{39}$$

and regular isoalgebra

$$[\hat{\sigma}_i, \hat{\sigma}_j] = \hat{\sigma}_i \hat{T} \hat{\sigma}_j - \hat{\sigma}_j \hat{T} \hat{\sigma}_i = i2\epsilon_{ijk} \hat{\sigma}_k,\tag{40}$$

establishing the isomorphism $\hat{SU}(2) \approx SU(2)$, and *isoeigenvalue equations* (where all products, thus including squares, are isotopic)

$$\hat{\sigma}_3 \hat{\times} |\hat{b}\rangle = \pm |\hat{b}\rangle,\tag{41}$$

$$\hat{\sigma}^2 = (\hat{\sigma}_1 \hat{T} \hat{\sigma}_1 + \hat{\sigma}_2 \hat{T} \hat{\sigma}_2 + \hat{\sigma}_3 \hat{T} \hat{\sigma}_3) \hat{T} |\hat{b}\rangle = 3 |\hat{b}\rangle.\tag{42}$$

thus confirming the spin 1/2 for *isoparticles*, namely, extended particles in deep EPR entanglement which are characterized by isounitary isoirreducible isorepresentation of the Lorentz-Poincaré-Santilli isosymmetry $\hat{P}(3.1)$ [66]-[69] (see Section 2 of paper [213] for a review).

Bell's proof of the lack of existence of classical images for two particles with spin 1/2 is reduced a quantum mechanical (qm) quantity D^{qm} which, when computed via the use of Pauli's matrices, verifies the expression

$$D^{qm} \leq 2.\tag{43}$$

Santilli conducted in paper [210] a step-by-step *non-unitary/isounitary*, transformation of Bell's derivation along rules (28) for two isoparticles described by hadronic mechanics (hm) with Bohm's hidden variables λ_1, λ_2 in realization (37), resulting in the following property

$$D^{hm} = \frac{1}{2} \left(\frac{\lambda_1}{\lambda_2} + \frac{\lambda_2}{\lambda_1} \right) D^{qm}.\tag{44}$$

Since the factor $\frac{1}{2}(\lambda_1/\lambda_2 + \lambda_2/\lambda_1)$ can assume values arbitrarily larger than 2, Santilli proved in Ref. [210] that *a systems of isoparticles with spin 1/2 in condition of mutual penetration, thus EPR entanglement and ensuing non-Hamiltonian interactions does admit classical images with specific examples identified in paper [210].*

The invariance of the results under the the time evolution of the theory was also proved according to rules (39).

7.2.3. Realization of the EPR entanglement. It should be noted that realization (28) of hidden variables is indeed the most elementary possible because the notion of EPR entanglement (Figure 2) is characterized by the isotopic element $\hat{T}$ with realization for the hadronic bound state of two isoparticles (such as the Deuteron) of the type

$$\hat{T} = \Pi_{k=1,2} \text{Diag.}(1/n_{1k}^2, 1/n_{2k}^2, 1/n_{3k}^2, 1/n_{4k}^2) \times \exp[-\Gamma(\psi, \hat{\psi}, \dots)] > 0, \quad (45)$$

where $n_{k\alpha}^2, \alpha = 1, 2, 3, k = 1, 2$, where: $n_{\alpha k}^2 = 1$ represents the deformable semi-axes of the extended k -particle normalized to the values for the sphere; n_{4k}^2 represents the density of the k -particle normalized to the value $n_{4k}^2 = 1$ for the vacuum; and the exponent is a positive-definite function representing non-linear, non-local and non-potential interactions with realizations of the type

$$\exp[-\Gamma(\psi, \hat{\psi}, \dots)] = \exp[-|\psi/\hat{\psi}| \int \hat{\psi}_1 \hat{\psi}_2^\dagger d^3r,] \quad (46)$$

where ψ is a quantum mechanical wave function, and $\hat{\psi}$ is the completed wavefunction under isotopy.

In conclusion, Ref. [210] has shown that the historical intuition of *hidden variable* by David Bohm [18] is truly fundamental because it characterizes the completion of quantum into hadronic mechanics resulting

in a basically new notion of particle entanglement with momentous applications indicated in the next sections.

The cosmological implication of the EPR argument based in the classification of elementary particles via Dinkins diagram was presented by the 2020 Teleconference [0] by E. Trell, See contributed paper [232].

7.3. Recovering of Einstein's determinism.

7.3.1. Rudiments of isomechanics. The proof of the recovering of Einstein's determinism achieved in the 2019 paper [211] requires the use of isomathematics outlined in the preceding section plus the use of isomechanics [24] whose basic elements can be briefly outlined for the non-initiated reader as follows:

1) The basic unit of isomechanics is given by the completion of Planck's unit $\hbar = 1$ into the isounit $\hat{I}$ which is the inverse of the iso' topic element (45)

$$\hat{I} = 1/\hat{T} = \Pi_{k=1,2} \text{Diag.}(n_{1k}^2, n_{2k}^2, n_{3k}^2, n_{4k}^2) > 0, \quad (47)$$

where exponent (46) is incorporated into the n -characteristic quantities. Isounit (47) represents the *volume* and *density* of isoparticles as well as the impossibility for *extended* protons in the core of a star to solely have discrete energy exchanges due to the extreme local densities and pressures, in favor of integro-differential energy exchanges;

2) The Lie-Santilli isothory has be formulated on a *Hilbert-Myung-Santilli isospace* [45] $\hat{\mathcal{H}}$ over the isofield of isocomplex isonumbers $\hat{\mathbb{C}}$ [33];

3) Isostates $\hat{\psi}(\hat{r}) \in \hat{\mathcal{H}}$ should have *isonormalization*

$$\begin{aligned} < \hat{\psi}(\hat{r}) | \hat{\times} | \hat{\psi}(\hat{r}) > = < \hat{\psi}(\hat{r}) | \hat{T} | \hat{\psi}(\hat{r}) > = \\ &= \int_{-\infty}^{+\infty} \hat{\psi}^\dagger(\hat{r}) \hat{T} \hat{\psi}(\hat{r}) d\hat{r} = \hat{T}, \end{aligned} \quad (48)$$

were one should note the need for isointegrals and isodifferentials (23);

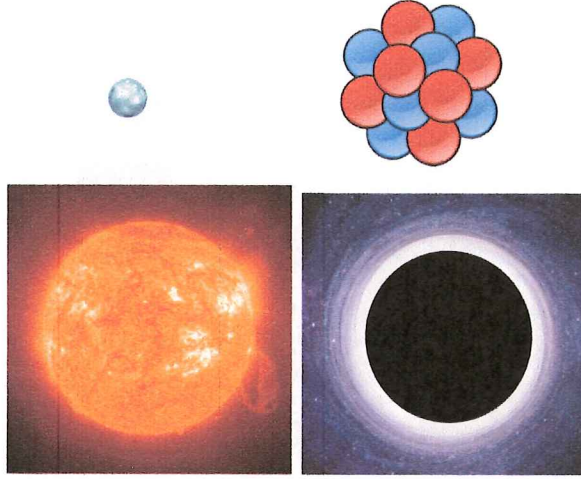


Figure 9: *Experimental evidence establishes that the wavepacket of particles has approximately the same size 10^{-13} cm as that of all hadrons (top left). Consequently, hadrons are composed by wavepackets in deep EPR entanglement (Sections 3.1 and 7.2.3) called 'hadronic medium.' A corresponding structure holds for nuclei, stars and black-holes due to partial or deep mutual penetration of hadron constituents. The realization via the isotopic element of hadronic mechanics of the pressure exercised by the hadronic medium on individual constituents (Figure 8) implies the progressive weakening of Heisenberg's uncertainties first identified in Ref. [47] of 1981, with progressive recovering of Einstein's determinism [1] in the structure of hadrons, nuclei and stars and its full recovering at the limit of gravitational collapse studied in detail in Refs. [210]-[214]. Note that Heisenberg's uncertainties remain valid for the center of mass of hadrons, but not so for black-holes, thus illustrating the indicated full recovering of Einstein's determinism.*

4) The *isoexpectation values* of an observable A are given by

$$\hat{\langle A \rangle} = \langle \hat{\psi}(\hat{r}) | \hat{\times} A \hat{\times} | \hat{\psi}(\hat{r}) \rangle = \langle \hat{\psi}(\hat{r}) | \hat{T} A \hat{T} | \hat{\psi}(\hat{r}) \rangle; \quad (49)$$

5) The realization of the linear momentum on isospaces over isofield via the conventional differential calculus leads to major inconsistencies. The correct formulation of the *isolinear isomomentum* is that via isoderivative (27) and it is given by

$$\hat{p} \hat{\times} \hat{\psi}(\hat{r}) = -i \hat{\times} \hat{\partial}_{\hat{r}} \hat{\psi}(\hat{r}) = -i \hat{I} \partial_{\hat{r}} \hat{\psi}(\hat{r}), \quad (50)$$

thus providing a forceful illustration of the need for the novel isodifferential calculus [34];

6) The *isocanonical isocommutation rules* are consequently given by

$$[\hat{r}^i, \hat{p}_j] = i \hat{\times} \hat{\delta}_j^i = i \hat{I} \delta_j^i, \quad [\hat{r}^i, \hat{r}^j] = [\hat{p}_i, \hat{p}_j] = 0, \quad (51)$$

by illustrating that isomechanics is *isolinear*, that is, linear on isospace over isofields because $[\hat{p}_i, \hat{p}_j] = 0$, although the theory is highly non-linear when *projected* into our conventional spaces over conventional fields because, in that case, $[p_i, p_j] \neq 0$;

7) The *iso-Schrödinger isoequation* is then given by [20]-[24]

$$\hat{H} \hat{\times} \hat{\psi}(\hat{r}) = \left[-\frac{1}{2m} \hat{\Delta}_{\hat{r}} + \hat{V}(\hat{r}) \right] \hat{\times} \hat{\psi}(\hat{r}) = \hat{E} \hat{\times} \hat{\psi}(\hat{r}) = E \hat{\psi}(\hat{r}), \quad (52)$$

and now represents extended, deformable and hyperdense particles at all levels, including isocoordinates $\hat{r}$, isostates $\hat{\psi}(\hat{r})$, isotopic element $\hat{T}$, isopotentials $\hat{V}(\hat{r})$ iso-Laplacian $\hat{\Delta}_{\hat{r}}$, etc. In particular, isostates verify the isosuperposition principle on isospaces over isofields

$$\hat{\psi}(\hat{r}) = \sum_{k=1,2} \hat{\psi}_k(\hat{r}), \quad (53)$$

because the theory is *isolinear* as indicated above. Therefore, hadronic mechanics resolves the inability by Heisenberg's non-linear completion

of quantum mechanics [16] to represent the constituents of a bound state of particles with non-linear internal forces discussed in Section 5.

7.3.2. Confirmation of the EPR argument. It is evident that the recovering of classical images in paper [210] established the foundation for the recovering of Einstein's determinism. These studies were initiated by Santilli in the 1981 paper [47], continued in various papers due to the need to achieve maturity in the formulation of isomathematics (see monographs [23] [24]), and concluded in the 2019 paper [211] via the following simple isotopy of the conventional derivation of Heisenberg's uncertainties

$$\Delta r \Delta p = \frac{1}{2} | \langle \hat{\psi}(\hat{r}) | \hat{\times} [\hat{r}, \hat{p}] \hat{\times} | \hat{\psi}(\hat{r}) \rangle \approx \frac{1}{2} \hat{T} \ll 1, \quad (54)$$

where the property $T \ll 1$ is established by all fits of experimental data to date [25] [26] (see Section 7.6 for the analytic aspects).

By recalling that the value of the isotopic element is inversely proportional to the increase of the density [211], isodeterministic principle (47) establishes *the progressive validity of Einstein's determinism in the interior of hadrons, nuclei and stars, and its full achievement in the interior of gravitational collapse* (Figure 9).

The latter result is due to the fact that the isotopic element admits a realization in terms of the space component of Schwartzchild's metric [211]

$$\hat{T} = \frac{1}{1 - \frac{2M}{r}} = \frac{r}{r - 2M}, \quad (55)$$

where M is the gravitational mass of the body considered with ensuing isodeterministic isoprinciple

$$\Delta \hat{r} \Delta \hat{p} \approx \hat{T} = \frac{r}{r - 2M} \Rightarrow_{r \rightarrow 0} 0. \quad (56)$$

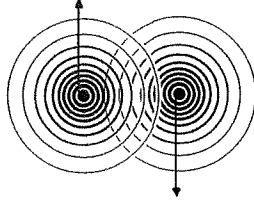


Figure 10: A conceptual rendering of the 'attractive' force between 'identical' electrons in valence pairs first identified in Chapter 4 of the 2001 Ref. [30] against their very big Coulomb repulsion, Eq. (6), thanks to the representation of the extended character of the electron wavepackets via isomathematics and isochemistry with ensuing total EPR entanglement of the electrons. Note that this achievement proves the last statement of the EPR argument to the effect that quantum wavefunctions cannot represent 'all elements of reality' [1].

Note that the center-of-mass of hadrons and nuclei verifies quantum uncertainties while that for stars and gravitational collapse verifies classical determinism.

Note also that isodeterministic principle (54) holds for the infinite class of isounitary time evolutions of the theory, Eqs. (29).

7.4. Achievement of attractive force between valence electrons.

Einstein, Podolsky and Rosen ended their historical paper [1] with the statement: "While we have thus shown that the wavefunction does not provide a complete description of the physical [and we add chemical] reality, we left open the question of whether or not such a description exists. We believe, however, that such a theory is possible."

In Section 3.5, we indicated that a primary insufficiency of quantum chemistry is the lack of an *attractive* force between the *identical* electrons in valence coupling, Eq. (5), and that their *repulsive* force is so big, Eq. (6), to prevent any realistic hope to achieve an attractive force via quantum

chemistry due to its linear, local, Hamiltonian and unitary structure.

In monograph [30] of 2001, particularly Chapter 4, Santilli achieved the needed attractive force between identical valence electrons via a *non-unitary/isounitary completion of the wavefunction of quantum mechanics* which is a direct verification of the final EPR statement quoted above (see Figure 10 and Section 5 of paper [214]).

The above result can be outlined as follows. Assume that the electrons are perfectly spherical for which $n_k^2 = 1$, $k = 1, 2, 3$, and that their density is ignorable for which $n_4^1 \approx 1$. Consider then a non-unitary transformation with the following simple realization of Eqs. (45)(46)

$$\begin{aligned}\hat{I} &= UU^\dagger = e^{\frac{V_{hm}}{V_{qm}}} \approx 1 + \frac{V_{hm}}{V_{qm}} \\ \hat{T} &= (UU^\dagger)^{-1} = e^{-\frac{V_{hm}}{V_{qm}}} \approx 1 - \frac{V_{hm}}{V_{qm}}\end{aligned}\quad (57)$$

where V_{hm} is a quantum mechanical (qm) potential, for instance, the *repulsive* Coulomb potential, and V_{hm} is a strongly *attractive* potential of hadronic mechanical (hm), such as the Hulthen potential,

$$\begin{aligned}V_{qm} &= +\frac{e^2}{r} \\ V_{hm} &= -W \frac{e^{-br}}{1 - e^{-br}},\end{aligned}\quad (58)$$

where W is a normalization constant.

It is then easy to see that a *non-unitary transformation can turn the quantum mechanical repulsion between valence electrons into the attraction needed to represent the "element of reality" given by valence bonds*

$$UV_{qm}U^\dagger \approx V_{qm}\left(1 + \frac{V_{hm}}{V_{qm}}\right) = V_{qm} + V_{hm} =$$

$$= +\frac{e^2}{r} - W \frac{e^{-br}}{1 - e^{-br}} \approx K \frac{e^{-br}}{1 - e^{-br}} \quad (59)$$

where the last step is due to the fact that the Hulthen potential behaves at short distances like the Coulomb potential [20]. However, the former is much stronger than the latter, by therefore allowing the absorption of the Coulomb potential into the Hulthen potential irrespective of whether the Coulomb potential is attractive or repulsive. In this way, the final equation solely shows the Hulthen potential with the new, positive, renormalization constant K (see page 833 of Ref. [20], Chapter 4 of Ref. [30], and paper [56]).

To achieve the corresponding iso-Schrödinger equation, recall rules (28) for the nonunitary transformation of the linear into the isilinear momentum

$$\begin{aligned} Up\psi(r)U^\dagger &= UpU^\dagger(UU^\dagger)^{-1}U\psi(r)U^\dagger = \hat{p}\hat{\times}\hat{\psi}(\hat{r}) = \\ &= -i\hat{\times}\frac{\hat{\partial}}{\hat{\partial}\hat{r}}\hat{\psi}(\hat{r}) = -i\hat{I}\partial_{\hat{r}}\hat{\psi}(\hat{r}). \end{aligned} \quad (60)$$

Under the above properties, the application of non-unitary transformation (57) to the conventional Schrödinger equation (5) for the valence electron pair with repulsive force then yields the iso-Schrödinger equation for their attraction as in the physical reality, which we write in its projection into the conventional Hilbert space over the field of complex numbers (see Section 2 of Ref. [214] for its detailed derivation)

$$\left(-\frac{1}{v\bar{m}}\hat{\Delta}_r - Ke^{\frac{-br}{1-e^{-br}}}\right)\hat{\psi}(r) = E\hat{\psi}(r), \quad (61)$$

where $v\bar{m}$ is the reduced form of isorenormalized masses, Eq. (56).

The above results confirm the final statement by Einstein, Podolsky and Rosen quoted above because the *wavefunction* $\psi(r)$ of quantum chemistry cannot represent the physical reality of the attraction between the

identical valence electrons in molecular bonds in favor of its completion into the isowavefunction $\hat{\psi}(\hat{r})$ of hadronic chemistry. A more recent derivation of the attraction between valence electrons is available in Section 2.8 of paper [214].

It should be indicated that *valence electron bonds appear to be one of the most significant realizations of the EPR entanglement (Figures 2 and Section 7.2.3) apparently occurring at the limit of null mutual distance with a considerable broadening of the implications of entanglement in physics, chemistry and biology.* In fact, by merely admitting the evidence, we learn that the entanglement of wavepackets allows particle to influence each other at a distance, we learn from isotopic element (37)-(39) that entanglement plays a crucial role in nuclear forces (see also Section 8), and we now learn that, at the limit of null mutual distance, entanglement may alter the very characteristics of particles, a feature called *mutation*, which is evidently needed to turn a natural repulsion into an attraction.

It may be of some interest to note that the indicated mutation of valence electrons can be quantitatively represented by the transition from *particles* characterized by a unitary irreducible representation of the spinorial covering of the Poincaré symmetry $\mathcal{P}(\cdot)$ into *isoparticles* characterized by isounitary isoirreducible isorepresentations of the Lorentz-Poincaré-Santilli-isosymmetry $\hat{\mathcal{P}}(\cdot)$ [66] - [69] (see reviews Ref. [213]).

In fact, the transition from $\mathcal{P}(\cdot)$ to $\hat{\mathcal{P}}(\cdot)$ implies, in general, a mutation of all *intrinsic* characteristics of entangled particles also called *isorenormalization* because caused by the isotopic element representing non-Hamiltonian interactions. When the entanglement is limited, as occurring in a nuclear structure, the spin 1/2 of the constituents is maintained, but when the entanglement is total, as occurring in valence bonds, all characteristics of the particles are expected to be mutated, including the charge.

7.5. Removal of quantum divergencies.

Recall that the divergencies of quantum mechanics originate on the sin-

gularity of Dirac's delta function

$$\delta(r) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{ikr} dk, \quad (62)$$

at the origin $r = 0$.

It is easy to see that there exist an isotopic element for which the singularity at the origin $r = 0$ is removed for the *Dirac-Myung-Santilli isodelta isofunction* [45] (see also Refs. [47] - [51]), as illustrated by the following simplest possible formulation

$$\hat{\delta}(r) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \hat{e}^{ikr} dk = \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{i\hat{T}kr} dk. \quad (63)$$

Similarly, recall that the isotopic product $A \star B = A\hat{T}B$ must be applied to the *totality* of the products, thus including all products appearing in perturbative series. But the isotopic element $\hat{T}$ has very small values in all known applications. Consequently, perturbative series that are generally divergent in quantum mechanics and chemistry are turned under isotopy into strongly convergent series. This is illustrated by fact that, given a divergent perturbative series

$$A(t) = A(0) + (AH - HA)/1! + \dots \rightarrow \infty, \quad (64)$$

there always exist a value of the isotopic element $\hat{T}$ causing the strong convergence of the isotopic series

$$A(t) = A(0) + (A\hat{T}H - H\hat{T}A)/1! + \dots = K \ll \infty, \quad \hat{T} \ll 1. \quad (65)$$

Consequently, the validity of Einstein's [determinism per Section 7.3 implies the removal of quantum mechanical divergencies (see Corollary 3.7.1, page 128, Ref. [213]).

The actual verification of the above important property has been provided by D. D. Shillady and R. M. Santilli in papers [31] [32] with the

proof that the perturbative series of hadronic chemistry converge at least one thousand times faster than the corresponding quantum chemical series.

In Santilli's view, the above rapid convergence of calculations for the EPR entanglement is an important advantage over the conventional quantum entanglement, particularly for applications such as that for computers based on the entanglement of electrons.

Note that the new computers are called in this Overview *EPR computers*, instead of 'quantum computers', due to the impossibility by quantum mechanics to provide a representation of the entanglement of electrons discussed throughout this work.

7.6. Representation of irreversible processes.

As indicated in Section 3.3 and Figure 5, the insufficiency of quantum mechanics that stimulated Santilli's lifelong research on the completion of quantum mechanics according to the EPR argument [1] is the inability to represent time-irreversible events, including energy-releasing processes, due to the invariance under anti-Hermiticity of the Lie brackets for Hermitean operators, $[A, B] = -[A, B]^\dagger$.

The above occurrence stimulated the construction of the Lie-admissible branch of hadronic mechanics [6]-[9], [19]-[25], [38]-[39], with time evolution (13)-(14) (see Section 2 of Ref. [212] for a review).

Discussions during the 2020 Teleconference [0] established that genotopic elements $\hat{R}$ and $\hat{S}$ represent the external terms $F > 0$ in Lagrange's and Hamilton's equations

$$\begin{aligned} \frac{d}{dt} \frac{\partial L(t, r, v)}{\partial v} - \frac{\partial L(t, r, v)}{\partial r} &= F(t, r, v), \\ \frac{dr}{dt} &= \frac{\partial H(t, r, p)}{\partial p}, \quad \frac{dp}{dt} = -\frac{\partial H(t, r, p)}{\partial r} + F(t, r, p). \end{aligned} \quad (66)$$

In fact, in his Ph. D. thesis of the mid 1960s [6]-[8], Santilli accepted the EPR argument [1] on the lack of completeness of quantum mechanics because quantum mechanics cannot represent the external terms F of

Lagrange's and Hamilton's analytic equations which is the analytic representation of the irreversibility of nature.

Santilli noted that the brackets (A, F, H) of the classical time evolution with the external terms

$$\frac{dA}{dt} = (A, H, F) = \frac{\partial A}{\partial r} \frac{\partial H}{\partial p} - \frac{\partial H}{\partial r} \frac{\partial A}{\partial p} + \frac{\partial A}{\partial r} F, \quad (67)$$

violate the right distributivity and scalar laws, thus prohibiting the construction of a completion of quantum mechanics characterized by an *algebra* as understood in mathematics.

Therefore, Santilli turned brackets (67) into the identical but bilinear form verifying the axioms of an algebra

$$\begin{aligned} \frac{dA}{dt} = (A, H) &= \frac{\partial A}{\partial r} \frac{\partial H}{\partial p} - \frac{\partial H}{\partial r} S \frac{\partial A}{\partial p}, \\ S &= 1 - \frac{F}{\partial H \partial r}. \end{aligned} \quad (68)$$

The algebra characterized by the new brackets (A, H) turned out to be Lie-admissible according to Albert [9].

In subsequent studies, Santilli constructed the operator image of brackets (57) with the following realization of the genotopic elements $\hat{R}$ and $\hat{S}$

$$\begin{aligned} i \frac{dA}{dt} = (A, H) &= A < H - H > A = \\ &= A \hat{R} H - H \hat{S} A = A H - H A - A F, \\ R &= 1, \quad \hat{S} = 1 - F/H, \end{aligned} \quad (69)$$

that was proved in paper [41] to be *directly universal* for the representation of all possible (non-singular) non-conservative, thus irreversible systems ("universality") directly in the coordinates of the experimentalist ("direct universality") without any need for the transformation theory.

Note that the isotopic element $\hat{T}$ is a particular case of the genotopic element $\hat{S}$ and that, in numeric values, $F < H$. Consequently, Eqs. (69) establish the analytic origin of the important property that the isotopic element is smaller than 1, $\hat{T} = 1 - F/H < 1$ which property is important for the regaining Einstein's determinism.

Additionally, Eqs. (69) provide a quantitative identification of the insufficiency of quantum mechanics for nuclear fusions, evidently given by the absence in the quantum mechanical time evolution of the extra term AF representing irreversibility.

It is important to clarify that, despite the lifting of Lie into Lie-admissible algebras, quantum mechanical axioms are indeed preserved. This important property can be best seen in the axiomatic representation of irreversible systems via Lie-admissible formulations whose main aspects are the following.

A tacit assumption of quantum mechanics is that the product of a quantity A times B to the right $A \rightarrow B$ is equal to the product of B times A to the left $A \leftarrow B = A \rightarrow B$ (which property is different than commutativity $AB = BA$), and the same property holds for isomechanics.

In Lie-admissible formulations, motion forward (backward) in time is represented by the axiom of restricting all products to be ordered to the right $A > B$ (to the left $A < B$). The representation of irreversibility is then assured when the numeric values of the two products are different, $A > B \neq A < B$. This results in the following ordered products and units called *genoproducts and genounits to the right and to the left*

$$\begin{aligned} A > B &= A\hat{R}B, \quad \hat{I}^> = 1/\hat{R}, \\ A < B &= A\hat{S}B, \quad \hat{I}^< = 1/\hat{S}, \\ \hat{R} > 0, \quad \hat{S} > 0, \quad \hat{R} &\neq \hat{S}. \end{aligned} \tag{70}$$

The above basic assumptions imply the duplication of isomathematics and related isomechanics, one for product to the right and one for

product to the left whose review is omitted for brevity [24]. The combination of forward and backward mathematics and mechanics are known as *genomathematics and genomechanics*.

An important point is that *the abstract axioms of quantum mechanics remain fully valid for each direction of time, including the preservation of the abstract axioms of numeric fields [33], functional analysis [23] differential calculus [34], etc, merely realized in a time-ordered isotopic form.*

It is generally believed that genomathematics and genomechanics are too complex for physicists. This is unfortunate because, as shown in Ref. [35], the construction of genomathematics is truly elementary. The completion of a quantum mechanical model of nuclear fusion into a form inclusive of irreversibility, is given by subjecting all quantum mechanical quantities and operations to the following *two* non-unitary transformations U and W , including Planck's unit $\hbar = 1$, products, etc.

$$1 \rightarrow \hat{I}^> = U1W^\dagger = 1/\hat{S},$$

$$1 \rightarrow \hat{I}^< = W1U^\dagger = 1/\hat{R},$$

$$AB \rightarrow U(AB)W^\dagger = (UAW^\dagger)(UW^\dagger)^{-1}(UBW^\dagger) = A\hat{S}B = A > B,$$

$$AB \rightarrow W(AB)U^\dagger = (WAU^\dagger)(WU^\dagger)^{-1}(WBA^\dagger) = A\hat{R}B = A < B. \quad (71)$$

As illustrated in the next section, the treatment of energy releasing processes via the Lie-admissible branch of hadronic mechanics has been done via the simple, dual, non-unitary transformation (71) of quantum mechanical models.

7.7. EPR completion for matter-antimatter annihilation.

As indicated in Section 3.4 and Figure 6, the additional insufficiency of quantum mechanics studied at the 2020 teleconference [0] has been the apparent impossibility of representing the mechanism of particle-antiparticle annihilation via the conventional charge conjugation, Eq. (4).

This insufficiency lead Santilli to propose in papers [62] [63] of 1985 a fifth EPR completion of quantum mechanics, that characterizes the *isodual map* (denoted with an upper symbol d) which is also an anti-Hermitean map like charge conjugation, but it is applied to the *totality* of the quantities Q representing particles and their operations with no known exception, such as

$$\begin{aligned} Q(t, r, E, \psi, \dots) &\rightarrow Q^d(t^d, r^d, E^d, \psi^d, \dots) = \\ &= -Q^\dagger(-t^\dagger, -r^\dagger, -E^\dagger, -\psi^\dagger, \dots). \end{aligned} \quad (72)$$

The above map implies that antiparticles have the *negative energy* $E^d = -E < 0$ predicted by Dirac [11] which is necessary to represent particle-antiparticle annihilation [29].

The novel mathematics characterized by isodual map (72) is known as *isodual mathematics* [29] [57], and it is based on the *isodual unit*

$$1^d = -1, \quad (73)$$

isodual numbers [33]

$$n^d = n1^d = -n, \quad (74)$$

isodual product

$$A \times^d B = AT^d B = -AB, \quad 1^d = 1/T^d = -1, \quad (75)$$

isodual spacetime

$$x^{d2d} = x^{d\mu} \times^d \eta_{\mu\nu}^d \times^d x^{d\nu} = -x^2, \quad (76)$$

and isodual image of 20th century applied mathematics [34] [32].

On axiomatic grounds, isodual mathematics resolves the violation of causality by particles with negative energy identified by Dirac [11] because *negative energies referred to negative units are as causal as positive*

energies referred to positive units, and the same holds for time and other physical quantities.

On experimental grounds, the isodual map is equivalent to charge conjugation at the particle level. Consequently, the isodual theory of antimatter represents all available experimental data in particle physics [32].

A basic novelty of the isodual theory of antimatter is that isoduality (72) predicts that antimatter emits a new light called *isodual light* [75] whose characteristics are *all* opposite to those of matter emitted light, thus implying new annihilations of the type

$$e^- + e^+ \rightarrow \gamma + \gamma^d. \quad (77)$$

which are independently requested by the symmetry of Dirac's equation and of the l. h. s. of the above decay known as *isoselfduality*, namely, the invariance under the isodual map (72) [32].

In particular, *light emitted by antimatter is predicted not to be visible by any Galileo-type refractive telescopes with convex lenses available on Earth or in terrestrial orbits*, because it requires for its detection a basically new type of telescopes currently under development. The cosmological implications of the isodual theory of antimatter are discussed in debate [76] and Ref. [163].

8. APPLICATIONS OF EPR COMPLETIONS

8.1. Foreword.

Following various requests, in this section we provide an outline of the novel applications permitted by the recent verifications [210]-[214] of the EPR argument [1] in: nuclear physics (Section 8.2), chemistry (Section 8.3), relativities (Section 8.4), and high energy scattering experiments (Section 8.5).

Since the outline presented in this section is mainly conceptual with a bare minimum of quantitative treatments, the study of the quoted literature is essential for a serious understanding of the treated topics. In

addition, the literature in the field is quite vast by therefore forcing us for brevity to quote only the originating papers. Comprehensive bibliographies can be obtained from the quoted reviews.

Needless to say, the fundamental methods of this section are the *iso-, geno- and hyper-mathematics*, with particular reference to the *iso-, geno- and hyper-completions of Lie's theory*, and corresponding *iso-, geno- and hyper-mechanics and chemistry* [21]-[30]. To assist the non-initiated reader, Tutoring Lectures in the above methods have been provided in Refs. [143]-[146].

In inspecting this section, interested readers should keep in mind that the basic axioms of 20th century theories are preserved in their entirety, and merely subjected to the broadest possible realization.

Consequently, this section is intended to illustrate the remarkable broadening of the prediction and representational capabilities of the basic axioms of 20th century theories, as well as to point out that technically unsubstantiated criticisms of the applications reviewed in this section are, in reality, criticisms on 20th century theories.

Another important aspect needed for the understanding of the applications herein reviewed is that 20th century theories have one single formulation, the conventional one. By contrast, their isotopic completions have *two* formulations, that on isospaces over isofield and their *projection* on conventional spaces over conventional fields.

Recall that 20th century Hamiltonian interactions between *point-like* particles cause alterations of the characteristics of particles called *renormalization*. It is known since the original 1978 proposal to construct hadronic mechanics that contact non-Hamiltonian interactions between *extended* particles causes different alterations of the characteristics of particles called *isorenormalizations* (see Section 5 of Ref. [20], and Refs [23] [24]). The latter renormalizations are triggered by the isotopic element $\hat{T}$. Consequently, the conventional notion of *particle* is no longer applicable under EPR completion in favor of the covering notion of *isoparticles* pointed out

in the preceding section [66]-[70].

As an illustration, the assumption that the particles in a valence bond are ordinary electrons leads to major inconsistencies that remain undetected by the non-initiated reader. Similarly, the assumption that the particles in the synthesis of the neutron from the hydrogen in the core of stars are conventional protons and electrons leads to inconsistencies so serious to prevent a consistent synthesis in disagreement with the experimental evidence on the existence of the neutron synthesis in nature.

8.2. Applications of EPR completions in nuclear physics.

8.2.1. Representation of nuclear magnetic moments. As it is well known, the magnetic moment of a rotating charged sphere increases (decreases) when said sphere is deformed into an oblate (prolate) spheroid while keeping constant its angular momentum. Enrico Fermi, Victor Weisskopf and other founders of nuclear physics suggested in the 1940's that the anomalous values of nuclear magnetic moments (Section 3.2 and Figure 3) may be due to a *deformation* of the charge distribution of protons and neutron caused by strong nuclear interactions.

Despite its manifestly plausible character and authoritative origination, the hypothesis of the deformability of nucleons has not yet been accepted by the mainstream physics community to date (with due exceptions) because it would require a modification of quantum mechanics essentially along the EPR argument [1]. In any case, point-like particles cannot be deformed.

The Lie-isotopic completion of quantum mechanics with two-body isounit (45), related deformation of charge distributions (Figure 14), and exact representation of the Deuteron magnetic moment first achieved in Ref. [40] of 1994, have been proposed to honor the legacy by Fermi, Weisskopf and others without altering the basic axioms of quantum mechan-

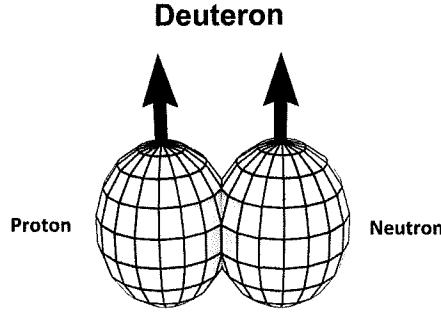


Figure 11: *A view of the deformation of the proton and the neutron in the Deuteron structure that permitted the first exact representation of the deuteron magnetic moment in Ref. [40] of 1994. Note that the triplet coupling used to represent the Deuteron spin 1 is unstable according to quantum mechanics. The resolution of this paradoxical occurrence is outlined in Section 8.2.5 [26] [213].*

ics.

However, in so doing the constituents of nuclei are no longer point-like protons and neutrons per 20th century definition, but extended, thus deformable *isoprotons* $\hat{p}^+$, *isoneutrons* $\hat{n}^0$, or *isonucleons*, as defined by the LPS isosymmetry. In fact, the 1998 proof of the EPR argument [210] applies the underlying isomathematics and isomechanics for the *exact* representation of the nuclear magnetic moment of the Deuteron $D = (\hat{p}^+, \hat{n}^0)_{hm}$, as well as of heavier nuclei, precisely along the legacy by Fermi, Weisskopf and others indicated above.

It should be noted that the deformability of nucleons was preliminarily confirmed by the 1981 neutron interferometric experiments [43] and the adjoining theoretical study on the Lie-admissible completion of the rotational symmetry [44].

Additionally, paper [210] used isomathematics and isomechanics for

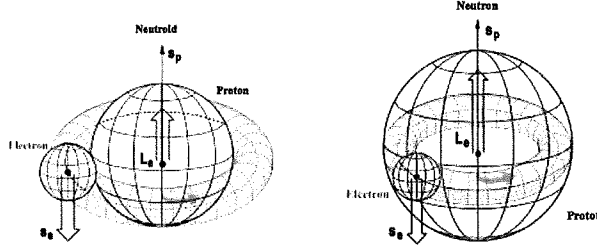


Figure 12: At the mutual distance of 10^{-13} cm, the proton and the electron experience the extremely big Coulomb attraction of 230 Newtons, Eq. (6), although quantum mechanics prohibits their bound state. In this figure we depict the two bound states predicted by hadronic mechanics [68] [84] [214]: the neutroid in the left for a singlet coupling with total spin 0 and 7 s mean-life, and the neutron in the right with spin 1/2 and 900 s mean-life under the total compression of the electron inside the proton in the core of stars (Section 8.2.2).

the reconstruction of the *exact* $\hat{SU}(2)$ -isospin symmetry when believed to be broken by electromagnetic interactions. This result was achieved via the representation of the proton-neutron mass difference with Bohm's hidden variable λ .

The systematic reconstruction of all space-times continuous and discrete symmetries at the isotopic level when believed to be broken is presented in Refs. [23] [24].

Note that, according to the basic axioms of quantum mechanics, the triplet coupling of the proton and the neutron of Figure 11 generally used to represent the spin 1 of the deuteron, is highly unstable, The only stable quantum mechanical bound state is the singlet, but in this case the total spin of the deuteron would be 0 contrary to experimental evidence. The resolution of this paradoxical situation via hadronic mechanics has been achieved in Refs. [26] [213] and will be outlined in Section 8.2.5.

8.2.2. Representation of the neutron synthesis. Nuclei are assumed to be bound states of protons and neutrons under strong interactions. However, isolated neutrons are naturally unstable. Therefore, a deeper representation of nuclear structures requires the understanding of the mechanism in which neutrons become stable when they are members of a nuclear structure. In turn, this understanding can best be achieved in the study of the neutron synthesis from a proton and an electron in the core of stars.

The problem is that the neutron synthesis is impossible for quantum mechanics despite the enormous Coulomb attraction between the proton and the electron at short distances per Eq. (6), because the mass of the neutron is 0.784 MeV bigger than the sum of the masses of the proton and of the electron, in which case the Schrödinger equation would require a *positive potential energy* producing a *mass excess* which is anathema for quantum mechanics.

The 1978 Harvard University paper [20] assumed the above insufficiency of quantum mechanics as a clear evidence on the lack of completion of quantum mechanics according to the EPR argument [1] and proposed the construction of the axiom-preserving hadronic mechanics precisely for the representation of the neutron synthesis from the hydrogen.

In view of the list of insufficiencies of the conjecture that the hypothetical and directly undetectable quarks are the physical constituents of hadrons [38], paper [20] presented in Section 5 a structure model of the octet of mesons as hadronic bound states of actual physical particles produced free in their spontaneous decays with the lowest mode. Note that the model achieved compatibility with the $SU(3)$ model of the time by merely restricting it to the sole quantum mechanical *classification* of mesons, and the use of hadronic mechanics for the *structure* of individual mesons due to the known inapplicability of quantum mechanics in the interior of hadrons, much along the dichotomy classification/structure

set in history for atoms and other structures.

Following the development of the Lie-Santilli isothory [22] and its application to the isotopies of the $SU(2)$ -spin symmetry [62] [63], hadronic mechanics was used in Ref. [84] for the first known non-relativistic representation of *all* characteristics of the neutron as a hadronic bound state of an isoelectron $\hat{e}^-$ and an isoproton $\hat{p}^+$

$$n = (\hat{e}^-, \hat{p}^+)_{hm}, \quad (78)$$

resulting in the prediction of *two* hadronic bound states: the first is called *neutroïd* and occurs for the singlet coupling isoelectron-isoproton with spin 0 and 7 s mean-life (left of Figure 12); the second is given by the neutron with spin 1/2 and 900 s mean-life, which occurs under the total compression of the isoelectron inside the isoproton (right of Figure 12).

The representation of the rest energy, mean life and charge radius of the neutron were represented by a non-unitary transformation of the conventional Schrödinger equation (1) with isotopic element of type (57) and ensuing iso-Schrödinger equation of type (61) in which the fine spectrum of the Hulthén potential is constrained to have one value only, the neutron.

The representation of the spin 1/2 required the irregular isotopies $\hat{S}\hat{O}(3)$ and $\hat{S}\hat{U}(2)$ to represent the *hadronic angular momentum* of the isoelectron when compressed inside the hyperdense isoproton, thus being forced to rotate with its spin, resulting in an *orbital* eigenvalue 1/2 which is impossible for the *unitary* $SU(2)$ symmetry of quantum mechanics, but readily possible for the *non-unitary* $\hat{S}\hat{U}(2)$ isosymmetry.

The representation of the anomalous magnetic moment of the neutron became possible thanks to the contribution from the rotation of the isoelectron inside the isoproton which is completely absent in quantum mechanics (see the review and upgrade in paper [214]).

The relativistic representation of *all* characteristics of the neutron in its synthesis from the hydrogen was achieved in the 1993 paper [68] written

at the Joint Institute for Nuclear Research, Dubna, Russia (see also the 1995 paper [69] published in China).

By ignoring in first approximation the negative binding energy due to the Coulomb interactions between the isoelectron and the isoproton (because the Coulomb attraction is absorbed by the Hulthén potential), iso-Schrödinger equation (62) admits physically meaningful solutions for the Hulthén potential only when the isorenormalized rest energy of the isoelectron, Eq. (36), acquires the value

$$\bar{m}_e = \frac{m_e}{n_4^2} = 0.511/n_4^2 = 1.295 \text{ MeV}, \quad (79)$$

from which we obtain the density value

$$n_4^2 = 0.394, \quad (80)$$

which is fully within the values of similar densities, such as that of the proton-antiproton fireball obtained from the fit of the experimental data of the Bose-Einstein correlation, which are given by the value $n_4^2 = 0.428$ of Ref. [85], Eq.(10.27a), page 127 and the value $n_4^2 = 0.364$ of Ref. [86], Table I, page 441.

As recalled earlier, the radial equation of iso-Kein-Gordon equation (35) or the iso-Dirac equation (163), do not admit physically meaningful solution in the event the rest energy of the neutron is *smaller* than the sum of the rest energies of the electron and of then proton, thus suggesting isorenormalization (79) that includes the missing 0.782 MeV.

A primary function of the iso-Minkowskian geometry [70] is the characterization of the isorenormalization of the rest energy of the electron caused by non-Hamiltonian interactions in the interior of the neutron in such a way to avoid the indicated "mass-energy defect" and allow isorelativistic equations to have physical solutions.

Needless to say, the above relativistic effects mandate the study of the laws applicable in the hyperdense medium inside the neutron, which

study is rudimentarily done in Section 8.4.4, see in particular Isoaxiom IV, Eqs. 133 (see also Volume IV or Ref. [87] for a 2008 review and Ref. [214] for a recent update).

In Section 8.2.5, we shall outline the significance of the neutron synthesis for the representation of nuclear data, such as the stability of nuclei as bound state of protons and neutrons despite the natural instability of the neutron when isolated.

8.2.3. Etherino or neutrino? Due to the respect toward the memory of W. Pauli and E. Fermi, it took years to indicate that in both, the non-relativistic [84] and relativistic [68] derivations, there is no energy available for the neutrino due to the fact that the neutrino is written in the *right-hand-side* of the synthesis,

$$p^+ + e^- \rightarrow n + \nu, \quad (81)$$

while 0.784 MeV are missing for the synthesis of the neutron itself.

Additionally, the isotopic $\hat{S}U(2)$ -spin isosymmetry established that there is no spin available for the neutrino because the total spin of the isoelectron inside the neutron must be 0. This is due to the fact that the orbital motion of the isoelectron inside the isoproton must be equal to the isoproton spin to avoid huge resisting forces and be opposite to the isoelectron sped for a stable singlet coupling (right of Figure 12).

These stability conditions confirm the original conception of the neutron synthesis by Rutherford in 1920 [88],

$$\hat{p}^+ \hat{e}^- \rightarrow n. \quad (82)$$

In summary, despite a widespread oblivion in the best Ph. D. schools of physics, the neutron synthesis is indeed the most fundamental event in nature with yet unsolved basic problems. For instance, it is generally believed that the missing energy of 0.784 MeV is provided by the

latent energy in the interior of stars. However, at the initiation of the production of light, our Sun synthesizes 10^{38} neutrons *per seconds*. In the event the missing energy is provided by the latent energy, stars would lose 10^{38} MeV per seconds, thus cooling down and not being able to produce light.

It is also believed that the missing energy in the neutron synthesis is provided by the $p^+ - e^-$ relative kinetic energy. However, the scattering cross section of protons and electrons at about 1MeV is so small, 10^{-20} Barns, to prohibit any synthesis.

Independently from the above aspects, in supernova explosions we have the emission of many times the entire energy released by the Sun in its entire life of 10 billion years. Calculations have shown that the emission of such enormous amount of energy in one single explosion cannot be consistently represented via the sole admission of quantum mechanical processes.

In view of the above, paper [89] of 2007 suggested that the missing energy in the neutron synthesis is provided by the *ether* as a universal substratum with extremely high energy density (to propagate light at $300K$ km/s) via a longitudinal, spin 0 impulse called *etherino*, (denoted with the letter "a" from the Latin *aether*) and placed in the *left-hand-side* of the synthesis,

$$p^+ + a + e^- \rightarrow n. \quad (83)$$

It appears that the experimental data of *indirect events* predicted by the neutrino hypothesis can be numerically interpreted via the etherino hypothesis, of course, jointly with corresponding reinterpretations in the standard model for its sole use for the classification of particles.

The historical criticism against the ether that it would violate special relativity has been long dismissed because we would never be able to identify a reference frame at rest with the ether. The additional historical criticism against the ether as a physical medium given by the *aethereal wind* that should be experienced by moving masses, has also been de-

bunked long time ago (see the 1956 paper [90]) because, from the known law $E = h\nu$, the electron is characterized by oscillations with the frequency of

$$Hz_e = 1.25 \times 10^{20} Hz. \quad (84)$$

The idea that the above oscillations are those of a tiny mass inside the electron has no scientific credibility. Hence, the sole known plausible hypothesis is that *the electron is characterized by oscillations of one point of the ether with frequency (84)*. When the electron moves, said oscillations are transferred to different points of the ether with no possible “aethereal wind,” and the same holds for all particles and, therefore, for matter. Inertia is the resistance by the ether against changes in the transfer speed of the oscillations to new points.

Note that, far from being pure philosophical considerations, the problem of the structure of the electron is crucial to achieve an understanding of the mechanism according to which two identical valence electrons bond themselves in molecular structures against their huge Coulomb repulsion (see Section 8.3.1).

Note also that the *transverse* character of electromagnetic waves (oscillations perpendicular to the direction of motion) mandates that the ether should have a “rigid-type” medium, while, by contrast, a “fluid-type” structure of the ether would be irreconcilable with the transverse characteristics of light.

To illustrate the implications of the problem here considered, the above view on the ether would imply that *matter is completely empty and space is completely full* to such an extent that, in case we could stop time, the entire universe would disappear.

Also, the etherino hypothesis provides a concrete example of the historical hypothesis of the *continuous creation of matter in the universe*, occurring not only for the neutron synthesis in the core of stars, but also for supernova explosions and other possible events.

Recall that, according to available measurements, the Sun loses via

radiation the rather sizable amount of

$$\Delta M_{sun}^{loss} = -4.26 \times 10^9 \text{ Kg per second.} \quad (85)$$

In the event the etherino hypothesis is correct, the above loss would be mostly counterbalanced by the synthesis in the Sun of 10^{38} neutrons per second implying an *increase* of its mass by about [163]

$$\Delta M_{sun}^{gain} = +2.87 \times 10^8 \text{ kg per second,} \quad (86)$$

resulting in the rather small loss by the Sun of about $3f73 \text{ Kg/s}$, which is well within current errors, plus secondary contributions, such as accretion of particles during travel in intergalactic space.

Rather than rejecting the above view due to its novelty, an important scientific question is whether current knowledge in planetary trajectory do set an upper value on the possible loss of mass by the Sun per second (additional aspects are treated in Ref. [163] and in the EPR Debate [91] and Ref. [136]).

8.2.4. Industrial synthesis of neutrons from the hydrogen. Tests on the laboratory synthesis of the neutron from the hydrogen were initiated in the late 1940's with support by Albert Einstein, but the related papers were never accepted for publication by scientific journals of the time. During the last of these initial tests, don Carlo Borghi detected nuclear transmutations apparently caused by particles he called *neutroids* emitted by the apparatus without the direct detection of synthesized neutrons (see the historical account [92]).

Following, and only following, the theoretical understanding of the neutron synthesis via hadronic mechanics (Section 8.2.2.), systematic tests on the laboratory synthesis of the neutron were initiated in the early 2000's. The first actual detection of the emission of neutrons synthesized from a hydrogen gas were presented in paper [93] of 2007, with the repeated detection of synthesized neutrons as well as the confirmation of

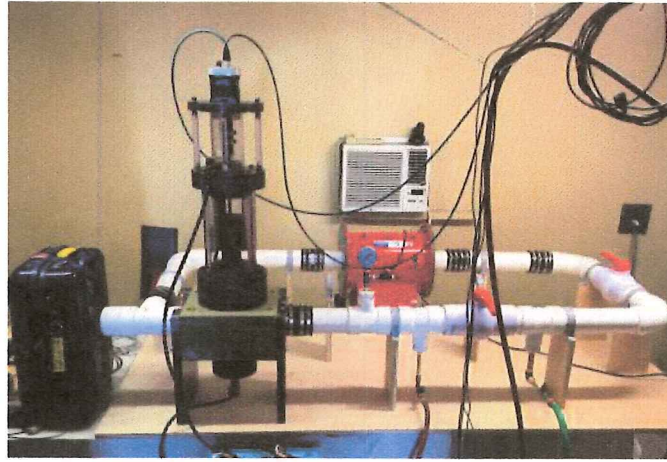


Figure 13: A view of the *Directional Neutron Source (DNS)* described in Section 8.2.4 which synthesizes on demand neutrons from the hydrogen in the desired direction, flux and energy.

the synthesis of don Borghi's neutroids via their stimulated nuclear transmutations (see paper [214] for a recent review).

The neutron synthesis was achieved via the use of a special electric arc submerged within a hydrogen gas which: ionizes the gas, synthesizes the neutroid and then synthesizes the neutron by "compressing" the isoelectron inside the isoproton much according Rutherford's original conception [88].

These tests led to the formation of the U. S. publicly traded company *Thunder Energies Corporation* (now *Hadronic Technologies Corporation*) for the manufacturing and sale of equipment called *Directional Neutron Source* (DNS) which produces on demand a neutron beam synthesized from the hydrogen gas in the desired direction, flux and energy (see Figure 13, the latest systematic tests [94] and the lectures by S. Beghella Bartoli in the 2020 Teleconference [0]).

Subject to appropriate funding, the DNS is intended to: 1) Provide the clear detection of nuclear weapons or materials smuggled in luggage; 2) Detect precious metals in mining operation; 3) Test weldings of thick plates in naval constructions; and other applications.

8.2.5. Representation of Deuteron data. As indicated in Section 3.2, a second important verification of the EPR argument in nuclear physics is the inability by quantum mechanics to represent the Deuteron magnetic moment, spin, stability and other data *for isolated Deuterons in their ground state* (because representations in excited orbits exist but they do not represent the physical reality).

The representation of all data of the neutron in its synthesis from an isoelectron and an isoproton allowed a new structure model of the Deuteron as a hadronic bound state of one isoelectron and two isoprotons

$$D = (\hat{p}_{\downarrow}^+, \hat{e}_{\uparrow}^-, \hat{p}_{\downarrow}^+)_{hm}, \quad (87)$$

first presented in Ref. [26], Section 2.5, page 181 on, continued in various works (see the references of paper [212]), and completed in Ref. [214].

These studies allowed the representation of all experimental data of the Deuteron *in its true ground state* as follows:

1) The representation of the total spin $J = 1$ of the deuteron by apparently confirming that *the spin 1 suggests the Deuteron to be a three-body bound state* according to the structure (87). Note that the sole spin representable via quantum mechanics for a *stable* bound state of a proton and a neutron *in the ground state* is the singlet

$$D = (p_{\uparrow}^+, n_{\downarrow}^0)_{qm}, \quad (88)$$

for which $J = 0$;

2) The representation of the anomalous magnetic moment of the Deuteron thanks to the anomalous magnetic of the neutron (Section 8.2.2.).

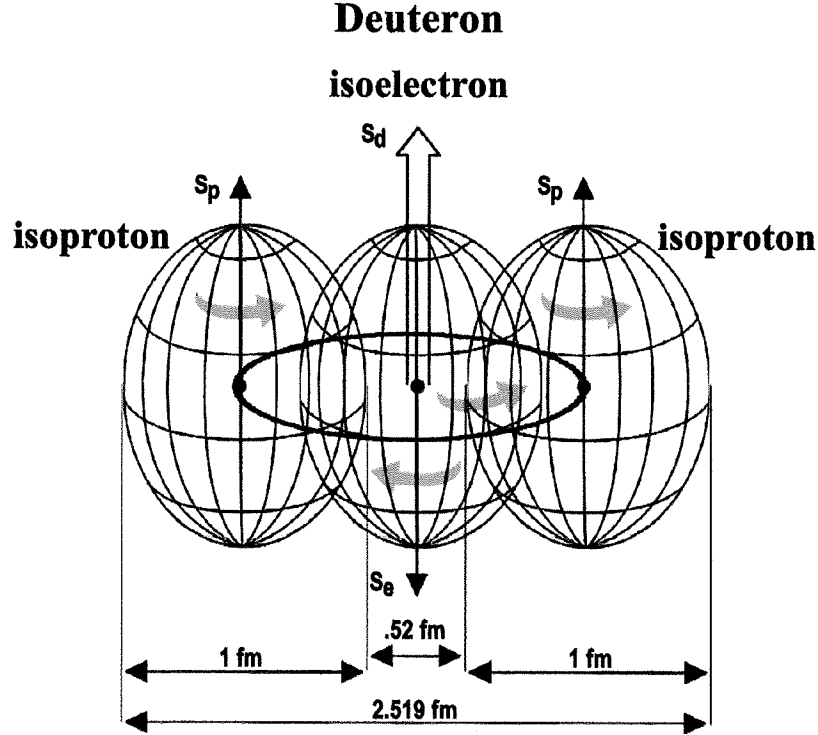


Figure 14: An illustration of the structure of the Deuteron following the neutron synthesis according to hadronic mechanics (Figure 12), composed by two isoprotons with parallel spin $1/2$ and one interconnecting isoelectron in singlet coupling with the isoprotons and orbital momentum inside the isoprotons constrained with total angular momentum 0 (Section 8.2.2). These features imply a very stable, gear-type, three-body configuration, Eq. (87), which has allowed the first known representation of all Deuteron data in its ground state, including magnetic moment, spin, stability, and other features [26] [214].

3) The representation of the stability of the Deuteron despite the natural instability of the neutron because isoprotons and isoelectrons are permanently stable particles.

Section 7 of paper [214] presents an upgrade of the Deuteron model of Ref. [26] with a more accurate representation of the magnetic moment and other nuclear data, thanks to the first known *explicit representation of strong nuclear interactions with isotopic element (38)(39)*.

Paper [95] of 2016 used the above results to achieve a representation of the magnetic moment and spin of all stable nuclei.

8.2.6. Reduction of matter to isoprotons and isoelectrons. The preceding results, with particular reference to the reduction of the neutron to a hadronic bound state of an isoproton and an isoelectron, as well as the ensuing representation of nuclear data in their ground state, imply the reduction of all matter in the universe to isoprotons and isoelectrons, including protons and electrons as a particular case [26]. Note that neutron stars are equally reduced to isoprotons and isoelectrons.

8.2.7. The synthesis of the pseudo-proton. Following the synthesis of the neutron via the compression of an electron within the hyperdense proton, by following knowledge established from the neutron synthesis (Figure 12), the Directional Neutron Source (DNS) of Section 8.2.4, progressively synthesizes, in a statistical lesser amount, *two negatively charged strongly interacting particles*. The first is given by a hadronic singlet coupling of an electron and a neutron called *protoid* and denoted $\bar{p}_1^-$ [96] with spin 0, mass essentially that of the neutron and predicted mean-life of about 5 s (left of Figure 12) whose existence is predicted from the fact that non-Hamiltonian interactions caused by deep EPR entanglement according to structure equation (61) are insensitive to charges. The second particle is given by the compression of the electron, this time, inside the hyperdense neutron, called *pseudo-proton* $\bar{p}_2^-$ [96] (Figure 15) with

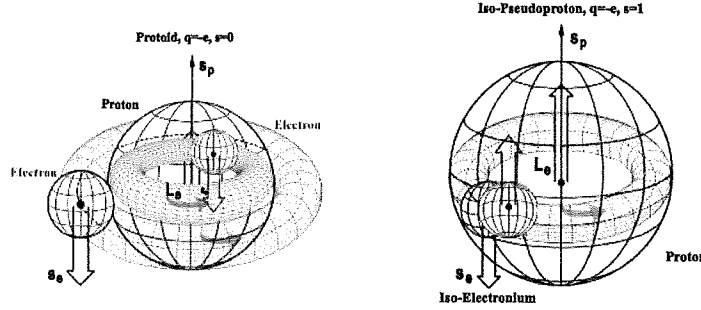


Figure 15: An illustration of two negatively charged strongly interacting particles predicted by the EPR completion of quantum into hadronic mechanics with mean-lives of the order of seconds and mass close to that of the neutron: the protoid (on the left) with spin 0, which is a hadronic singlet state of an electron and a neutron, and the pseudo-proton (on the right) which occurs when the electron is totally compressed inside the neutron with spin 1 (Section 8.7).

spin 1, mass essentially that of the neutron and mean-life of the order of 7 s (see also [214] for a recent review).

In regard to the spin of the pseudo-proton, we recall that electrons couple in singlet. Therefore, when the external electron is compressed inside the neutron, it couples in singlet with the pre-existing internal electron, resulting in a singlet electron pair with total spin 0 which is *constrained* to rotate by the hyperdense medium inside the neutron with the neutron spin 1/2, resulting in the total spin 1. Different models are faced with the extreme resistance that would be experienced by an extended wavepacket to rotate inside and against the motion of the hyperdense neutron medium.

The emerging new technology, called *pseudo-proton irradiation* is made possible by the fact that, even though the pseudo-proton is evidently unstable, its mean-life is nevertheless of the order of a few seconds (a frac-

tion of the 900 s mean life of the neutron), thus being suitable for applications.

On industrial grounds, by noting that *pseudo-protons are strongly attracted by nuclei*, pseudoproton irradiation is significant to study new clean nuclear energies, recycling of nuclear waste, and other intriguing applications studied in Section 8.2.10.

On medical grounds, pseudo-proton irradiation is under consideration for cancer treatment via a localized low energy beam in replacement of proton irradiation due to its excessive invasive character caused by the high energies needed to overcome proton-nucleus repulsion.

It should be noted that the existence of the pseudo-proton, let alone its long mean life for particle standards, is impossible without the EPR completion of quantum into hadronic mechanics.

8.2.8. The synthesis of the pseudo-Deuteron. The Directional Neutron Source of Section 8.2.5 synthesizes neutrons and pseudo-protons (as well as their intermediate states) when filled up with a commercial grade hydrogen gas.

When filled up with Deuterium gas, the same DNS is predicted to synthesize a *negatively charged nucleus* called the *pseudo-Deuteron* [96] [214] with the structure

$$\hat{D} = [\hat{p}_{\uparrow}^+, (\hat{e}_{\downarrow}^-, \hat{e}_{\uparrow}^-), \hat{n}_{\uparrow}^0]_{hm}, \quad (89)$$

namely, via the compression of an electron pair $(\hat{e}_{\downarrow}^-, \hat{e}_{\uparrow}^-)$ inside the Deuteron $D = (p_{\uparrow}^+, n_{\uparrow}^0)$ resulting in a new nucleus with spin 1 that (in nuclear symbols A = atomic number, Z = charge and J = spin) is denoted $\hat{D}(-1, 2, 1)$ with evident decay $\hat{D} \rightarrow D + 2\beta$.

To understand the above synthesis, let us recall that the valence electrons of the deuterium molecule are bonded into the valence pair $(\hat{e}_{\downarrow}^-, \hat{e}_{\uparrow}^-)$ called *isoelectronium* [30] (Section 8.3.1) with charge $2e$, total spin 0, total angular momentum 0 and total magnetic moment 0.

Under sufficient power, a submerged DC arc separates Deuterium

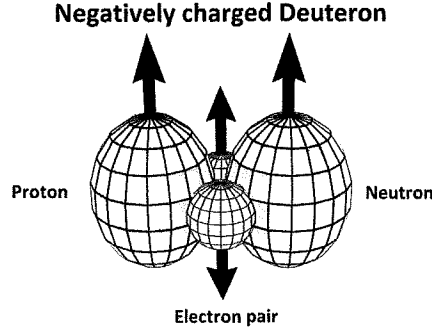


Figure 16: *An illustration of the negatively charged Deuteron, called pseudo-Deuteron denoted $\hat{D}(-1, 2, 1)$, according to structure (89), which is given by the compression within a Deuterium gas of a valence electron pair (known as isoelectronium) inside a natural Deuteron via a sufficiently powerful DC electric arc.*

molecules by forming a plasma mostly composed by protons and valence pairs due to their strong bond (Section 8.1). Next we should recall the *very big Coulomb attraction between valence electron pairs and natural Deuteron* according to Eq. (6) which prepares synthesis (89) by forming an intermediate state called *Deuteroid* [96]. The same DC arc then compresses the valence pair inside the Deuteron according to the configuration of Eq. (89) and Figure 16.

This process creates a negatively charged nucleus which is evidently unstable, but its main-life (computed via hadronic mechanics) is of the order of *seconds*, thus allowing industrial applications.

Recall that the structure $D = (p_{\uparrow}^+, n_{\uparrow}^0)$ is *unstable* due to the triplet coupling of the proton and the neutron. Therefore, the compression of the electron pair $(\hat{e}_{\downarrow}^-, \hat{e}_{\uparrow}^-)$ inside the Deuteron *stabilizes* the structure due to the physical separation of the proton and the neutron.

Needless to say, the plasma created by the submerged DC electric arc

also contains isolated electrons that are also very strongly attracted by Deuterons, and can form other hadronic bound states with smaller statistical probabilities and mean-lives [96].

Again, the pseudo-Deuteron is unstable, but again its mean life is estimated to be of the order of seconds, thus being suitable for applications.

8.2.9. New clean nuclear energies. In this section, we provide a conceptual outline of new nuclear energies without harmful radiations called *hadronic energies* originally proposed in paper [97] of 1994, which are not possible for quantum mechanics, but they are indeed possible for the completion of quantum into hadronic mechanics [23]-[25] according to the Einstein-Podolsky-Rosen argument [1] following the exact representation of nuclear spin, magnetic moments, stability and other data presented in the preceding sections.

The *existence* of the new energies in parts per million (ppm) has been experimentally verified by various independent laboratories thanks to funds provided by Magnegas Corporation (now Taronis Corporation), but no funds could be located for their development up to the level of *industrial* production of new nuclear energies, which development is therefore left to interested scientists and governmental officers.

Among the variety of hadronic energies predicted in Ref. [97] (thanks to the Lie-admissible completion of quantum mechanics for irreversible processes) we outline the following three clean energies (see Refs. [25] and [98] for reviews):

8.2.9-I: Nuclear energies via stimulated neutron decays. Incontrovertible experimental evidence establishes that the neutron is synthesized from the proton and the electron and decays into the proton and the electron. Hadronic mechanics has permitted the representation of *all* characteristics of the neutron at both non-relativistic and relativistic levels (Section 8.2.3) when assumed to be a hadronic bound state of an isoproton $\hat{p}^+$ with conventional rest energy of 938.272 MeV and an iso-

electron $\hat{e}^-$ with isorenormalized rest energy of 1.295 MeV , Eq.(80) due to its immersion within the hyperdense medium inside the proton with density (79).

In view of the above data, Ref. [97] submitted the hypothesis that *when the member of a suitably selected, light, natural, and stable nucleus, the neutron can be stimulated to decay via a resonating frequency which is an integer multiple or submultiple of $\hat{\gamma} = 1.295 \text{ MeV}$ corresponding to the resonating frequency (see Eq. (3.6), page 326, Ref. [87])*

$$\nu_{\text{reson}} = 3.1289 \times n \times 10^{20} \text{ Hz}, \quad n = 1, 2, 3, \dots \quad (90)$$

or

$$\nu_{\text{reson}} = 3.129/n \times 10^{20} \text{ Hz}. \quad (91)$$

By recalling that $1.295 \text{ MeV} = 0.0014 \text{ amu}$, and by ignoring integer multiples or submultiples for a first analysis, the indicated hypothesis can be written

$$\begin{aligned} & \hat{\gamma}(0, 0, 1, 0.0014 \text{ amu}) + N(Z, A, J, \text{amu}) \rightarrow \\ & \rightarrow \hat{N}(Z + 1, A, J + 1, \text{amu}) + \hat{e}^-(-1, 0, 0, 0.0014 \text{ amu}). \end{aligned} \quad (92)$$

Note that the energy supplied by the resonating photon is reemitted by the isoelectron which has the rapid spontaneous decay in vacuum

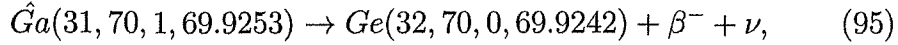
$$\hat{e}^- \rightarrow e^- + \nu \text{ (or } a), \quad (93)$$

where the etherino alternative a is outlined in Section 8.2.4.

Among various possible hadronic energies, Ref.[97] suggested in page 340 the test of the stimulated decay of a neutron of $30 - Z_n - 70$ according to the reaction (see Ref.[99] for nuclear data)

$$\begin{aligned} & \hat{\gamma} + Zn(30, 70, 0, 69.9253) \rightarrow \\ & \rightarrow \hat{Ga}(31, 70, 1, 69.9253) + \hat{\beta}^-, \end{aligned} \quad (94)$$

where $\hat{Ga}$ is a highly unstable pseudo- nuclide due to the missing energy of 0.0007 amu to have the tabulated mass for $31 - Ga - 70$ of 69.9260 amu , with spontaneous decay



where $32 - Ge - 70$ is a stable light natural element. Note that the indicated transmutation triggered by the stimulated neutron decay provides *two* different new clean hadronic energies without the emission of harmful radiation or release of radioactive waste: 1) Heat produced in the amount of $0.0011 \text{ amu} = 1.024 \text{ MeV}$ per reaction, and 2) Electricity generated by the production of two electrons per reaction which can be easily trapped via a thin metal shield to form a *nuclear battery* (see Sections 4-I and 4-II of Ref. [97] for details.).

The stimulated double beta decay was tested in Ref. [100] (see also the subsequent presentation [121]) with preliminary positive results reported in Ref. [87].

The tests can nowadays be done by irradiating a plate of the selected isotope (e.g., $30 - Zn - 70$) with resonating frequency available from radioactive isotopes for a few days, and then have comparative mass spectroscopic analyses of the untreated and the treated plate to ascertain the possible presence in the treated plate of the predicted new isotope $32 - Ge - 70$ in ppm.

The direct costs for the repetition of the tests are approximately given by: 1) \$1,200 for a sample of the needed pure $30 - Zn - 70$ isotope with certified content; 2) \$2,200 for the purchase of the needed radioactive isotope; and 3) \$800 for mass spectroscopic analyses, for the total direct cost of 4,200.

It should be noted that there exist a considerable amount of scientific literature on double beta decays (which is easily identifiable via an internet search), although none of them, to our knowledge, considers the *stimulated* double beta decay proposed in Ref. [97] of 1994 and tested in

Ref. [100] of 1996. The study of the triggering of known double beta decays with resonating photons to search for new clean energies is left to interested colleagues.

8.2.9-II: Intermediate Controlled Nuclear Fusions. It is generally believed in mainstream physics circles that *nuclear fusions cannot occur without the emission of neutrons*. It is important for our environment to disregard such a view because generally based on the dismissal of the insufficiencies of quantum mechanics in nuclear physics (Section 3.2), and expressed in oblivion of Einstein's view on the limitation of quantum mechanics. This section is devoted to physicists interested in testing nuclear fusions without the emission of neutrons, as predicted by the completion of quantum into hadronic mechanics according to the EPE argument.

Additionally, this section is intended for physicists interested in verifying or dismissing in refereed scientific publications that the sole scientific, i.e., quantitative interpretation of thunder is that via nuclear fusions of light, natural and stable elements triggered by lightning without the emission of neutrons.

As it is well known, the *existence* of nuclear fusions (also called syntheses) at low energies has been established by the so-called *cold fusions*, but the energy available turned out to be insufficient to power all engineering means needed for industrial production of clean energy. The existence of nuclear fusions has also been established by the so-called *hot fusions*, but their energies and temperatures are so big to create uncontrollable instabilities at the time of the initiation of the fusion process.

Despite the investment of billions of dollars of public funds over half a century, cold and hot fusions have been unable to achieve the much needed, industrially viable, clean nuclear energies.

The apparent primary reason is that all research in the field was based on quantum mechanics in oblivion of the EPR argument [1] as well as

of the known impossibility for quantum mechanics to provide an exact representation of nuclear data, as well as the impossibility to provide a correct representation of energy releasing processes due to their time irreversibility (Section 3.3).

In view of the above occurrence, systematic research was initiated via funds provided by MagneGas Corporation in the search of new clean nuclear energies based on the completion of quantum mechanics into the irreversible, Lie-admissible branch of hadronic mechanics. This research can be briefly outlined as follows.

Following, and only following the achievement of sufficient mathematical [23] and physical [24] maturity on the Lie-admissible branch of hadronic mechanics (see also lecture [101]), a study was conducted on the formulation of the *new* physical laws of nuclear fusions according to hadronic mechanics, which laws were presented in the 2008 paper [102] to characterize the new *Intermediate Controlled Nuclear Fusions* (ICNF), namely, nuclear fusions with energy intermediate between the cold and hot fusions.

The main differences between quantum fusions and ICNF are: 1) Nuclei are massive points for quantum fusions, while nuclei are extended for ICNF; 2) Quantum fusions solely admit action-at-a-distance potential interactions, while ICNF are primarily based on contact non-potential interactions; and 3) Quantum fusions are solely based on physical processes, while ICNF are based on physical as well as certain crucial chemical processes indicated in Section 8.3, thus requiring both hadronic mechanics and chemistry (see the review in Section 3.25 of Ref. [25]).

A hadronic reactor for the test of ICNF was built in 2009 comprising:

- 1) A metal vessel housing in its interior a pair of Carbon electrodes whose gap is electronically controlled from the outside;
- 2) A selected light, natural and stable gas called *hadronic fuel* which is stored at pressure in the metal vessel;
- 3) A DC arc between a pair of electrodes submerged within the hadronic



Figure 17: *A view of the hadronic reactor used for the Intermediate Controlled Nuclear Fusion (ICNF) of the Nitrogen from Deuterium and Carbon without the emission of neutronic or other harmful radiations first achieved in Ref. [103] of 2010.*

fuel powered by a commercially available 50 KW DC welder produced by Miller Electric corporation which delivers up to 1K A in the arc between the submerged electrodes with about 7 mm gap;

4) Means for the control of the internal nuclear fusions including the control of the DC power, gas pressure, electrode gap, and other engineering means;

5) The hadronic reactor is then completed with a number of peripheral equipment, such as: a variety of radiation detectors with alarms preset at minimal reading for safety; vacuum and pressure pumps; vacuum and pressure gages; internal and external temperature gages; electric panel for the monitoring and control of the pressure, temperature, radiation; and other equipment (see Figure 19).

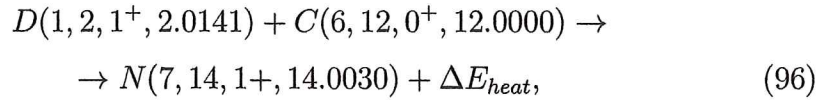
Samples of the gaseous hadronic fuel are taken before and after the activation of the reactor whose operation is limited to a few minutes for evident safety due to a generally rapid increase of the temperature. ICNF occur when laboratory analyses signed by the laboratory director estab-



Figure 18: A view of the team of scientists who confirmed the ICNF of the Deuterium and Carbon into the Nitrogen without the emission of harmful radiations and without the release of radioactive waste [100]-[112].

lish the presence of new light, natural and stable elements at least in ppm, under the condition that said new elements do not exist in the original hadronic fuel.

Following extensive tests, Ref. [103] of 2010 announced the ICNF in ppm of the Nitrogen from the Carbon of the electrodes and a commercial grade Deuterium gas as hadronic fuel (Figure 17) according to the ICNF



confirmed by chemical analyses [104]-[107] signed by the laboratory director, where the released heat is given by

$$\Delta E_{heat} = 10.2231 \text{ MeV} = 1.57 \times 10^{-15} \text{ BTU} \quad (97)$$

per fusion.

It should be stressed that the indicated Nitrogen synthesis can only occur without any production of neutron or other harmful radiation or release of radioactive waste wince it deals with the synthesis

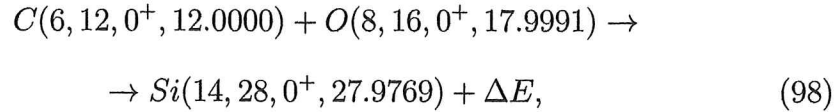


Figure 19: A view of the third hadronic reactor for the ICNF of the Carbon and Oxygen into the silicon successfully tested in 2011 [109].

of two light, natural and stable elements into a third light, natural and stable element with a smaller mass, as well as due to the limited power of the DC source (50 KW) which is clearly insufficient to crack nuclei for the production of the popularly expected neutrons. A recording of the sound of the hadronic reactor during operation is available from Ref. [108].

The above results were all confirmed by systematic tests and verifications [109] - [112] conducted by an independent team of scientists (Figure 18).

Among a number of additional hadronic reactors build for the test of ICNF, we mention the third hadronic reactor (Figure 16) built in 2011 for a Chinese client for the ICNF of Carbon and Oxygen into the Silicon,



were

$$\delta E = 2.0222 \text{ amu} = 1,882.6682 \text{ MeV} = 2.8604 \times 10^{-13} \text{ BTU/reaction}, \tag{99}$$

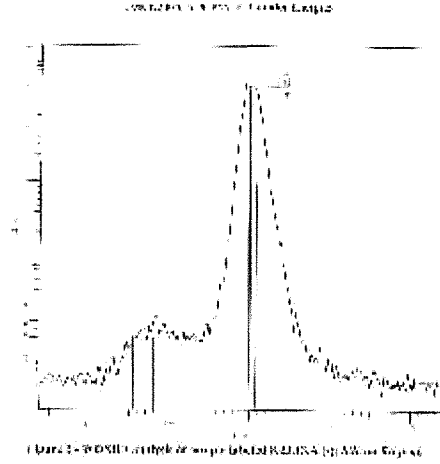
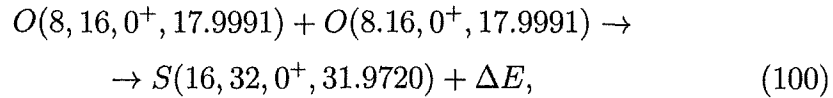


Figure 20: A view of one of the analyses [116]-[118] signed by laboratory directors showing the pick of the Silicon synthesized from Carbon and Oxygen.

the ICNF



where

$$\begin{aligned} \Delta E &= 4.0262 \text{ amu} = 3,7483,3922 \text{ MeV} = \\ &= 5.6950 \times 10^{-13} \text{ BTU per synthesis,} \end{aligned} \quad (101)$$

and other ICNF that were announced in video [113] and paper [114] of 2011 (see also lecture [115]) following systematic analyses [116] - [120] also signed by the laboratory director (see the sample analysis of Figure 20).

It should be stressed that the above tests were primarily done to establish the *existence* and *control* of ICNF in parts per millions under the

expectation that such an evidence was sufficient for funding the construction of industrial hadronic reactors, by keeping in mind that the efficiency of the reactors was limited by the low operating pressure, the low power of the DC generator and other reasons.

The rather voluminous amount of information collected on ICNF indicates that the achievement of a an *industrial* hadronic reactor capable of producing truly controlled and radiation free nuclear energy is indeed within current technological reach via the use of: a specially built, 100 KW DC power unit of the particular type used for the synthesis of the neutron (Section 8.2.3); the use of bigger pressures; and other engineering means. To see it, recall that the Nitrogen synthesis of Ref. [103] produces $1.57 \times 10^{-15} BTU \text{ per fusion}$. The plasma surrounding a '1 cm DC arc contains about 10^{30} atoms. The extrapolation of available data indicates the realistic possibility of producing of 10^{21} fusions *per hour* with the ensuing delivery of $10^6 BTU/h$ which are fully sufficient to generate the needed electric power plus the delivery of clean energy of nuclear origin. Alternatively, as shown in video [113], the third hadronic reactor for the synthesis of the Silicon was already producing electricity when operated at 1,000 *psi* pressure under 50 KW power. The production of free, clean, nuclear energy when operated at 5,000 *psi* pressure with 100 KW DC power is rather plausible.

A study of ICNF presented at the Teleconference is available in Ref. [219] and in the lecture recorded by I. B. Das Sarma [0].

It is regrettable for the environment that billions of dollars of public funds have been and continue to be invested in conventional nuclear fusions despite known insufficiencies of quantum mechanics, while it has not been possible to secure until now comparatively small funds for the test of the indicated ICNF.

8.2.9-III. HyperFusions of natural nuclei and pseudo-nuclei. The biggest problem that has prevented the achievement to date

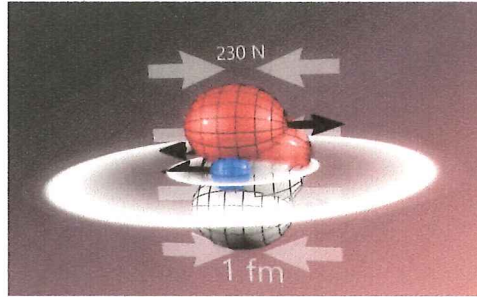


Figure 21: *A first serious environmental problem facing mankind is the lack of achievement of 'new' clean nuclear energies (Section 8.2.9). In this picture, we illustrate the new HyperFusion without harmful radiation between natural Deuterons and synthesized negatively charged pseudo-Deuterons solely permitted by the EPR completion of quantum into hadronic mechanics.*

of controlled nuclear fusions is the *Coulomb barrier*, namely, the *repulsive Coulomb force* between natural nuclei caused by their positive charge which, from Eq. (6), has the extremely big value for nuclear standards of hundreds of Newtons. In this section, we report the research done to date on a basically new type of nuclear fusion, called *HyperFusion* occurring between natural, positively charged nuclei and synthesized negatively charged nuclei, by therefore turning the repulsive Coulomb force into an attraction [96] [214].

The principle of HyperFusion is elementary. Recall from Section 8.2.2 that at 10^{-13} cm the electron and the proton experience the extremely big *attractive Coulomb force* of 230 Newtons. Nevertheless, quantum mechanics admits no bound state between the electron and the proton at short distance, by therefore confirming Einstein's view on the lack of completeness of quantum mechanics beyond scientific doubts [1]. The completion of quantum into hadronic mechanics then allowed the identification of all engineering needs for the synthesis of the electron and the

roton into an unstable particle with 900 seconds mean life, the neutron.

The principle of the new HyperFusion is essentially the same. At 10^{13} cm mutual distance, electrons are attracted by nuclei with a Coulomb force of hundreds of Newtons. The technology established for the Rutherford compression of the electron inside the proton is also applicable to the compression of electrons inside nuclei, resulting in the synthesis of negatively-charged nuclei, called *pseudo-nuclei* [214] that are evidently unstable, but possess nevertheless mean lives of the order of seconds, thus being usable for industrial applications. HyperFusion is then completed by established technologies for the controlled separation of pseudo-nuclei from their originating plasma, and controllable engineering means for their natural attraction by natural nuclei, activation at contact of strongly attractive nuclear forces, and then inevitable HyperFusion.

As indicated in Section 8.2.4, when filled up with hydrogen, the Directional Neutron Source (DNS) produces neutroids, neutrons, and pseudo-protons. When the same DNS is filled up with a commercially available Deuterium gas, it produces negatively charged pseudo-Deuterons via the synthesis [96] [214]

$$D(1, 2, 1) + (e_{\downarrow}^-, e_{\uparrow}^-) \rightarrow \hat{D}(-1, 2, 1). \quad (102)$$

The existence of the pseudo-Deuteron, and its separation from the DNS plasma via standard technologies, allow the test of the HyperFusion between natural Deuterons and pseudo-Deuterons according to the reaction [96] [214]

$$\begin{aligned} D(1, 2, 1_{\uparrow}^+, 2.0141 \text{ amu}) + \hat{D}(-1, 2, 1_{\downarrow}^-, 2.0141 \text{ amu}) \rightarrow \\ \rightarrow He(2, 4, 0, 4.0026 \text{ amu}) + 2\beta^- + \Delta E, \end{aligned} \quad (103)$$

where [*loc. cit.*]

$$\Delta E = 0.0222 \text{ amu} = 20.67 \text{ MeV}. \quad (104)$$

Note that the above fusion not only eliminates the *repulsive* Coulomb force between natural nuclei, but turn the repulsion into an *attraction* due to the opposite charges, as a result of which Deuterons and pseudo-Deuterons are naturally attracted to the mutual distance of 10^{-13} cm needed to activate nuclear forces.

Recall that a controlled quantum mechanical fusion of two Deuterons into the Helium has been essentially impossible due to the need for a *controlled* singlet coupling of the two Deuterons which is needed for the conservation of the angular momentum (see the hadronic laws for nuclear fusions in Ref. [102]). By comparison, the coupling of a Deuteron and a pseudo-Deuteron is naturally singlet as expressed in reaction (103) due to their opposite charges and magnetic moments, by therefore providing a second facilitation for a controlled nuclear fusion over conventional attempts.

Note finally that the fusion $D + \hat{D} \rightarrow He + 2\beta^0$ can be initiated and stopped on demand, and can be controlled via the control of: the pressure of the Deuterium gas; the energy of the DC arc; its voltage; the flow of the Deuterium gas through the arc; and other engineering means.

8.2.10. The problem of recycling nuclear waste. As it is well known, automotive production is under reorganization in the U.S.A. and in other developed countries to replace fossil fuels operated cars with electrically operated cars. This reorganization implies a consequential, not sufficiently spoken, exponential increase of electricity produced from nuclear power plants and other sources. As a consequence, the problems of recycling highly radioactive nuclear waste and the achievement of clean combustion of fossil fuels are becoming some of the most serious environmental problems facing mankind today.

In this section we shall indicate the implications of the EPR argument for the recycling of nuclear waste, while the achievement of full combustion of fossil fuels is treated in the next section.

Due to various oppositions, it has not been possible to store radioactive nuclear waste in depositories such as that in the Yucca Mountain in the U.S.A. Consequently, nuclear waste is stored nowadays by the nuclear power plants themselves in their own facilities. Such a storage has already reached safety limits with an expected unreassuring surpassing of safety limit under the ongoing automotive reorganization from fossil fuels to electric cars.

Also, all attempts at an effective recycling of nuclear waste via quantum mechanical technologies have failed to date. Therefore, a most unreassuring aspect of the current condition is the lack of a visible search for *new* forms of waste recycling, despite the availability of large corporate and governmental funds, and the continuation of the century old oblivion of the EPR argument [1].

By recalling that any transportation of radioactive nuclear waste is prohibited by popular opposition, the indicated unreassuring condition mandates at least the *search* by responsible societies of the only viable solution, that of developing new technologies for the recycling of nuclear waste by the nuclear power plants themselves in their own facilities. Among all possible solutions, the most desirable recycling is that via the *stimulated decay of radioactive waste* in such a manner of reducing mean lives from thousands of years down to days or minutes.

It is known that such a recycling is impossible for quantum mechanics because the alteration of the mean-life of radioactive nuclei would imply a violation of the Poincaré symmetry which is at the foundation of special relativity.

In this section, we outline the following three recycling of nuclear waste proposed in Ref. [97], Section 4-III-A, page 342 on, which are made possible by the completion of quantum into hadronic mechanics:

8.2.10-I. Recycling of radioactive hospital waste. Tests conducted in 2011 at the U. S. publicly traded company MagneGas Cor-

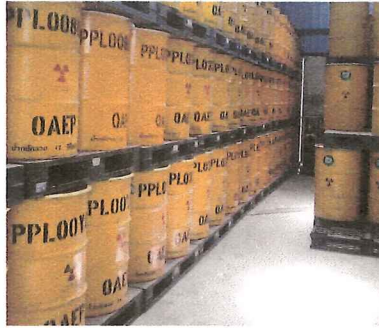


Figure 22: *A second serious environmental problem facing mankind is the recycling of radioactive nuclear waste now stored by nuclear power plants in their facilities. It is unfortunate for mankind that the century old opposition to the the EPR argument by mainstream physics generally discredits the search for stimulated recycling of nuclear waste because not permitted by quantum mechanics.*

poration have indicated that a 300 KW PlasmaArcFlow Hadronic Reactor (Figure 22) can apparently recycle lightly radioactive liquid waste from hospitals by triggering their decays via the deformation of their nuclei when exposed to intense electric and magnetic fields into such a prolate shape to cause the disintegration of naturally unstable nuclei due to internal Coulomb repulsion between aggregated protons at the extremities.

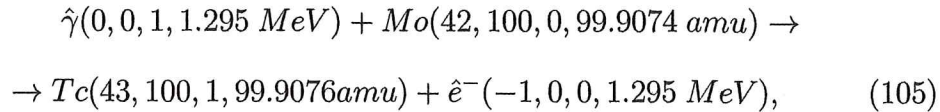
Note that such a deformation is impossible for quantum mechanics due to the representation of nuclei as massive points. By contrast, said deformations are fully possible for hadronic mechanics due to the representation of nuclei as extended and, therefore, deformable according to Eqs, (45)-(46) for which the stimulated decay of nuclear waste is reduced to the intensity of the electric and magnetic fields.

Jointly, the indicated 300 KW Plasma-Arc-Flow hadronic reactor sterilizes the liquid waste due to its exposure to the high temperature of the arc.

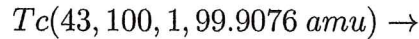


Figure 23: In this figure, we show a 300 kw PlasmaArcFlow Recycler produced and sold by Magnegas Corporation (now Taronis Corporation) which is recommended for the recycling of radioactive hospital waste, since the extreme electric and magnetic fields of the submerged electric arc predict their stimulate decay.

8.2.10-II. Recycling nuclear waste via stimulated neutron decays. A primary reason for suggesting the rather inexpensive test of the stimulated neutron decay in Section 8.2.9-I is that, in the event successful, it would allow an effective recycling of radioactive nuclear waste via the stimulated decay of some of its peripheral neutrons. This possibility is illustrated in Ref. [97], page 340, with the predicted stimulated decay of $42 - Mo - 100$, which is an unstable nuclide with mean -life of 10^{19} years. Yet, a stimulated neutron decay would imply the transmutation of $42 - Mo - 100$ into $43 - Tc - 100$ according to the reaction



by turning the mean life of 10^{19} years of $42 - Mo - 100$ into the mean-life of 18.5 seconds for $43 - Tc - 100$, with natural decay



$$\rightarrow Ru(44, 100, 0, 99.9042 \text{ amu}) + e^- + \nu \text{ (or } a), \quad (106)$$

where one should note that the final nuclide is stable and there is no emission of neutronic or other damaging radiation.

From data [99], we know that the indicated transmutation produces heat for 3.475 MeV per reaction plus electricity generated by easily trap-pable electrons.

For brevity, interested readers may inspect Ref. [97], page 342 for a possible industrial recycling of nuclear waste via a beam of resonating photons causing two or more stimulated neutron decay per nucleus.

8.2.10-III. Recycling nuclear waste via pseudo-proton irradiation. The 2020 Teleconference [0] included discussions on the possible recycling of radioactive nuclear waste via their irradiation with pseudo-protons (Section 8.2.7). In this case, unlike stimulated decay (105)-(105), we have the predicted reaction via the use of two pseudo-protoids with spin 0

$$\begin{aligned} 2\bar{p}^-(-1, 1, 01.0073 \text{ amu}) + Mo(42, 100, 0, 99.9074 \text{ amu}) \rightarrow \\ \rightarrow Zr(40, 102, 0, 101.9229 \text{ amu})n + \Delta E, \end{aligned} \quad (107)$$

in which case the 10^{19} years main-life of $40 - Zr - 102$ is reduced to the of 2.9 seconds mean-life of $40 - Zr - 102$.

Since the corporate handling of nuclear waste is prohibited by law, systematic research on the suggested stimulated recycling of radioactive nuclear waste by nuclear plants themselves is left to governmental laboratories of countries interested in developing new technologies permitted Einstein's legacy.

In closing this section, it should be indicated that mainstream physicists generally attempt to discredit the search for the new clean nuclear energies on grounds that they are not predicted by quantum mechanics, without any repetition of the numerous inexpensive tests done to date,

by ignoring the gross insufficiencies of quantum mechanics in nuclear physics (Section 3.2), and in oblivion of Einstein's view that "quantum mechanics is not a complete theory" (Figure 21).

Due to its importance for society, the problem of the recycling of nuclear waste was addressed in the opening lecture of the 2020 Teleconference by J. Dunning-Davies (see the recorded lecture [0]) and Ref. [220].

8.3. Applications of EPR completions in chemistry.

8.3.1. Representation of valence electron bonds. There is no doubt that, beginning with the 1916 pioneering contribution by G. N. Lewis, followed by numerous advances, the 20th century valence bond theory has allowed historical discoveries in chemistry.

Yet, it is well known that science at large, and chemistry in particular, will never admit final theories. Advances generally depend on the identification of open problems followed by due scientific process in the proposed solutions.

One of the best kept secrets of the best Ph. D. schools in chemistry is that there is no possibility for quantum mechanics and, therefore, for quantum chemistry, to represent the *attraction* between the *identical* electrons in valence bonds because the Schrödinger equation for an electron pair, Eq. (1), can only predict the extremely big *repulsion*, of 230 Newtons at the 10^{-13} cm mutual distance of a valence electron bond (Eq. (6) and Section 3.5).

A truly *attractive* force between identical valence electron pair was achieved in 2001 (see Chapter 4 of Ref. [30]) inspired by the last statement of the 1935 paper by Einstein, Podolsky and Rosen [1] according to which the *wavefunction* of quantum mechanics and chemistry cannot describe all "elements of reality."

As reviewed in Section 7.4, the much needed attractive force was achieved thanks to the completion of quantum into hadronic chemistry,



Figure 24: *The third serious environmental problem facing mankind is the lack of full combustion of fossil fuels. It is unfortunate for our environment that mainstream chemistry generally discredits its solution via the EPR completion of quantum chemistry in oblivion of the inability by quantum chemistry to represent the attraction between valence electron pairs and Einstein's legacy that quantum wavefunctions cannot represent all "elements of reality."*

resulting in a Hulten-type new bond, of type (61) called *strong isovalence bond*, which is so strongly attractive to generate a quasi-particle called *IsoElectronium* (IE)

$$IE = (\hat{e}_{\downarrow}^-, \hat{e}_{\uparrow}^-)_{hm}, \quad (108)$$

with double elementary charge, null spin and null magnetic moment.

In this section, we hope to indicate the unreassuring implications for our environment of molecular models without a clear attraction between valence electron pairs (Figure 19).

8.3.2. Representation of molecular data. Historical advances have been made during the 20th century in the collection and representation of molecular data. Yet, a belief in the achievement of final knowledge would imply the existing from the boundaries of science since so much remains to be understood.

Quantum mechanics allowed the representation of the experimental

data of the Hydrogen atom from first axiomatic principles with an incredible accuracy. By comparison, when two Hydrogen atoms are bonded together in the Hydrogen molecule, quantum chemistry misses 2% of the molecular binding energy from first axiomatic principles without adulterations, which percentage is far from being ignorable because it corresponds to about 950 *BTU*.

The inability by quantum chemistry to achieve an exact representation of the binding energy of the Hydrogen and other molecules is clear evidence of the appearance in valence electron bonds of short range non-linear, non-local and non-potential, thus non-Hamiltonian interactions typical of the EPR entanglement (Section 7.2.3) that are absent at the large mutual distances of the atomic structure.

In fact, the EPR completion of quantum into hadronic chemistry, and the achievement of a strong valence bond, have allowed the following advances:

1) The representation of the binding energies of the Hydrogen [31] and water [32] molecules which is *exact to the desired decimal value*.

2) Perturbative calculations used in the above results that have a convergence at least one thousand times faster than the convergence of the corresponding quantum chemical calculations, thanks to the very low value of the isotopic element $\hat{T}$ inserted in all associative products $A \hat{\times} B = A \hat{T} B$ (Section 7.5).

3) The representation of the diamagnetic character of the Hydrogen molecule which is not possible for 20th century weak valence bonds.

4) The explanation of the reason why the Hydrogen molecule can only accommodate *two* Hydrogen atoms in its stable configuration, resulting in a *restricted three-body model of the hydrogen molecule*, with the consequential, first known admission of analytic solutions that have evident importance for the environment, e.g., to achieve full combustion of fossil fuels.

5) Consequential advances in other molecules, such as the *iso-Helium*

model [122] and other advances [123].

8.3.3. The new chemical species of magnecules. Nowadays, we release in our atmosphere about *35 billion tons of contaminants per year* mostly composed by Carbon Monoxide $CO = C - O$ (where $-$ represent valence bond) and HydroCarbons HC . As it is well known, this environmental problem is caused by the incomplete combustion of fossil fuels (Figure 24). By noting that CO and HC are themselves combustible, the indicated environmental problem is caused by the lack of complete combustion of fossil fuels. In turn, this is due to a combustion insufficient for the dissociation of the valence bonds of fuels such as gasoline or diesel. Note that the dissociation of fuel molecules into their atomic constituents is a necessary pre-requisite for the chemical synthesis of CO and HC .

In view of the above unreassuring data, a considerable scientific and industrial effort was done in early 2000 to conceive, test and produce *fuels with the new chemical species of magnecules* (see Chapter 8 of Ref. [30] and the U. S. patent [123]). The new species of magnecules (to distinguish them from molecules) has essentially the same *atomic* components of fossil fuels and it is stable in pressure thanks at ambient temperatures, yet *the binding energy of magnecules is much smaller than that of molecules* as a necessary condition to achieve full combustion, namely, a combustion without detectable CO and MHC in the exhaust.

The new species of magnecules is essentially composed by clusters of atoms (such as H , C , etc.), dimers (such as $H - O$, $C - H$, etc.), and ordinary molecules (such as $H - H$, $C - O$, etc.) bonded together by the attractive force called *magnecular bond* between opposing magnetic polarities of toroidal configurations of the orbits of peripheral electrons, which magnecular bond is stable under ambient temperature.

As an illustration, by denoting the magnecular bond with the symbol

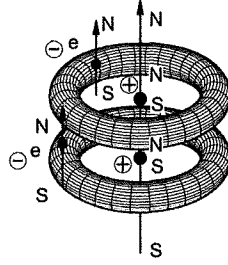


Figure 25: *An illustration of the elementary magnecule such as $H \times H$, $O \times O$, $C \times O$, etc., where one can see: the toroidal polarization of atomic orbitals; the attraction between opposing magnetic polarities; and the exposure of nuclei out of their electron cloud with the proper spin coupling, which features are essential for the ICNF of Section 9.2.9-II.*

" $\times$ ", samples of elementary magnecules are given by (see Figure 25)

$$H \times H, O \times O, C \times O, \text{etc.} \quad (109)$$

By noting that electron orbitals can be controlled under very big electric and magnetic fields [30], new fuels with magnecular structure are produced via specially designed hadronic reactors converting fossil fuels into their gaseous magnecular form via a submerged electric arc. Water is added to fossil fuels to increase the content of Hydrogen and Oxygen of the magnecular species by therefore improving its environmental qualities [124].

Stock cars produced to run on compressed natural gas, but operated on compressed magnegas release an exhaust containing no appreciable Carbon Monoxide CO or HydroCarbon HC ; and Oxygen O_2 up to 14% (see Figure 23 and the documentation presented at the 2016 International Summit on the Environment, Hainan Island, China [126]).

In view of the importance of the above features for the environment, the publicly traded company *Magnegas Corporation* was organized for

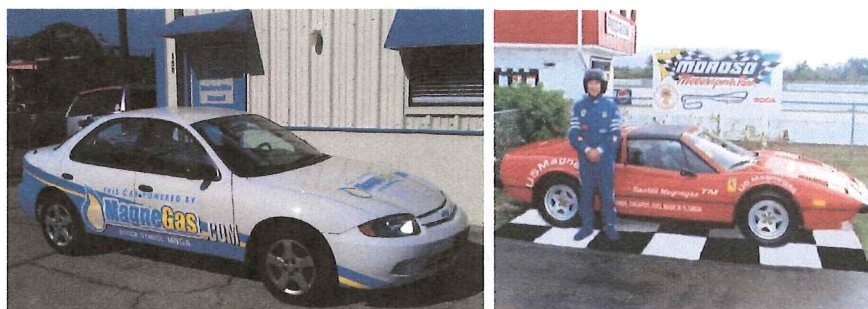


Figure 26: A view in the left of a Ford Cavalier produced to run on compressed natural gas and run on compressed magnegas with no appreciable CO and HC in the exhaust, and a view in the right of a Ferrari 308 GTS 1981 converted to run on compressed magnegas and shown at the Moroso International Race Track, Florida, to be competitive with same Ferraris running on gasoline.

the production and sale world wide of the gasification of fossil fuels into the environmentally friendly *magnegas* (chemical symbol *MG*, Figure 26) [125].

It should be indicated that *the author does not recommend the large scale use of Hydrogen as an alternative automotive fuel to gasoline because:* 1) Cars running on compressed Hydrogen are expected to produce neutrons (Section 8.2.3); 2) The large scale use of Hydrogen would cause a prohibitive Oxygen depletion; 3) Hydrogen seeps through walls and immediately raises to the Ozone Layer with ensuing rapid chemical reaction $H_2 + O_3 \rightarrow O_2 + H_2O$; and for other environmental problems. [30].

8.3.4. Experimental verifications of the new species of magnecules. The chemical composition of gases is nowadays analyzed via Gas Chromatographers Mass Spectrometers (GC-MS), InfraRed Detectors (IRD) and other equipment all designed, specifically, to detect *molecules*, that is, atoms bonded together by valence bonds with ensuing

large binding energy. In said instruments, molecules are first exposed to an ionization beam whose energy is below the molecular dissociation energy; the molecules are then processed by a reactant in the column; and then eluded with a speed inversely proportional to their masses for identification.

By noting that magnecules have a binding energy which is about 10% that of molecules, GS-MS are basically unable to detect magnecules because their ionization beams destroy the very species to be detected, since they terminate all magnecular bonds and reduce the species to conventional molecules.

In view of the above, magnecules can only be indirectly detected via GC-MS/IRD and the findings confirmed via direct measurements of the characteristics of the magnecular gas, such as flame temperature, BTU content, etc. [126].

The instruments used so far for the analysis of fuels with magnecular structure are GC-MS equipped with IRD (GC-MS/IRD) so as to test first the gas on the GC-MS used with special provisions, such as the lowest possible ionization energy, the lowest possible column temperature, and the longest possible elution time so as to minimize the destruction of the species to be detected. The clusters identified in the GC-MS are then tested via the IRD *without transferring the gas to a separate IRD because of the impossibility to combine with certainty results from different tests*. Magnecules are detected when the clusters identified in the GC-MS have no IR signature because lack of IR signature implies the lack of existence of internal molecular bonds.

The GC-MS/IRD tests that were originally done by the author in 1998 are reported in Chapters 8 and 9 of Ref.[30], Figures 8.7 on). More recent independent experimental verifications via GC-MS/IRD are available in Refs. [127] [128]. Comparison of the original tests [30] with then more recent tests [127] [128] shows that the detection of magnecular clusters is decreased in the latter tests due to the increase of the ionization energy

of the GC-MS which was confirmed by comparison of data in the relative manuals.

The best evidence establishing the existence of a *new* chemical species is given by direct measurements of the characteristics of magnegas since said characteristics cannot be explained with quantum chemistry.

The first of these anomalous characteristics is given by extensive tests conducted by scientists of the *Institute for Ultra Fast Spectroscopy and Laser of the City College of New York*, [129] [130] which established that *magnegas synthesized from fossil fuels has a flame temperature in air of 10,597 F = 3,647 C which is 56% bigger than the hydrogen flame temperature in air 2,045 C*. The increased flame temperature explains the absence of *CO* and *HC* in magnegas exhaust due to their combustion. Additionally, the increased flame temperature makes magnegas a fuel particularly suited for steel mills, refineries and other applications [124]-[126].

An intriguing feature of magnegas is its energy content because *magnegas cuts a 12" thick steel slab faster than acetylene (C₂H₂) which possesses 1,498 BTU/cf* [132]. This anomaly property is established by the fact that a conventional chemical analysis of magnegas done at maximal ionization energy and column temperature reveals that magnegas is composed by

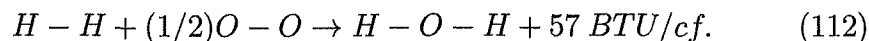
$$MG : 54\% H_2, 31\% CO, 15\% HC, \quad (110)$$

with corresponding energy content of 325, 323, 1,500 BTU/cf. Hence, according to quantum chemistry, magnegas synthesized from petroleum should contain a maximum of

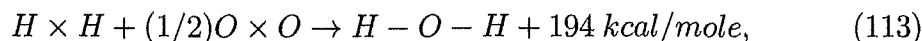
$$54\% 325 + 31\% 323 + 15\% 1,500 \text{ BTU/cf} = 431 \text{ BTU/cf}, \quad (111)$$

namely, an amount basically insufficient to cut any metal, let alone 12" thick steel plates, thus confirming that *magnegas cannot be quantitatively described via quantum chemistry*.

The description of the energy content of magnegas according to hadronic chemistry is essentially the following. Let us denote the molecular bond with the symbol "–" and the magnecular bond with "×." It is generally believed that the combustion of Hydrogen and Oxygen according to the reaction



However, the two Hydrogen atoms are separated in the $H - O - H$ molecule. Therefore, in order to verify the principle of conservation of the energy, the combustion of Hydrogen and Oxygen must produce 104 kcal/mole for the $H - H$ dissociation, plus 45 kcal/mole for the $O - O$ dissociation, plus 57 kcal/mole of produced heat, for a total of 206 kcal/mole. In the event the Hydrogen and Oxygen species have a magnecular structure $H \times H$ and $O \times O$, the binding energy is only 10% of the molecular value. In this case, the combustion of the magnecular species $H \times H$ and $O \times O$ produces the total energy of $97 + 40 + 57 \text{ kcal/mole} = 194 \text{ kcal/mole}$



which is 3.4-times the energy produced by the molecular species, Eq. (113), resulting in the value of 1,465 BTU/cf which is comparable to the energy content of acetylene 1,498 BTU/cf, by providing in this way the only known quantitative explanation of the reason magnegas cuts faster than acetylene under essentially the same atomic structure.

In conclusion, the energy content of molecular gases is constant while, by comparison, the energy content of magnecular gases is variable because it depends on the original liquid feedstock, the power of the hadronic reactor, and other engineering data.

8.3.5. Magne-Hydrogen and Magne-Oxygen. The anomalous temperature and energy content of magnegas is due to the anomalous character of its primary components known as *Magne-Hydrogen* (symbol MH), *Magne-Oxygen* (MO) and *Magne-Carbon-Monoxide* (MCO) [133].

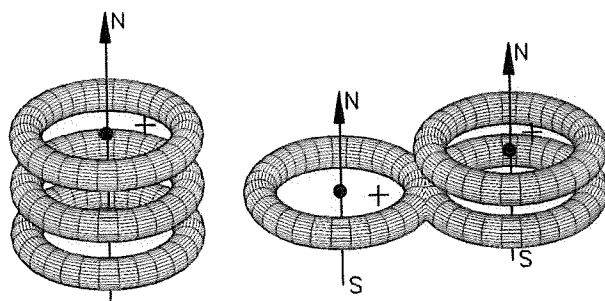


Figure 27: An illustration of the magnecule $3H = H \times H \times H$ that should be compared with the molecule $H_3 = H - H - H$. The main difference is that the latter is unstable, while the former is stable at ambient temperature according to repeated detections of 3 amu in the same MH gas. Such a difference is due to the independence of the magnecular bond from the number of bonded atoms compared to the lack of existence of a valence electron triplet.

The indicated new species have been separated from magnegas via Pressure Swing Adsorption (PSA) equipment, also called molecular sieving, and their anomalous specific weight has been confirmed by independent measurements [134]-[137]. By recalling that MH constitutes more than 50% of MG, the anomalous energy content of MH is established by that of MG. Tests on the increase of the efficiency of fuel cells via the use of MH and Oxygen are reported in Ref. [124]. The case of a 3-atomic magnecule or molecule is indicated in Figure 27.

An important experimental evidence in the above tests is that the specific weight of the magnecular species MH , MO and MCO increases with the number of times the separated gas is passed again through the PSA equipment. This is a clear indication of the effect known as *magnecular accretion*, namely, the increase of the mass of magnecule with the increase of pressure and other treatments.

In turn, the above effect indicates that *the Avogadro number is not expected to be a constant for magnecular gases* [30], which feature is suggested for test by interested chemists jointly with other innovative measurements.

An understanding of the above intriguing feature is the following. Recall from Section 3.5 that another insufficiency of quantum chemistry is the impossibility of achieving an *attractive* force between the water molecules if the liquid state due to their diamagnetic and dielectric characters. It appears that such an attractive force is of magnecular type since the Hydrogen atoms H in the water molecule $H - O - H$ has a toroid polarization of its orbit for the proper bonding to the corresponding valence electron of the Oxygen, thus allowing a magnecular attraction of the type [138]

$$H - O - H_{\downarrow} \times H^{\uparrow} - O - H^{\uparrow} \times H_{\downarrow} - O - \dots \quad (114)$$

According to this view, the magnecules of the new species MH , MO and MCO are *quasi-liquid* with consequential magnecular accretion and the predicted lack of constancy of the Avogadro number.

Chemical analyses have repeatedly established that MH is composed by atomic masses from 1 *amu* to hundreds of *amu*, thus suggesting a structure of the type

$$MH: \{H, H - H, H \times H, H \times H, \\ H - H \times H, H - H \times H, \dots\}, \quad (115)$$

with a corresponding structure for MO and MCO .

In view of its special features, including the increased specific weight, increased energy content and predicted increase of the liquefaction temperature, MH is expected to be particularly significant for the aerospace industry as well as for gasoline refineries, fertilize production and other fields. MO is expected to be significant for medical applications, e.g., for use in ventilators for persons affected by the Corona Virus because its

magnecular structure is expected to decompose at contact with lungs, with ensuing release of sterilizing UV radiation as it is the case for the decomposition of Ozone. *MCO* has been conceived and tested for the primary aim of developing the HyperCombustion indicated (Section 8.3.7).

8.3.6. The HHO combustible gas. One of the most intriguing fuels with magnecular structure is the *HHO gas* [139] which is produced in the gasification of water via a specially designed water electrolyzer developed by the U. S. company *Hydrogen Technologies Applications, Inc.*

Some of the unique features of the HHO gas, which are manifestly outside any serious representational capability by quantum chemistry, are:

- 1) The HHO gas instantly cuts bricks, tungsten and other hard material at content in air or under water;
- 2) The combustion of the HHO gas occurs without any need for atmospheric Oxygen, since HHO is composed by a stoichiometric ratio of Hydrogen and Oxygen; and
- 3) The combustion exhaust of the HHO gas are composed by water vapor without any contaminant.

By using a cautious scientific language, Ref. [139] (see also Ref. [124]) presents a series of *measurements* on the HHO gas conducted with various instruments and their tentative representation with the new chemical species of magnecules.

It should be indicated that all hadronic reactors indicated in this section have a *negative* energy balance in the sense that they require energy to produce a gaseous, environmentally friendly fuel, unless, jointly with the gasification of the liquid feedstock, the hadronic reactors synthesize new elements, such as the synthesis of $Si - 28$ from $C - 12$ and $O - 16$, in which case the energy balance is positive even for nuclear synthesis in ppm.

8.3.7. Hyper-Combustion. At the dawn of the third millennium,



Figure 28: A view of the equipment developed by Hadronic Technologies Corporation to test Hyper-Combustion in a four cylinder electric generator showing (from bottom right) Control Module, Variac and four ICNF Activation Banks (Section 3.3.7).

the combustion of fossil fuels is essentially the same as it was some fifty thousands years ago, because in our civilian, industrial and military engines we essentially strike a spark and lit the fuel.

This section is intended for scientists interested in the search for a basically new form of combustion as a necessary condition for the future achievement of a sustainable life of our planet.

Recall from Section 8.3.3 that magnegas does achieve *full combustion*, namely a combustion without appreciable combustible contaminants in the exhaust.

Based on such a result, a main drive of the physical and chemical studies reported in this section has been the achievement of full combustion, this time, for fossil fuels as commercially available, including gasoline, diesel, methane, acetylene, et al.

Recall that our combustion of about 36 *billion* barrels of crude oil per year releases in the environment about 10 billion barrels per year of con-

taminants such as CO and HC . However, it is important to note that CO and HC are themselves combustible. This establishing that *the temperature of combustion in our civilian, industrial and military engines is insufficient to achieve full combustion.*

The studies reported in this section have led to the formulation of a new form of combustion known as *Hyper-Combustion* [140] under development by the privately held *Hadronic Technologies Corporation* which can be defined as follows: *the hyper-combustion of Carbon with atmospheric Oxygen comprises the conventional chemical combustion plus a controllable number of engineering means causing the synthesis of Silicon and other light, natural and stable elements without the emission of harmful radiation and without the release of radioactive waste.*

The engineering realization of the hyper-combustion is based on the use of a specially designed DC power unit delivering an arc that: 1) Ionizes the fuel; 2) Creates magnecules $C \times O$ between toroidally polarized stable isotopes of Carbon and Oxygen; and 3) Triggers Intermediate Controlled Nuclear Fusions (ICNF, Section 8.2.9) in parts per million (ppm) of the indicated magnecule Carbon and Oxygen into the Silicon.

Recall from Nuclear Data [99] that Carbon has the following two stable isotopes with relative abundance $6-C-12$, 98.898%, $6-C-13$, 1.11% and that the Oxygen has the following three stable isotopes with relative abundance $8-O-16$, 99.76%, $8-O-17$, 0.037%, $8-O-18$, 0.200%. Consequently, when dealing with nuclear fusions in ppm, the significant isotopes are $6-C-12$ and $8-O-16$ with related nuclear fusion into the $14-Si-28$ studied in Section 9.2.9-II, Eqs. (98)-(99). This fusion has been experimentally verified with the Third Hadronic Reactor (Figures 16 and 17, video [113], sound [108], and verifications [114]-[120]).

Note that the aim of hyper-combustion *is not* that of replacing fossil fuels with controlled nuclear fusions, but that of using Carbon and Oxygen as fuels for nuclear fusions mainly intended to maximize the energy output of crude oil and achieve full combustion without harmful radia-

tions or radioactive waste.

To minimize possible misrepresentations, it should be reported that Hadronic Technologies Corporation has manufactured an equipment (Figure 28) testing Hyper-Combustion in a four cylinders electric generator with tungsten tip spark (patent pending) which comprises: 1) A control module to set maximal power and arc settings per selected engine; 2) A variac to operate with minimal and maximal power; and 3) Four ICNF Activation Banks, one per cylinder. The equipment produced the desired increase of the combustion temperature at which *CO*, *HC* and other combustible contaminants burn by producing a first increase of power output. The same equipment has produced a second increase of energy output due to the activation of ICNF in ppm of Carbon and Oxygen into Silicon within the thermodynamical limit of the electric generator.

In closing this section, it should be reported that mainstream chemists generally dismiss the *anomalous characteristics* of magnegas as being due to *anomalous measurements*, without repeating the measurements and without considering the fact that said measurements were done by the highly authoritative Institute for Ultra Fast Spectroscopy and Laser of the City College of New York [130] [131].

Additionally, the same chemists generally dismiss the existence of the new species of magnecules on grounds that the identified *anomalous clusters* can be reduced by the GC-MS to their *molecular* constituents, by ignoring that: 1) Stable clusters systematically detected by GC-MS must have an internal atomic or molecular bond to exist; 2) The ionization energy used in the decomposition of the clusters to their molecular constituents destroys the very species to be detected; and 3) The dismissal is done in oblivion of Einstein's view on the limitations of quantum mechanics and chemistry, as well as without the proof in refereed journals that quantum chemistry provides a quantitative representation of magnegas anomalous features.

Objections against magnegas and magnecules voiced in social media,

rather than via publications in refereed journals, has significantly undermined and delayed the solution to our environmental problems. In fact, as it is well known, social media plays in the hands of those who profit by shorting the stock of publicly traded companies, by therefore causing significant losses to investors eager to support the development of new environmental technologies for a sustainable future of our planet.

8.4. Applications of EPR completions to special relativity.

8.4.1. Foreword.

In this section, we show that the methods developed for the proof of the EPR argument [1] imply a *geometric unification of Einstein's special and general relativities for the exterior problem of point-like particles in vacuum, as well as their extension for interior dynamical problems of extended particles within physical media* .

8.4.2. Applications of the EPR completion to Galileo's relativity. As it is well known, the Galileo symmetry $G(3.1)$ and relativity are exactly valid for the description of non-relativistic conservative systems of point-like particles moving in vacuum (*non-relativistic exterior dynamical systems*), thus without any resistive force or contact interaction.

Since point-like particles are an approximation of the physical reality, the verifications of the EPR argument reported in Section 7 mandate the completion of Galileo symmetry and relativity for *non-relativistic interior dynamical systems* comprising extended particles in deep EPR entanglement, with ensuing resistive, as well as non-linear, non-local and non-Hamiltonian interactions (Figure 29).

The above need suggested the construction of the Lie-isotopic completion of the Galileo symmetry $\hat{G}(3.1)$ and relativity, called *Iso-Galilean isorelativity*, for the axiom-preserving representation of extended masses moving within physical media, thus experiencing resistive as well as con-

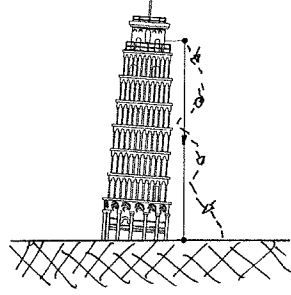


Figure 29: A schematic view of the experiments done by Galileo Galilei at the end of the 16th century to measure the acceleration due to gravity by dropping from the top of the Pisa tower balls with different masses assumed to be point-like, thus ignoring the resistance due to our atmosphere (vertical line). The axiom - preserving iso-Galilean isorelativity aims at the dynamics of extended bodies, thus including atmospheric resistance (wiggly line).

tact interactions. Note that the axiom- preserving condition restricts the system to have a conserved total energy, as it is the case for the non-relativistic description of an isolated nucleus with contact internal forces.

Since conservative systems are an evident particular case of nonconservative/irreversible systems, the studies here considered required the construction of the broader Lie-admissible completion of the Galileo symmetry $\hat{G}^>(3.1)$ and relativity for the description of extended particles in nonconservative conditions, as it is the case for the non-relativistic representation of nuclear fusions.

Since non-relativistic studies are an evident pre-requisite for relativistic counterparts, we regret not to be able to review them to avoid a prohibitive length of this Overview. Nevertheless, we indicate for interested colleagues that non-relativistic studies were initiated in paper [19] of 1978 with the Lie-admissible completion of Galileo symmetry and relativity for non-conservative and Galileo- non-invariant systems.

The first direct study of Galileo's isosymmetry was done in Section 5.3, pages 225 on, of the 1983 monograph [22] formulated over conventional fields. These isotopies were then systematically studied and upgraded in the 1991 volumes [71] [72]. The formulation of Galilean isosymmetries with the full use of isomathematics was done in the 1995 monographs [23] [24] [73].

The above studies attracted the attention of Abdus Salam, founder and President of the *International Center for Theoretical Physics* (ICTP), Trieste, Italy, who invited Santilli in 1991 to deliver at his Center a series of lectures in the isotopies of the Galileo symmetry and relativity, said invitation being apparently the last one by Salam prior to his death. During his visit at the ICTP, Santilli wrote papers [147]-[153]. The notes from Santilli's lectures at the ICTP were collected by some of the attendees and published in volume [154] of 1992.

8.4.3. Special Relativity (SR). Mainly for the sake of notation, we recall that special relativity (SR) is defined on a Minkowski space $M(x, \eta, I)$ over the field of real numbers $\mathcal{R}$ with local coordinates $x = (x, \rho)$, $\rho = 1, 2, 3, 4$, $x^4 = ct$, metric $\eta = \text{Diag.}(1, 1, 1, -1)$, unit $I = \text{Diag.}(1, 1, 1, 1)$, and spacetime interval

$$(x - y.)^2 = (x - y)^\mu \eta_{\mu\vartheta} (x - y)^\nu = (x_1 - y_1)^2 + (x_2 - y_2)^2 + (x_3 - y_3)^2 - (t_1 - t_2)^2 c^2, \quad (116)$$

which is left invariant by the Lorentz symmetry $SO(3.1)$, the Lorentz-Poincaré symmetry $P(3.1)$ and its spinorial covering $\mathcal{P}(3.1)$.

The above methods uniquely and unambiguously characterize the following basic axioms for a *time-reversal invariant relativistic system of point-like particles and electromagnetic waves propagating in vacuum*:

AXIOM I: The speed of light c is the maximal causal speed for point-like particles propagating in vacuum

$$V_{max} = c \quad (117)$$

AXIOM II: The addition of speeds follows the relativistic law

$$V_{tot} = \frac{v_1 + v_2}{1 + \frac{v_1 v_2}{c^2}}. \quad (118)$$

AXIOM III: The dilation of time, the contraction of lengths, the variation of mass and the mass-energy equivalence follow the relativistic laws:

$$t' = \gamma t, \quad (119)$$

$$\ell' = \gamma^{-1} \ell, \quad (120)$$

$$m' = \gamma m, \quad (121)$$

$$E = mc^2 \quad (122)$$

where

$$\beta = \frac{v}{c}, \quad \gamma = \frac{1}{\sqrt{1 - \beta^2}}. \quad (123)$$

AXIOM IV: The frequency shift due to relative speed follows the law (for null aberration)

$$\omega' = \gamma [1 - \beta \cos(\alpha)] \omega. \quad (124)$$

The above axioms are hereon assumed to be exactly valid for the assumed time-reversal invariant systems of point-like particles and electromagnetic waves in vacuum.

8.4.4. Special isorelativity (SIR). As a consequence of the widely assumed reduction of the entire universe to point-like particles, it is generally believed that special relativity and the constancy of the speed of light c , are valid for whatever conditions exist in the universe.

In the preceding sections we have shown that such a conception can be considered as being *approximately valid*, although it implies a number of insufficiencies, such as the inability to achieve exact representations of

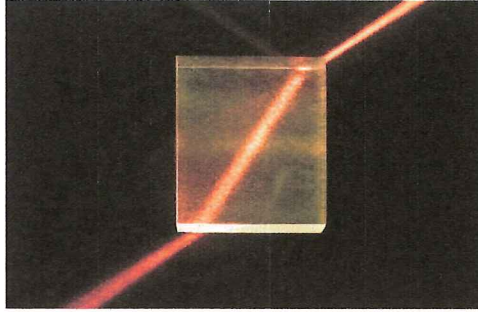


Figure 30: *The light beam passing through a glass of water depicted in this figure is generally reduced to photons for the intent of maintaining special relativity in interior conditions. However, such a reduction is afflicted by the widely propagated (yet ignored) Inconsistencies 1) to 7) of Section 8.4.1 whose resolution mandates the completion of SR for interior dynamical conditions.*

nuclear and molecular data due to the extended character of particles at a mutual distances of the order of 10^{-13} cm (relativistic interior dynamical problems).

When the reduction of the universe to point-like constituents is assumed as being *exactly valid*, it leads to serious inconsistencies that remain generally ignored by the mainstream physics community, although they need to be brought to the attention of the scholars in the field. For instance, the belief that electromagnetic waves propagating in water can be reduced to photons propagating in vacuum and scattering among the water molecules is afflicted by the following [24]:

INCONSISTENCY I: The reduction of light to photons cannot represent the angle of refraction of light in water, evidently because photons will scatter in all directions at the impact with the water surface.

INCONSISTENCY II: The reduction of light to photons cannot represent the decrease of the speed of light in water by 100,000 km/s , be-

cause calculations have shown that the scattering of photons among water molecules can at best represent a reduction of the speed of light of about 7,000 km/s .

INCONSISTENCY III: The reduction of light to photons cannot represent the propagation of light in water as a beam, again, because a beam of photons will scatter among the water molecules and disperse in water.

INCONSISTENCY IV: The reduction light to photons has no physical sense for infrared light or radio waves with 1 m wavelength that experience the same phenomenology as that for electromagnetic waves at large.

INCONSISTENCY V: The reduction of light to photons scattering among water molecules, thus propagating in vacuum at speed c , cannot represent the Cherenkov light because said light can only occur when electrons travel *faster* than the local speed of light.

INCONSISTENCY VI: The reduction of light to photons cannot resolve the inapplicability of the relativistic sum of speeds in water, Eq. (118), since the sum of two light speeds in water does not yield the light speed in water.

INCONSISTENCY VII: The reduction of light to photons cannot be tested experimentally due to the lack of an inertial frame in water.

Following the construction of the axiom-preserving isotopies of the various branches of Lie's theory in the 1983 monograph [22], Santilli constructed the axiom-preserving isotopies of special relativity for extended particles with interval (25) in Ref. [60] for the classical part and Ref. [61] for the operator counterpart, with the first known construction of the the Lorentz-Isotopic symmetry $\hat{S}\hat{O}(3.1)$, today known as the *Lorentz-Santilli isosymmetry*, with isotransformations (30).

Subsequently, Santilli conducted systematic studies [62]-[74] of the isotopies of all conventional spacetime symmetries resulting in the Lorentz-Poincaré- Santilli isosymmetry $\hat{P}(3.1)$ [67] and related isospinorial covering $\hat{P}(\cdot)$ [68] [69] which are formulated on an iso-Minkowskian isospace first introduced in Ref. [60] (see Ref. [70] for detailed studies) $\hat{M}(\hat{x}, \hat{\Gamma}, \hat{I})$ over the isofield of isoreal isonumbers $\hat{\mathcal{R}}$ with isospacetime isocoordinates $\hat{x} = x\hat{I}$, $\hat{y} = y\hat{I}$, isometric correctly written as having elements in the isofield $\hat{\mathcal{R}}$

$$\hat{\Gamma} = \{\hat{\eta}_{\mu\nu}\}\hat{I} = (\hat{T}\eta)\hat{I}, \quad (125)$$

and isounit ($\hat{I} = 1/\hat{T} > 0$ given in Eq. (34), with invariant (25), which we now rewrite in the form

$$\begin{aligned} (\hat{x} - \hat{y})^2 &= (\hat{x} - \hat{y})^\mu \hat{\times} \hat{\Gamma}_{\mu\nu} \hat{\times} (\hat{x} - \hat{y})^\nu = \\ &= [(x - y)\hat{I}]\hat{T}(\hat{\eta}\hat{I})\hat{T}[(x - y)\hat{I}] = [(x - y)^\mu \hat{\eta}_{\mu\nu} (x - y)^\nu] \hat{I} = \\ &= \left[\frac{(x_1 - y_1)^2}{n_1^2} + \frac{(x_2 - y_2)^2}{n_2^2} + \frac{(x_3 - y_3)^2}{n_3^2} - (t_1 - t_2)^2 \frac{c^2}{n_4^2} \right] \hat{I}, \end{aligned} \quad (126)$$

where: the multiplication of the interval by the isounit $\hat{I}$ is necessary for its value to be an element of the isofield; and the characteristic quantities n_ρ , $\rho = 1, 2, 3, 4$ are solely restricted by the condition of being positive-definite $n_\rho > 0$, but possess otherwise an unrestricted functional dependence on all needed local variables (which shall be hereon tacitly assumed) such as local coordinates x , velocity v , momentum p , energy E , density ρ , temperature τ , pressure γ , frequency ω , wavefunctions ψ , their derivatives $\partial_r \psi$, etc. $n_\rho = n_\rho(t, r, v, p, E, \rho, \tau, \gamma, \omega, \psi, \partial\psi, \dots)$.

A feature important for the understanding of this section is that all (non-singular) infinitely possible realizations of the iso-Minkowski isospace $\hat{M}$ on an isoreal isofield $\hat{\mathcal{R}}$ are locally isomorphic to the conventional space M over the reals $\mathcal{R}$. This property was first proved in the 1983 paper [60] (see Refs. [23] [72] for a detailed treatment) and can be seen

from the fact that the Minkowski metric η is completed into the isometric $\hat{T}\eta$ while, jointly, the basic unit I is completed by the *inverse* amount $\hat{I} = 1/\hat{T}$, thus preserving the original metric η .

Alternatively, one can see from Eq. (125) that the numeric value of the *Minkowskian* metric η is preserved under isotopies since $(\hat{T}\eta)\hat{I} \equiv \eta$.

Note the duality of the formulation, namely, the iso-Minkowski isospace can be first written on isospace over isofields (see the first line of interval (126)), in which case SR applies identically, and then *projected* on the conventional Minkowski space over a conventional field (see the third line of interval (126)) where novelties appear.

Recall that the Minkowskian geometry represents a *homogeneous and isotropic 3-dimensional space* while, by comparison, the iso-Minkowskian isogeometry represents an *inhomogeneous space*, due to the local variation of the density, as well as an *anisotropic space*, due to the change of geometry with the change of the direction.

Consequently, axioms (117)-(124) of SR do not need the identification of the selected direction, while such an identification is necessary for the SIR due to the indicated variation of physical characteristics with the variation of the space direction.

Under the above clarifications, special isorelativity can be defined as the *axiom-preserving formulation of special relativity on iso-Minkowski isospaces over isoreal isofields*. Its universal LPS isosymmetry characterizes uniquely and unambiguously the following isoaxioms first formulated systematically in Refs, [72] of 1991 on conventional fields and completed in Ref.[24] of 1995 over isofields, here expressed for the selected k -direction, e.g., that of the third space component,

ISOAXIOM I: The speed of light within (transparent) physical media is given by the locally varying speed:

$$C = \frac{c}{n_4}. \quad (127)$$

ISOAXIOM II: The maximal causal speed within physical media is given by:

$$V_{max,K} = c \frac{n_k}{n_4}. \quad (128)$$

ISOAXIOM III: The addition of speeds within physical media follows the isotopic law:

$$V_{tot} = \frac{\frac{V_{1,k}}{n_k} + \frac{c_{2,k}}{nn_k}}{1 + \frac{v_1 v_2}{c^2} \frac{n_4^2}{n_k^2}}. \quad (129)$$

ISOAXIOM IV: The isodilation of time, the isocontraction of lengths, the isovariation of mass and the mass-energy isoequivalence (isorenormalization) within physical media follow the isotopic laws:

$$t'_k = \hat{\gamma}_k t, \quad (130)$$

$$\ell'_k = \hat{\gamma}_k^{-1} \ell, \quad (131)$$

$$m'_k = \hat{\gamma}_k m, \quad (132)$$

$$\hat{E}_k = m V_{max,k}^2 = \hat{m}_k c^2, \quad \hat{m}_k = m \frac{n_k^2}{n_4^2}, \quad (133)$$

where, from Eqs. (31)

$$\hat{\beta}_k = \frac{v_3/n_k}{c_o/n_4}, \quad \hat{\gamma}_k = \frac{1}{\sqrt{1 - \hat{\beta}_k^2}}. \quad (134)$$

ISOAXIOM V: The frequency shift within physical media follows the isotopic law (for null aberration)

$$\omega' = \hat{\gamma} [1 - \hat{\beta} \cos(\hat{\alpha})] \omega. \quad (135)$$

Note that *the maximal causal speed in SIR is no longer given by the speed of light*, and it is given instead by value (128), because physical media are generally opaque to light, thus requiring the broader geometric notion derivable from the expression in $(k - 4)$ -space

$$\frac{dx_k^2}{n_k^2} - dt^2 \frac{c^2}{n_4^2} = 0. \quad (136)$$

Among a variety of verifications of Isoaxioms I to V, we indicate here the following representative examples:

8.4.4-I. Verification of SIR within liquid media. It is an instructive exercise for the interested colleague to verify that Isoaxioms I to V resolve all Inconsistencies I to 7 (Figure 30) in the use of conventional axioms.

This includes the verification via isoaxiom (118) that the sum of two local speeds of light $C = c/n_4$ yields the local speed of light $C = c/n_4$. Moreover, water can be assumed to be homogeneous and isotropic for which $n_\mu = 1$, $\mu = 1, 2, 3, 4$ for which interval (126) becomes

$$\begin{aligned} (\hat{x} - \hat{y})^{\hat{2}} &= (x - y)^2 \hat{I} = \\ &= [(x_1 - y_1)^2 + (x_2 - y_2)^2 + (x_3 - y_3)^2 - (t_1 - t_2)^2 c^2] \hat{I}, \end{aligned} \quad (137)$$

in which case the maximal causal speed *in water* is the speed of light c *in vacuum*, by therefore providing the first known quantitative representation that *electrons can indeed travel in water faster than the local speed of light*.

Additionally, the iso-Minkowskian geometry [70] provides a geometric representation of the difference between the actual size d_{act} of an object in water and that perceived by an external observer d_{ext} . For this purpose, note that in water we have the value $\hat{I} = n_4^2 = 9/4 > 1$. The use of the basic law for isotopy [23] [24] $d_{act}^2 \hat{I} = d_{ext}^2 I$, then yields the value

$$d_{ext} = n_4 d_{act} = \frac{3}{2} d_{act} \quad (138)$$



Figure 31: A view of the apparent increase of size of objects submerged in water as seen by an outside observer. This increase is quantitatively represented by Special IsoRelativity (SIR) via Eq. (138) of Section 8.4.4-I with intriguing geometric implications.

which is essentially verified by visual inspection as one can see from Figure 31.

Intriguing similar properties occur for other external and internal characteristics. For instance we have a similar distinction between the observer time called *external time* t and related unit $I_t = 1$ (which is the time of the external observer), and the *intrinsic time* $\hat{t}$ with related isounit $\hat{I}_t$ (which is the time for an internal observer). Said two times are interconnected by the isotopic law $tI_t = \hat{t}\hat{I}_t$.

The above notion of *isotime* appears to be significant for biological structures [165] because we generally assume that the time felt by a seashell $\hat{t}$ is identical to our time t , while in reality the external and internal times may be different.

8.4.4-II. Verification of SIR within gaseous media. Recall that the Doppler law in vacuum, Eq. (124), can be written in first order

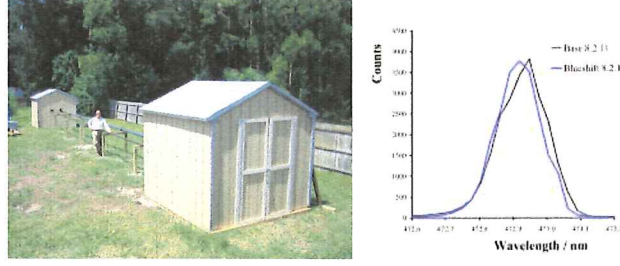


Figure 32: A view on the left of the 60'-long Isoshift Testing Station that established the energy loss (gain), thus frequency decrease (increase) without any relative motion of a blue laser light passing through air at 1000 psi and at -10^0 C ($+150^0$ C) called isoredshift (isoblueshift), which was first predicted in the 1991 Refs. [71] [72], and first detected in the 2010 paper [155] via the scan shown in the right view.

approximation

$$z = \frac{\omega'}{\omega} - 1 \approx \pm \frac{v}{c}, \quad (139)$$

where the minus (plus) sign occurs for the source moving away (toward) the observer. The corresponding expansion for the iso-Doppler law (135) yields the expression

$$\hat{z} = \frac{\omega'}{\omega} - 1 \approx \pm \frac{v_k}{c} \frac{n_4}{n_k}, \quad (140)$$

where the factor n_4/n_k can be expanded in terms of $(c/v_k)d$, d being the travel of light in the medium

$$\frac{n_4}{n_k} \approx 1 \pm S(\rho, \tau,) \frac{c}{v_k} d, \quad (141)$$

where S is a function of the local density ρ , temperature τ and possibly other local variables.

The iso-Doppler law (135) then assumes the form in first order approximation, first derived in the 1991 Refs. [71] [72],

$$\hat{z} = \frac{\omega'}{\omega} - 1 \approx \pm \frac{v}{c} \pm S(\rho, \tau, \dots)d, \quad (142)$$

which represents two independent frequency shifts, the conventional Doppler's shift $\pm v/c$ due to relative motion between observer and source, and the isoshift $\pm S(\rho, \tau, \dots)d$ due to loss (absorption) of energy $E = h\nu$ to (from) a gas. at temperatures lower (bigger) than 0 C, called *isoredshift* (*isoblueshift*), in which the prefix "iso" denotes the sole possible derivation via isomathematics.

Following a decade of failed attempts to locate a physics laboratory interested in proving or disproving the prediction of the isoshift [72], Santilli conducted systematic tests in collaboration with the technicians of *Magnegas Corporation* at its former laboratories located at 150 Rainville Rd, Tarpon Springs, Florida. An *isoshift Test Station* (left view of Figure 32) was built consisting of a front (rear) air-conditioned cabins containing a blue laser light (wavelength analyzers), the two cabins being interconnected by a 60 ft long tube containing air at 1,000 psi. The air temperature was varied from $-30\text{ }^{\circ}\text{C}$ to $+200\text{ }^{\circ}\text{C}$ via commercially available cooling or heating means..

Systematic tests reported in the 2010 paper [155], established that a blue laser light loses (gains) energy, thus decreasing (increasing) its frequency, when passing through air contained in the indicated tube at 1,000 psi and $-10\text{ }^{\circ}\text{C}$ ($+150\text{ }^{\circ}\text{C}$).

Since none of the measured frequency shifts occurred with any relative motion between the source and the observer, tests [155] established the existence of the isoshift as predicted in the 1991 monograph [72].

Systematic additional measurements were conducted in the USA and Europe via the best available wavelength analyzers by following Sunlight with a telescope from the Zenith to the horizon. These tests established

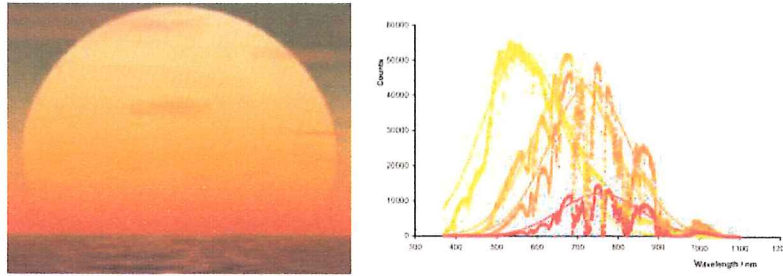


Figure 33: *In order to maintain the validity of special relativity within our inhomogeneous and anisotropic atmosphere, it is generally believed that the redness of Sunlight at Sunset is due to the absorption of blue light resulting in the residual red light, against evidence known since Newton's time that blue light is the most penetrant and red light is quickly absorbed by media. Systematic measurements [159] done in the U.S.A. and in Europe established that the redness of the Sun at Sunset (left view) is due to loss of energy to our atmosphere with ensuing isoredshift of blue light into the red red light (right view).*

that the redness of the Sun at the horizon (left view of Figure 33) is due to a loss of energy to our atmosphere such to cause the isoredshift of the blue light into the red light (right view of Figure 33) [157] (see also lecture f). It should be noted that the scattering of photons among the molecules of our atmosphere is basically insufficient to represent the large redshift from the blue all the way to the red.

8.4.4-III. Verification of SIR in astrophysics. Measurements [159] provided an experimental verification on Earth of Zwicky's hypothesis of *Tired Light* [160], according to which galactic light loses energy to the very cold intergalactic medium (mostly composed by Hydrogen at absolute zero degree temperature) in a way essentially proportional to the covered distance d in a static universe without motion of galaxies at such a velocity v/c to provide a contribution to the redshift.

In particular, SIR provides a direct representation of Hubble's law with the identification of the S quantity of Eq. (142) with the Hubble constant H_0 without any contribution from the Doppler shift

$$\hat{z} = \frac{\omega'}{\omega} - 1 = -\frac{H_0}{c}d. \quad (143)$$

The above isolaw provides an excellent representation of astrophysical data on cosmological redshift as shown in Ref.[161], including a proportionality from the distance, the representation of the very large redshift of light from galaxies at the end of the known universe without any need for superluminal speeds of entire galaxies or the hyperbolic and unverifiable assumption that space itself is expanding.

The numerical representation of internal galactic redshift anomalies occurs via the use of the isoredshift for star light propagating through the cold peripheral intergalactic medium, and the isoblueshift for star light passing through hot intergalactic medium near a black hole [162].

The acceleration of the cosmological redshift with the distance d can be beautifully represented via a gravitational contribution to the redshift of galactic light passing near stars or galaxies in their long travel to Earth [163].

The above results set the foundations of the new *Tired Light Cosmology representing a static universe in total EPR entanglement* (Figure 2) according to the view preferred by Einstein, Hubble, Hoyle, Zwicky, Fermi, and others who died without accepting the expansion of the universe.

Note that for about one century the cosmological redshift z has been interpreted to be a direct "measurement" of the speed v of galaxies in an expanding universe according to the SR law (139), while in reality the identification $z = v/c$ is an *assumption* due to the existence of the Tired Light and other interpretations of the cosmological redshift.

SIR has established that the identification $z = v/c$ is an experimentally unverifiable *assumption* since the cosmological redshift z can be equally

interpreted via Zwicky's Tired Light without any expansion of the universe, and via other models such as that of the *Tired Time* [85].

Note that the assumption $z = v/c$ implies

$$\frac{v}{c} = \frac{H_o}{c}d, \quad (144)$$

from which

$$v = H_o d, \quad (145)$$

which establishes the *radial* character of the conjectured expansion of the universe necessarily implying Earth at its center [163].

Following one century of oblivion of Einstein's rejection of the expansion of the universe as well as the oblivion of Einstein's view that "quantum mechanics is not a complete theory, " it is hoped that cosmologists will compare the new Tired Light Cosmology with EPR entangled universe [155]-[162] with the forgotten conceptual, geometrical and physical insufficiencies or sheer inconsistencies of the unverifiable conjecture of the expansion of the universe [163] [164] (see also debate [76] for details).

8.4.4-IV. Verification of SIR with the mean-life of unstable hadrons. Ref.[166] of 1964 suggested that non-linear and non-local effects in the interior of hadrons caused by their high density can manifest themselves in the outside via deviations of the behavior with energy of their mean lives τ from time evolution law (119), i.e.,

$$\tau' = \frac{\tau}{\sqrt{1 - \frac{v^2}{c^2}}}. \quad (146)$$

Numerous generalizations of said law were then proposed, but Ref. [167] showed that, in view of the universality of the Lorentz-Santilli isosymmetry $\hat{SO}(3.1)$ for all possible symmetric space-times (116), all said generalized time evolution laws are particular cases of the SIR isolaw (130),

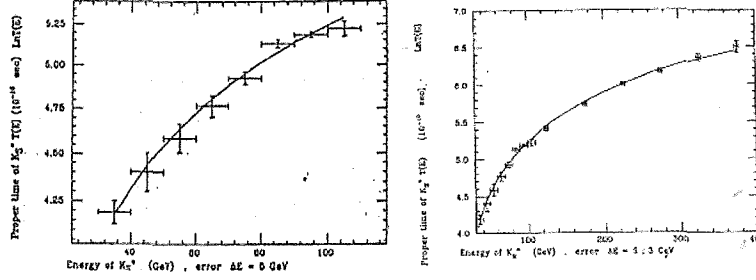


Figure 34: In this figure, we show the exact fit by special isorelativity experimental data [168] showing deviations from special relativity law (119) in the behavior of the mean-life of Kaons from 1 to 100 GeV (left fit) as well as experimental data [168] joined with those of Ref. [169] claiming the confirmation of special relativity law (119) between 100 to 400 GeV (right fit) [170]-[172].

i.e.

$$\tau' = \frac{\tau}{\sqrt{1 - \frac{v_k^2}{c^2} \frac{n_4^2}{n_k^2}}}. \quad (147)$$

since they can be all obtained via different expansions of the term n_4^2/n_k^2 in different variables and with different truncations. Consequently, Ref. [167] provided a significant clarification for experiments since they can be all restricted to test isolaw (130).

Experiments on the indicated prediction of deviation from law (146) were conducted in 1983 [168] for the behavior of the unstable kaons with speed and established deviations from law (119), consequently in favor of isolaw (130) from 1 to 100 GeV. A counter-experiment was done in 1987 [169] claiming the confirmation of law (119) between the *different* range of 100 to 400 GeV.

Refs. [170] [171] showed that the data from both experiments [168] and [169] can be exactly fit with the iso-Minkowskian geometry of relativistic hadronic mechanics [60] [70] (Figure 34). The 1992 Ref. [172]

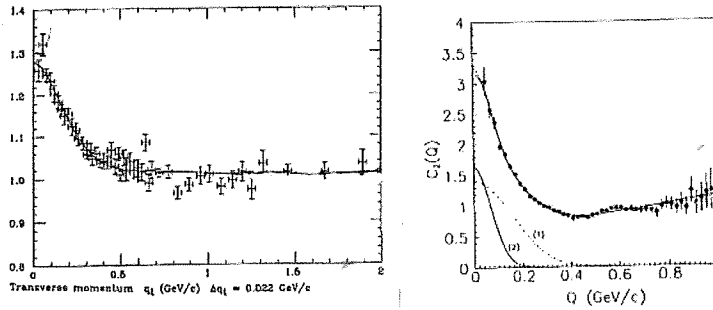


Figure 35: *The fit of experimental data for the two-point correlation function of proton-antiproton annihilation in the Bose-Einstein correlation requires 'four' arbitrary parameters of unknown origin [173]. Refs. [85] [86] (see also review [174]) have shown the inapplicability of special relativity for the proton-antiproton annihilation in favor of the exact fit via special isorelativity at high energy (left plot) and low energy (right plot) from which the four characteristic quantities n_μ of the iso-Minkowskian geometry represent the very elongated fireball of proton-antiproton annihilations, Eqs. (150)-(151).*

confirmed the findings of Refs. [170] [171], by indicating flaws in the theoretical elaboration of the form factors of counter-experiments [169]. A detailed presentation is available in Ref. [74], Vol. IV, Section 9.

It is unfortunate for our scientific knowledge that, according to the official position of the editors of the journals of major physical societies, SR is claimed to be exactly valid in the interior of hadrons following the 1987 counter-experiment [169] despite the fact that the deviations from SR laws remained established between 100 to 400 *GeV* and in complete oblivion of refs. [170] [171], while requests made to various particle physics laboratories over decades to run the experiment again from 1 to 400 *GeV* have been generally discredited.

8.4.4-V. Verification of SIR in the Bose-Einstein cor-

relation. As it is well known (see, e.g., Ref. [173]), the fit of the experimental data of the *two*-point correlation function of the Bose-Einstein correlation for proton-antiproton annihilation requires *four* different arbitrary parameters of unknown origin or meaning called the *chaoticity parameters*. It has been shown in Refs. [85] [86] that this occurrence is clear evidence on the lack of exact character of SR for proton-antiproton annihilation for a number of reasons, including the fact that the vacuum expectation value of the two-point correlation operator $C_{2 \times 2}$ is given by the known quantum mechanical expression

$$\langle C_{2 \times 2} \rangle = \langle \psi | C_{2 \times 2} | \psi \rangle, \quad (148)$$

which, being two-dimensional and real-valued, can at best allow *two* parameter following due manipulation of the basic axioms.

By contrast, the use of relativistic hadronic mechanics [24] implies the following isoexpectation value

$$\langle \hat{C}_{2 \times 2} \rangle = \langle \hat{\psi} | \hat{X} C_{2 \times 2} \hat{X} | \hat{\psi} \rangle = \langle \hat{\psi} | \hat{T}_{2 \times 2} C_{2 \times 2} \hat{T}_{2 \times 2} | \hat{\psi} \rangle. \quad (149)$$

It is then easy to see that the positive-definite 2×2 -dimensional isotopic element $\hat{T}_{2 \times 2} > 0$ does indeed allow in its non-diagonal realization the introduction of four characteristic quantities n_μ^2 , $\mu = 1, 2, 3, 4$ from first axiomatic principles without any adulteration, whose value is fit from the experimental data resulting in the values (Ref. [85], Eqs. (10.27a), page 127, Ref. [86], Table I, page 441, and review [174])

$$n_1^2 = n_2^2 = 10.666, \quad n_3^2 = 0.355, \quad (150)$$

$$n_1 = n_2 = 3.265, \quad n_3 = 0.595 \quad (150)$$

$$n_4^2 = 0.429, \quad n_4 = 0.654. \quad (151)$$

One can see from values (150) that the fit of the experimental data (Figure 35) characterizes the known prolonged toroid of the proton-antiproton fireball, while value (151) characterizes its density (for details, see Ref. [75]. Volume IV, Section 10, page 802-809 and review [174]).

8.4.4-VI. Verification of SIR with superluminal speeds. 8.4.4.VIA. Solution of the historical Lorentz problem.

The most important implication of the axiom-preserving isotopies of the Lorentz symmetry achieved in the 1983 paper [60] is the *solution of the historical Lorentz problem*, namely, the invariance of the locally varying speeds of light $C = c/n$ that Lorentz could not solve due to the linearity of Lie's theory, by being therefore forced to restrict his invariance to the particular case of a constant c .

In the author's view, the most important implication of the EPR argument [1] applied to isosymmetry [60] can be expressed with the following property from Ref. [175] [176]: :

LEMMA 8.4.1. The axioms of Einstein's special relativity provide a representation invariant over time of speeds of light through transparent media that can be arbitrarily bigger or smaller than the speed of light in vacuum,

$$C = \frac{c}{n_4} \begin{matrix} \leq \\ \geq \end{matrix} c. \quad (152)$$

PROOF:

Isosymmetries $\hat{SO}(3.1)$, $\hat{P}(3.1)$ and $\hat{\mathcal{P}}$ of isospacetime(126) [60]-[70] (see Refs. [24] and [213] for a review) require no restrictions on the value and functional dependence of the characteristic quantities of the medium, except for the condition of being positive definite, $n_\mu > 0$, $\mu = 1, 2, 3, 4$. Consequently, said isosymmetries provide a characterization of arbitrary local speeds of light, Eq. (152), in a way fully compatible with the axioms of SR because said isosymmetries are locally isomorphic to the corresponding conventional spacetime symmetries $SO(3.1)$, $P(3.1)$, $\mathcal{P}$. The proof of the invariance over time of arbitrary speeds (152) can be done via a simple isotopy of the invariance over time of the speed of light c under the conventional symmetries $SO(3.1)$, $P(3.1)$, $\mathcal{P}$.

Q.E.D.

Alternatively, SR can be fully realized on the Minkowski-Santilli isospace [70] $\hat{M}(\hat{x}, \hat{\Gamma}, \hat{I})$ over the isoreal isofields $\hat{\mathcal{R}}$ with isounit $\hat{I} = 1/\hat{T} > 0$, in which case the line element given by the top line of Eq. (126).

The realization of the isocoordinates

$$\hat{x}^\mu = \frac{x^\mu}{n_\mu} \hat{I}, \quad (153)$$

then yields isosymmetries $\hat{S}O(3.1)$, $\hat{P}(3.1)$, $\hat{\mathcal{P}}$ with arbitrary speeds (152).

The propagation of light within transparent liquids with densities $n_4 > 1$ and light speeds *smaller* than that in vacuum, $C = c/n_4 < c$, has been known for centuries (Section 8.4.4-I), e.g., for the case of water with density characterized by $n_4 = 1.5$ for which we have (Figures 30, 31)

$$C = \frac{c}{1.5} = 0.666c = 200K \text{ km/s}. \quad (154)$$

Note that the characteristic quantity n_4 provides a *geometric characterization* of the density of the medium whose actual value is given by $D = 1/n_4$.

Note also Inconsistencies 1) to 7) in the event mainstream physicists attempt to maintain in water the speed of light in vacuum via the usual reduction of electromagnetic waves to photons.

Recall that the Cherenkov light we see in the pools of nuclear power plants establishes the propagation of electrons in water at speed *bigger* than the local speed of light.

SIR has generalized the above Cherenkov effect to hyperdense physical media with geometrized densities $n_4 < 1$ resulting in arbitrary speeds $C = c/n_4 > 1$. This result is implicit in all experimental fits of particle physics experiments to date via the SIR (e.g., Figures 34, 35). The result is also implicit in all structure models of hadrons, nuclei and stars based on the EPR completion of quantum into hadronic mechanics [36] [74].

As an illustration, recall from Section 8.2.2 that the *sole* known possibility to represent *all* characteristics of the neutron in its synthesis from

the hydrogen is that the energy of the isoelectron is isorenormalized according to Eq. (36) from $m_e = 0.511 \text{ MeV}$ to $m_{\hat{e}} = 0.511 + 0.784 = 1.295 \text{ MeV}$, by therefore yielding the expected geometrized density of the proton

$$n_4 = \frac{0.511}{1.295} = 0.394 < 1, \quad (155)$$

whose order of magnitude is confirmed by the geometrized density of the proton-antiproton fireball in the Bose-Einstein correlation (Ref. [85], Eqs. (10.27a), page 127).

This implies that *the isoelectron is rotating inside the hyperdense proton (Figure 12) with the superluminal tangential speed*

$$C = \frac{c}{0.394} = 2.538 \text{ } c. \quad (156)$$

Superluminal speeds are generally obtained within the π^0 meson [20] and hold for all remaining hadrons since they all have masses bigger than that of the π^0 , while having essentially the same size of the π^0 . Consequently, SIR suggests that *particles travel at a superluminal speed that begins with the interior of the π^0 , and increases with the increase of the mass of the hadrons in a way parallel to the progressive regaining of Einstein's determinism with the increase of the density of hadrons [210]-[214].*

8.4.4.VIB. Geometric locomotion. It may be of some interest to mention the *mathematical* prediction by SIR of spaceships traveling at speeds much bigger than the speed of light in vacuum which speeds are evidently necessary for any interstellar travel. The prediction was submitted in Section 4.3.3, page 281 on (see also Figure 4.5) of the 2006 monograph [29] under the name of *geometric locomotion*, also known as *isolocomotion* due to the need of its treatment via isomathematics. Isolocomotion is an intrinsic feature of the iso-Minkowskian geometry [70] and its Lorentz-Santilli isosymmetry [60].

Recall that the main notion of any geometry dating back to Euclid is the distance d between two points, which we write $d1$, where 1 is the unit of measurement (as well as the unit of the basic numeric field), e.g. 1 m . Recall the preservation of numeric values under axiom-preserving isotopies [24], which we write $d1 \equiv \hat{d}\hat{1}$. Isolocomotion is based on means capable of altering the local geometry in such a way to *increase* the value of the isounit $\hat{1}$ in the desired direction, with consequential *decrease* of the distance $\hat{d}$ in the selected direction. Isolocomotion then occurs without any possible geometric singularity, such as instantaneous accelerations, sharp changes of direction, or arbitrary speeds because the interior observer in the iso-Minkowski isospace is at rest since locomotion is achieved via the change of the geometry in its environment. By contrast, an observer in our external Minkowski space may see the distance d being covered at a multiple the speed of light c .

Alternatively, the aim of isolocomotion is to achieve arbitrary speeds without violating SR. This is *mathematically* achieved by turning a spaceship into rather complex *interior* conditions via the mutation of the surrounding geometry, with ensuing local applicability of SIR and related arbitrary speeds.

8.4.4.VIC. Interstellar travel. Recall that no interstellar travel is nowadays possible because of: 1) The need for superluminal speeds, 2) The impossibility of carrying along the necessary fuel, 3) The inability to change directions to avoid collisions under very high speeds.

The possible achievement of superluminal speeds has been discussed in the preceding section. Studies on a future resolution of the fuel problem have been initiated in Refs. [29] [175]. The main result is that quantitative studies for future interstellar travels can indeed be conducted provided that space is conceived as a universal substratum (ether) with an extremely big energy density for the characterization and propagation of particles and electromagnetic waves (Section 8.2.3).

In different words, the sole known possibility for interstellar travel is that the needed fuel has to be extracted from the ether.

The complexity of the problem soon emerges when considering the need for *two* co-existing ethers, one with positive energy for the characterization of matter, and one with negative energy for the characterization of antimatter [29]. This structure is of such a complexity that, in the author's view, can be best represented via the hyperstructural branch of hadronic mechanics [24].

Despite its complexity, the above conception of a universal substratum implies that immense values of positive and negative energy are available everywhere in the universe. Future attempts at the realization of isolocomotion are then reduced to engineering means for the directional transfer of *negative* energy from the ether to the spaceship. Under the indicated conditions, isolocomotion occurs via matter-antimatter repulsion.

The third problem (how to avoid collisions with astrophysical objects) is perhaps the most complex of all problems connected with interstellar travel since its sole solution is a change of the very structure of the spaceship in such a way to pass through planets without any damage, as shown in reports by the U. S. Navy of UFO entering in the sea at high speeds without causing any wave. Conceivably, the alteration of the geometry of the ether can *mathematically* be such to produce the needed "evanescence."

In short, the ether appears to have a truly fundamental role for interstellar travel due to various reasons treated in Ref. [175], as a result of which the ether is considered by an increasing number of scientists as the most important frontier of the third millennium.

In closing, we point out that the entire content of this section, including superluminal speeds, is fully compatible with the axioms of Einstein's special relativity, only realized in their most general possible form [175] [176].

An additional study of the implications of the EPR argument for superluminal speeds was presented at the 2020 Teleconference [0] by Y. F. Chang from China and it is available in Ref. [218].

8.5. Application of EPR completions to exterior gravitational problems.

8.5.1. Universal invariance of Riemannian intervals.

As it is well known, Lie's theory can only provide the invariance of *linear* theories and this explains the reason for the inability to construct Lie symmetries of Riemannian intervals due to their non-linearity.

A primary motivation for the construction of the *non-linear* completion of Lie's theory into the covering Lie-Santilli isothory [21]-[25] [50] [55] [201] [226] has been the construction of the universal isosymmetry of Riemannian intervals.

The above objective was studied for the particular case of isointerval (126) in which the isometric $\hat{\eta}_{\mu\nu}$ coincides with a Riemannian metric $g(x)$, and resulted in the following basic isosymmetries (see the review in Ref. [213]): 1) The iso-Lorentz symmetry, nowadays called the *Lorentz-Santilli (LS) isosymmetry* $\hat{SO}(3.1)$ first achieved at the classical level in the 1983 paper [60] with operator counterpart in Ref. [61];

2) The *iso-Poincaré isosymmetry*, nowadays called the *Lorentz-Poincaré-Santilli (LPS) isosymmetry* $\hat{P}(3.1)$, first achieved in the 1993 paper [67] written at Moscow State University; and

3) The spinorial covering of the LPS isosymmetry $\hat{P}(3.1)$, first achieved at the 1994 paper [68] written at the JINR, Dubna, Russia (see also the 1995 paper [69] published in China).

The understanding of the remaining parts of this section requires a technical knowledge that the isotopic symmetries formulated on iso-Minkowskian isospaces over isofields are locally isomorphic to the corresponding conventional symmetries formulated on a Minkowski space over a numeric field, i.e., $\hat{SO}(3.1) \approx SO(3.1)$, $\hat{P}(3.1) \approx P(3.1)$, $\hat{\hat{P}}(3.1) \approx$

$\mathcal{P}(3.1)$.

8.5.2. Isogeometric unification of special and general relativity. As indicated earlier, the *iso-Minkowskian geometry* [60] [70] includes as particular cases all possible geometries with symmetric space-time intervals (126), thus including the Minkowskian, Riemannian, Fynslerian and other geometries.

Consequently, the iso-Minkowski geometry is particularly suited for the *isogeometric unification of Einstein's special and general relativities under the universal LPS isosymmetry*, namely, a unification based on isomathematics while maintaining identically Einstein's field equations

$$R_{\mu\nu} - \frac{1}{2}R_{\mu\nu}R = 0. \quad (157)$$

Consider an arbitrary non-singular Riemannian metric $g(x)$ where x are the conventional space-time coordinates. Its *identical* reformulation via isomathematics requires the decomposition of $g(x)$ into the product of the Minkowski metric η multiplied by a 4×4 positive-definite *gravitational isotopic element* $\hat{T}_{gr}(x)$ [177] [178],

$$g(x) = T_{gr}(x) \times \eta = \hat{\eta}_{gr}. \quad (158)$$

The resulting iso-Minkowskian isospace $\hat{M}(\hat{x}_{gr}, \hat{\eta}_{gr}, \hat{I}_{gr})$ is then formulated over an isoreal isofield $\hat{\mathcal{R}}$ with *gravitational isounit* given by the *inverse* of the gravitational isotopic element,

$$\hat{I}_{gr}(x) = 1/\hat{T}_{gr}(x), \quad (159)$$

and local isocoordinates

$$\hat{x}_{gr} = x\hat{I}_{gr}. \quad (160)$$

Exterior general isorelativity (EGIR) [177]- [182] can be defined as the *formulation of general relativity on isospace* $\hat{M}(\hat{x}_{gr}, \hat{\eta}_{gr}, \hat{I}_{gr})$ over an isoreal

isofield $\hat{\mathcal{R}}$ under the condition that its projection on a Riemannian space over a conventional field recovers general relativity uniquely and identically. Consequently, by construction, isogravitation is a mere reformulation of general relativity via the use of isomathematics. The terms *exterior isogravitation* are used to recall that general relativity describes the exterior gravitational field in vacuum, in preparation of the complementary interior gravitational problem studied in the next section.

Note that, in view of the dependence of the isometric $\hat{\eta}(x)$ on space-time coordinates, the iso-Minkowski isogeometry is formulated with the entire mathematical machinery of the Riemannian geometry, although expressed in terms of the isodifferential isocalculus [70].

Note also that isogravitation provides the reformulation of general relativity with the axioms of *special* relativity in their isotopic formulation, Isoaxioms I to V.

The isogeometric unification of special and general relativities is then assured by the fact that isogravitation is an identical reformulation of general relativity while admitting special relativity at the simple limit

$$\text{Lim} \hat{I}_{gr} I. \quad (161)$$

8.5.3. Resolution of century-old controversies in gravitation? As it is well known to experts in gravitation, although rarely admitted, general relativity has been plagued by a host of controversies that have not been resolved in about one century of studies, such as the apparent incompatibility of general relativity with special relativity, quantum mechanics, grand unifications and other 20th century theories.

In the author's view, protracted *physical* controversies are generally due to the fact that the used *mathematics* is insufficient for the solution of the problem considered. For the case of general relativity, it has been shown that the origin of the controversies can be reduced to the incompatibility of the Riemannian geometry with the axiomatic structure of

20th century theories, since the latter are all defined on flat spaces, while the former is defined on a curved space.

Isogravitation (GIR), that is, the reformulation of general relativity via the iso-Minkowskian geometry, appears to offer realistic possibilities of resolving the indicated century-old controversies, as illustrated by the following comments:

1) **Isogravitation is isoflat**, namely, it is flat on isospace over isofield. This important feature can be seen from the fact that the isogravitational isometric $\hat{\eta}_{gr}$ is given by the Minkowski metric η multiplied by the isotopic element $\hat{T}_{gr}$ according to Eq. (158), while jointly the basic unit of the Minkowski geometry $I = \text{Diag.}(1, 1, 1, 1)$ is completed by the *inverse* amount, Eq. (150), resulting in no curvature. In the author's view, the isoflatness of GIR is important for the achievement of compatibility between gravitation and 20th century theories for the indicated reason that the latter are formulated in flat spaces.

2) **Invariance under isosymmetries that are locally isomorphic to the corresponding conventional symmetries** (Section 8.5.1). Recall that another reason for the incompatibility of GR with 20th century theories is the invariance of the former under the Lorentz and Poincaré symmetries compared to the lack of invariance for GR. It is then evident that the reformulation of gravitation in a form admitting symmetries locally isomorphic to the conventional symmetries provides serious support for the compatibility of isogravitation and 20th century theories.

3) **Unique and unambiguous limit of isogravitation into special relativity** [177]. Recall that the limit of general into special relativity has remained controversial for a century due to numerous reasons [181], such as the impossibility of recovering from the Riemannian geometry the *Poincaré symmetry* of special relativity, let alone its generators (conserved quantities). These controversies appear to be resolved by isogravitation due to its formulation via the axioms of *special* relativity. Additionally, SR is recovered from GIR in full via simple limit (161).

4) **Unique and unambiguous operator formulation of gravitation** [177]. An additional reason for controversies is the lack of a clear and unambiguous quantum mechanical formulation of GR. This is due to the fact that GR is a *non-canonical* theory whose consistent operator image is then given by *non-unitary* theories. But the latter theories formulated in conventional fields violate causality as indicated various times in this Overview, resulting in the lack of a unique, physically consistent operator image of GR. This additional, century-old controversy is resolved by the fact that the operator image of GR is uniquely and unambiguously given by relativistic hadronic mechanics [36] when characterized by the gravitational isounit $\hat{I}_{gr}$, Eq. (159). Additionally, relativistic hadronic mechanics is *isounitary*, that is, unitary on isospaces over isofields, thus recovering causality.

5) **Unique and unambiguous grand unification** [178]. Recall the impossibility to achieve a consistent grand unification of gravitation with other interactions beginning with Einstein's own failed attempts. In addition to a number of problematic aspects [181], this impossibility is primarily due to the *curvature* of the Riemannian geometry because, when combined with electromagnetic and/or weak interactions, curvature causes the collapse of their axiomatic structure, beginning with the loss of space-time, gauge and other symmetries. This additional century-old controversy appears to be resolved by isogravitation because of its isoflatness, as studied in detail in monograph [29]. Note that the resulting grand unification required, for consistency, the addition of the gravity of antimatter via the isodual of completion of charge conjugation (Section 7.7).

6) **Historical objections against the curvature of space.** As it is well known, when looking at the Sun at Sunset, the Sun is already below the horizon due to the *refraction* of Sun light in our atmosphere without any possible curvature of space. The historical, well known (but rarely mentioned) objection against the curvature of space is that the bending of light passing near our Sun is due to its refraction of light within the Sun

chromospheres. Additional controversies occur from the fact that space is assumed to be empty. It is therefore counter-intuitive for a number of physicists to accept the idea that the curvature of an empty space can control the trajectory of large planets such as Jupiter. Additional controversies on curvature can be found in the debate [192].

7) **Lack of time-invariant numerical predictions.** Recall that a majestic feature of special relativity is the preservation over time of numerical prediction due to the *invariance* of the theory under the Lorentz-Poincaré symmetry. The canonical character of SR then assures the *uniqueness of the space-time metric for all experimental verifications*. As it is well known to historians (but also rarely spoken), the lack of Lie symmetries in the Riemannian geometry mandated the replacement in GR of the notion of invariance with that of *covariance*. However, such a replacement triggered a number of controversies, the first controversy being due to the the lack of uniqueness of the Riemannian metric for all experimental verifications in exterior conditions, with ensuing lack of final experimental results. Secondly, the use of covariance instead of invariance implies that numerical predictions of GR are not preserved over time, with the ensuing additional reason for the lack of final character of experimental verifications [161]. Additional controversies have occurred for experimental verifications of GR due to the apparent *ad hoc* selection of the PPN approximation for the selected experiment. It appears that GIR offers realistic possibility of resolving the indicated controversies on experimental verifications in case seeded in a receptive scientific environment (for additional controversies, see next section and Ref. [182]).

8.6. Application of EPR completions to interior gravitational problems.

8.6.1. **Exterior and interior gravitational problems.** In the first part of the 20th century, dynamical problems were called *exterior problems* when dealing with point-particles in vacuum and *interior*

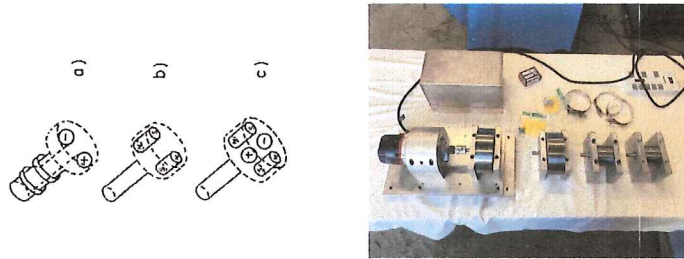


Figure 36: On the left of this figure, we reproduce the proposal in page 145 of the 1974 MIT paper [185] to test the Poincaré hypothesis that the exterior gravitational field of a mass is entirely due to the electric and magnetic fields of the charged constituents in high dynamical conditions. In the right of this figure, we show the Gravity Generator Equipment (GGE) manufactured by Hadronic Technologies Corporation to test the Poincaré hypothesis in the expectation of its conduction by a physics laboratory with a sufficiently sensitive gravity meter.

problems when dealing with particles in the interior of physical media. For instance, Schwarzschild wrote *two* important papers: the first paper [183] in the *exterior gravitational problem* that became justly famous, and the second paper [184] on the *interior gravitational problem* that continues to remain vastly ignored because not compatible with Einstein general relativity.

This view is essentially based on the fact that the reduction of matter to *point-like* constituents according to quantum mechanics, essentially eliminates any major distinction between exterior and interior problems.

The admission of the physical reality that particles in general, and hadrons in particular, are *extended*, and the ensuing verifications [210]-[214] of the EPR argument [1], reinstate the structural difference between exterior and interior gravitational problems adopted by Schwarzschild [183] [184], due to the very complex interactions occurring for particles

in interior conditions, ultimately reducible to deep EPR entanglements (Figure 2 and Section 7.2.3), which internal interactions are completely absent for the same particles in exterior conditions.

8.6.2. Interior General Isorelativity (IGIR). As noted in Section 8.5, general isorelativity (GIR) is characterized by an isometric solely dependent on space-time isocoordinates $\hat{\eta}(\hat{x})$, in which case the theory can solely represent the exterior gravitational field.

In general, the isometric has an unrestricted dependence on all needed local variables [70]

$$\hat{\eta} = \hat{\eta}(x, v, p, E, \mu, \tau, \rho, \dots) = \hat{T}_{gr}(x, v, p, E, \mu, \tau, \rho, \dots)\eta. \quad (162)$$

Consequently, isogravitation may indeed allow the study of interior gravitational problems, resulting in a theory called *interior general isorelativity* (IGIR).

One of the best illustrations of IGIR is given by the *iso-Dirac isoequation*, also known as *Dirac-Santilli isoequation*, on an iso-Minkowskian isospace $\hat{M}(\hat{x}, \hat{\eta}, \hat{I})$ first introduced in Ref. [70] of 1995

$$(-i\hat{I}\hat{\eta}^{\mu\nu}\hat{\gamma}_\mu\partial_\nu + mC)|\hat{\psi}(\hat{x})\rangle = 0. \quad (163)$$

In this case, the *Dirac-Santilli isogamma isomatrices* $\hat{\gamma}\hat{I}$ are given by

$$\hat{\gamma}_k = \frac{1}{n_k} \begin{pmatrix} 0 & \hat{\sigma}_k \\ -\hat{\sigma}_k & 0 \end{pmatrix}, \quad \hat{\gamma}_4 = \frac{i}{n_4} \begin{pmatrix} I_{2 \times 2} & 0 \\ 0 & -I_{2 \times 2} \end{pmatrix}, \quad (164)$$

where $\hat{\sigma}_k$ are the *regular Pauli-Santilli isomatrices* used for the EPR verification [211] (see realization(39) via Bohm's hidden variables and Section 3.3 of Ref. [213] for the general case), with *anti-isocommutation rules*

$$\{\hat{\gamma}_\mu, \hat{\gamma}_\nu\} = \hat{\gamma}_\mu \hat{T} \hat{\gamma}_\nu + \hat{\gamma}_\nu \hat{T} \hat{\gamma}_\mu = 2\hat{\eta}_{\mu\nu}. \quad (165)$$

The simpler case of GIR holds when the isometric coincides with a Riemannian metric, $\hat{\eta}_{\mu\nu} = g(x)_{\mu\nu}$, including the Schwartzschild metric, by

therefore describing an isoelectron-isopositron pair under an external gravitational field.

The isogeometric unification of this section is illustrated by the fact that Eq.(163) allowed the representation of *all* characteristics of the neutron in its synthesis from the hydrogen with the sole change of the metric into one used for SIR, such as that of the EPR entanglement of Section 7.2.3 [70].

8.6.3. Black or brown holes? The study of IGIR has been rudimentarily initiated in Ref. [180] with intriguing outcome, such as the apparent *reformulation of black holes into brown holes* in representation of the lack of existence of singularities in nature.

Alternatively, the reformulation of black into brown holes appears to indicate the existence of a limit in the compressibility of protons at the divergence of their entangled number

8.6.4. Studies in the origin of gravitation. Recall that the gravitational field of a mass originates in the interior of the mass. Hence, a historical open problem is that of the *origin* rather than the sole description of a gravitational field.

It may be of some interest to know that IGIR can be defined as *a representation of the origin of the gravitational field with particular reference to the study of matter-matter gravitational attraction and matter-antimatter gravitational repulsion.*

As an example, a typical problem of the IGIR is the study of Poincaré's hypothesis that the exterior gravitation field of a mass is entirely generated by the electric and magnetic fields of its charged constituents.

The Poincaré hypothesis has been studied in detail in the 1974 MIT paper [185] via advanced and retarded formulations of quantum field theory. This study confirmed that the electromagnetic field of all charged components of a mass, including atomic and nuclear constituents, can indeed be the complete source of the exterior gravitational field of a mass

even when the total charge is zero.

This study implies the existence of a gravitational source $\hat{F}_{\mu\nu}^{elm}$ representing the origin of the gravitational field, with ensuing completion of field equations (157) [180] into the form

$$\hat{R}_{\mu\nu} - \frac{1}{2}\hat{R}_{\mu\nu}\hat{R} = k\hat{F}_{\mu\nu}^{elm}, \quad (166)$$

which does verify the forgotten Freud identity of the Riemannian geometry [181].

Note that Eq. (163) no longer represent gravitation via curvature and this explains the reason for which the study of the Poincaré hypothesis in the origin of the gravitational field has been vastly ignorewd by mainstream physics for one full century (see debate [91] for details) despite Einstein's doubts on the r.h.s. of field equations (162) which he called "*a house made of wood*" while called the l.f.s. "*a house made of marble*."

For non-initiated readers, we should recall that Eq. (163) is fully admitted by conventional GR although, in this case, $\hat{F}_{\mu\nu}^{elm}$ represents the field of the *total charge* of the mass considered, with ensuing extremely small contribution to the gravitational field that, as such, remains represented by curvature.

By contrast, in field equation (163) according to the 1974 paper [185], the term $\hat{F}_{44}^{elm}$ represents the *total mass of the body assumed to be neutral*, the extension to the a non-null total charge being trivial. In this case, the gravitational field cannot be represented via curvature, because it is correctly represented by the iso-Minkowskian geometry that, as indicated earlier, is flat.

The basic open problems of the IGIR can then be tentatively formulated as the study of: the gravitational attraction of two masses represented with two equations of type (163)a; and the study of the gravitational repulsion between one mass represented with Eqs. (163) and its antimatter image represented via the isodual iso-Minkowskian geometry

and isodual image of Eqs.m(163) [29] [70].

Additional studies in the general relativistic formulation of the electromagnetic field and its implications for quantum gravity are available from contributions [225] and [234] of the 2020 Teleconference [0].

8.6.5. Experimental test of the origin of gravitation. An experiment to prove or disprove the Poincaré hypothesis was suggested in page 145 of the 1974 MIT paper [185] (Figure 36). The U. S. *Hadronic Technologies Corporation* has constructed the equipment according proposed in Ref. [185] called *gravity generation equipment* (GGE), essentially consisting of a series of discs with *null* total charge of 1" thickness and diameter varying from 3" to 5". The discs are composed by various material, such as aluminum or Iron, which discs are put in rotation up to 100,000 *rpm* by a specially designed sequence of spindles (Figure 35).

The proposed experiment essentially consists in: 1) placing the indicated GGE next to a highly sensitive gravity detector; 2) measuring the local gravitational field when the GGE is disconnected; and 3) measuring the local gravitational field when the GGE is progressively activated up to the maximal 100,000 *rpm*. The detection of any gravitational field when the GGE discs are rotating would establish the first known *creation in laboratory of a gravitational field*. Note that there is no need for the GGE discs to be charged (see Ref.[185] for details).

In the author's view, Einstein's geometric conception of gravitation as being entirely due to curvature is one of the most beautiful and important discoveries by the human mind. However, such a conception has merely initiated, rather than ended, the studies in gravitation in view of century-old controversies yet to be resolved, and so much remains to be discovered theoretically and experimentally.

8.7. Application of EPR completions to high energy scattering experiments

The non-relativistic and relativistic elaboration of high energy scattering

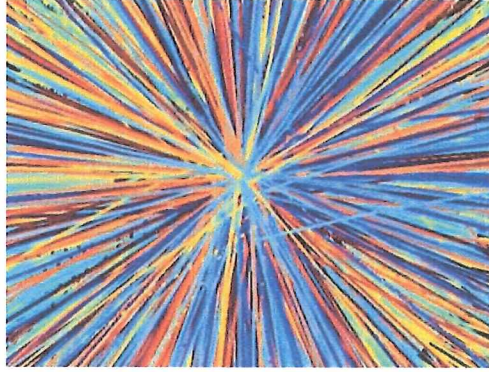


Figure 37: *It is generally accepted in the physics community that quantum mechanics is inapplicable in the interior of a black hole. Consequently, quantum mechanics should not be expected to be applicable in the interior of the scattering region of current high energy particle experiments in accordance with the EPR argument [1].*

experiments with the EPR completion of the quantum into hadronic mechanics is known as:

- i) *isoscattering theory*, when possessing a Lie-isotopic algebraic structure for the representation of *elastic*, thus time-reversal invariant scattering experiments on an iso-Hilbert isospace $\hat{\mathcal{H}}$ over the isocomplex isofield $\hat{\mathbb{C}}$;
- ii) *genoscattering theory*, when possessing a Lie-admissible algebraic structure for the representation of *inelastic*, thus time irreversible scattering experiments on an geno-Hilbert genospace $\hat{\mathcal{H}}^>$ over the genocomplex genofield $\hat{\mathbb{C}}^>$; and
- iii) *hyperscattering theory*, when elaborated via hyperstructures for the representation of time irreversible multi-particle scattering experiments requiring a three-dimensional multi-valued representation on a hyper-Hilbert hyperspace over the hypercomplex hyperfield [24] [233].

Studies in the field were initiated by R. Mignani in 1984 [186] and continued by other authors (see the 1995 review in Chapter 12 of Ref. [24]). Subsequent detailed studies have been conducted by A. K. Aringazin *et al.* [54], A. O. E. Animalu *et al.* [187] [215].

Numerous additional works exist in the formulation of *non-unitary scattering theories*, but they are formulated on a conventional Hilbert space $\mathcal{H}$ over the conventional field of complex numbers $\mathbb{C}$, thus generally violating causality [23] [212]. Nevertheless the latter scattering theories are significant because they can be easily reformulated on the appropriate iso-, geno- or hyper-space over a corresponding iso-, geno- or hyperfield, by becoming iso-, geno- or hyper-unitary theories and regaining in this way causality.

By recalling the preservation of quantum mechanical axioms by hadronic mechanics (Section 8.4.4-I), and by remembering the dual formulation of hadronic mechanics on isospaces over isofields and their projection on conventional spaces over conventional fields, to the author's best knowledge the application of the EPR completions to high energy scattering experiments essentially implies the following:

- a) The simplest possible Copenhagen realization of quantum mechanical axioms for point-particles in vacuum is no longer exactly valid for the hyperdense interior of high energy scattering regions due to their approaching the density of black holes, in favor of the broadest possible realization of quantum mechanical axioms according to the iso-, geno- and hyper-structural branches of relativistic hadronic mechanics (Figure 37).
- b) All theoretical as well as numerical results of high energy scattering experiments released by particle physics laboratories to date remain valid when formulated on an iso-, geno-, or hyper-Minkowskian isospace over an iso-, geno-, or hyper-complex fields.
- c) The projection of the preceding formulations on our physical Minkowski space-time over the field of real numbers requires isorenormalization (133) of Isoaxiom IV of the energy and other isorenormalizations

of the characteristics of intermediate particles which have been crucial for the exact representation of the neutron synthesis (Section 8.2.2), nuclear data (Section 8.2.5), molecular data (Section 7.2), and other data.

9. CONCLUDING REMARKS

The author has stated various times in his works that *the basic axioms of quantum mechanics and special relativity are majestic in view of their mathematical consistency, predictive power, and preservation over time of numerical results for the conditions of their original conception (point particles in vacuum)*.

A central feature of the research outlined in this Overview is that the basic axioms of quantum mechanics are preserved identically in the verifications and applications [210]-[214] of the EPR argument [1], and they are merely realized in their most general possible form.

A similar situation occurs for special relativity because, when equally realized in their most general possible form, its axioms appear to allow: a geometric unification of Einstein's special and general relativities; their extension to interior dynamical problems; the apparent resolution of century-old controversies in gravitation; the study of the *origin* (rather than the sole description) of the gravitational field; the study of the mechanism of matter-matter gravitational attraction and matter-antimatter gravitational repulsion; and other intriguing open problems.

Acknowledgements.

The author would like to reinstate the rather long acknowledgements expressed in Refs. [36] [214], and additionally thank all participants of the 2020 Teleconference [0] for penetrating criticisms and comments. Special thanks are due to Carla Santilli, Founder and President of *Hadronic Press, Inc.*, for her crucial role in recording the laborious efforts over the past half a century in the construction of hadronic mechanics and chemistry and making the publications available in free pdf download for this

Overview. Special thanks are finally due to Mrs. Sherri Stone for an accurate linguistic control of the entire manuscript. Needless to say, the author is solely responsible for the content of this Overview due to numerous revisions done in the final version.

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Isorepresentations of the Lie-Isotopic $SU(2)$ Algebra with Applications to Nuclear Physics and to Local Realism

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(Received: 1 February 1994)

Abstract. In this note, we study the nonlinear-nonlocal-noncanonical, axiom-preserving isotopies/ Q -operator deformations $S\hat{U}_Q(2)$ of the $SU(2)$ spin-isospin symmetry. We prove the local isomorphism $S\hat{U}_Q(2) \approx SU(2)$, construct and classify the isorepresentations of $S\hat{U}_Q(2)$, identify the emerging generalizations of Pauli matrices, and show their lack of unitary equivalence to the conventional representations. The theory is applied for the reconstruction of the exact $SU(2)$ -isospin symmetry in nuclear physics with equal p and n masses in isospaces. We also prove that Bell's inequality and the von Neumann theorem are inapplicable under isotopies, thus permitting the isotopic completion/ Q -operator deformation of quantum mechanics studied in this note which is considerably along the celebrated argument by Einstein, Podolsky and Rosen.

Mathematics Subject Classification (1991). 57R52.

Key words: isotopies, isorepresentations, Lie-isotopic algebras.

1. Statement of the Problem

According to current knowledge (see, e.g., [1, 4]), the $SU(2)$ spin or isospin symmetry can solely characterize the familiar eigenvalues $j(j+1)$ and m , $j = 0, \frac{1}{2}, 1, \dots$, $m = j, j-1, \dots, -j$.

In this note, we show that the isotopic generalization of $SU(2)$, herein denoted $S\hat{U}_Q(2)$, while being locally isomorphic to $SU(2)$, can characterize more general eigenvalues of the type

$$j^2 \Rightarrow f(\Delta)j[f(\Delta)j+1], \quad J_3 \Rightarrow f(\Delta)m, \quad (1.1)$$

where j and m have conventional values and $f(\Delta)$ is a real valued, positive-definite function of the determinant of the background metric $\Delta = \text{Det } g = \text{Det } Q\delta$ such that $f(1) = 1$.

For the two-dimensional case, the condition $\det g = 1$ for $g = \text{diag}(g_{11}, g_{22})$ is realized by $g_{11} = g_{22}^{-1} = \lambda$. This implies the preservation of the conventional value $\frac{1}{2}$ of the spin, but the appearance of a nontrivial generalization of Pauli's matrices, herein called *iso-Pauli matrices*, with an explicit realization of the 'hidden variable' λ in the structure of the spin $\frac{1}{2}$ itself.

Reprinted by permission from Accademia Piceno Aprutina dei Velati (APAV), "Overview of Historical and Recent Verifications of the EPR Argument and their Applications to Physics, Chemistry and Biology", R.M. Santilli, in press at APAV.

As a first application, we construct the isotopies of the conventional isospin (see, e.g., [2, 6]), and show that the iso-Pauli matrices permit the reconstruction of an *exact* $S\hat{U}(2)$ -isospin symmetry *under electromagnetic and weak interactions* because protons and neutrons acquire equal masses in the underlying isospace.

As a second application, we show that Bell's inequality and the von Neumann theorem (see, e.g., review [7]) are inapplicable under isotopies, thus permitting the isotopic completion of quantum mechanics studied in this note, which is considerably much along the lines of the celebrated Einstein-Podolski-Rosen (EPR) argument.

It should be noted that, at the International Workshop on Symmetry Methods in Physics held at the JINR in July 1993, Lopez [9] showed that the so-called *q-deformations* (see, e.g., [17, 45]) can be put in an axiomatic form precisely via the isotopic Q -operator deformations studied in this note.

One isotopy of Pauli matrices was first presented by this author at the Third International Wigner Symposium (held at Oxford University in September 1993, [13]). In this note, we present, apparently for the first time, a systematic study and classification of the fundamental (adjoint) isorepresentation of the Lie-isotopic $S\hat{U}_Q(2)$ algebra, their applications to the reconstruction of the exact isospin symmetry as well as to the limitation of Bell's inequality and von Neumann's theorem. Additional applications to nuclear magnetic moments, particle physics and other fields will be presented elsewhere.

2. Isotopies of $SU(2)$ Symmetry

The understanding of this note requires a knowledge of the nonlinear-nonlocal-noncanonical, axiom-preserving isotopies of the theory of numbers [11] and of Lie's theory as reviewed in the article [8] in this issue and studied in detail in the monographs [16, 18].

The fundamental notion is the *isotopy of the unit* of the theory considered [4, 6-14], in this case, the unit $I = \text{diag}(1, 1)$ of $SU(2)$, into a two-dimensional matrix $\hat{I}$ whose elements have the most general possible dependence on complex coordinates $z, \bar{z}$ of the underlying carrier space of $SU(2)$, their derivative with respect to time of arbitrary order, the wave functions $\psi, \psi^\dagger$ and their derivatives also of arbitrary order, and any needed additional quantity, subject to the condition of preserving the original axioms of I (smoothness, boundedness, nonsingularity, Hermiticity and positive-definiteness, as a necessary condition for isotopy),

$$I = \text{diag}(1, 1) > 0 \Rightarrow \hat{I} = \hat{I}(t, z, \bar{z}, \dot{z}, \dot{\bar{z}}, \psi, \psi^\dagger, \partial\psi, \partial\psi^\dagger, \dots) > 0. \quad (2.1)$$

The isotopy of the unit then demands, for consistency, a corresponding, compatible lifting of all associative products AB among generic QM quantities A, B , into the isoproduct

$$AB \Rightarrow A * B := AQB, \quad Q \text{ fixed}, \quad (2.2)$$

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where the isotopic character of the lifting is established by the preservation of associativity by the isoproduct, $A * (B * C) = (A * B) * C$.

The assumption $\hat{I} = Q^{-1}$ then implies that $\hat{I}$ is the correct left and right unit of the theory, $\hat{I} * A = A * \hat{I} \equiv A$, in which case Q is called the *isotopic element*, and $\hat{I}$ is called the *isounit*. Note the invariant appearance of q -deformations in their Q -operator form at the very foundation of the theory, provided that they are reformulated with respect to the new unit $\hat{I} = q^{-1}$ ([10, 11]).

The isotopies of the unit $I \Rightarrow \hat{I}$ and of the product $AB \Rightarrow A * B$ then imply the necessary lifting of all mathematical structures of quantum mechanics (QM) into those of a covering discipline called *hadronic mechanics* (HM) [16]. Here we mention the lifting of the field of complex numbers $C(c, +, \times)$, with elements c , ordinary sum $+$ and multiplication $c \times c' = cc'$, into the infinitely possible isotopes $\hat{C}_Q(\hat{c}, +, *)$, with *isocomplex numbers* $\hat{c} = 'C\hat{I}$, conventional sum $+$ and isomultiplication $\hat{c}_1 * \hat{c}_2 = \hat{c}_1 Q \hat{c}_2 = (c_1 c_2) \hat{I}$. Note for future use that, for an arbitrary quantity A , $\hat{c} * A = \hat{c} \hat{I} Q A \equiv cA$.

The isotopies of the unit, multiplication and fields then demand, for mathematical consistency, corresponding compatible isotopies of the basic carrier space, the two-dimensional complex Euclidean space $E(z, \bar{z}, \delta, C)$ with familiar metric $\delta = \text{diag}(1, 1)$ into the complex two-dimensional *iso-Euclidean spaces* introduced in [15, 16]

$$\hat{E}_Q(z, \bar{z}, \hat{\delta}, \hat{C}): z = (z_1, z_2), \quad \hat{\delta} = Q\delta \equiv g = \text{diag}(g_{11}, g_{22}) = g\hat{I} > 0, \quad (2.3a)$$

$$z \dagger_i g_{ij}(t, z, \bar{z}, \dots) z_j = \bar{z}_1 g_{11} z_1 + \bar{z}_2 g_{22} z_2, \quad (2.3b)$$

where the assumed diagonalization of Q is always possible (although not necessary) from its positive-definiteness.

The isotopic character (as well as novelty) of the generalization is established by the fact that, under the *joint* lifting of the metric $\delta \Rightarrow \hat{\delta} = Q\delta = g$ and of the field $C \Rightarrow \hat{C}_Q$, $\hat{I} = Q^{-1}$, all infinitely possible isospaces $\hat{E}_Q(z, \bar{z}, \hat{\delta}, \hat{C})$ are locally isomorphic to the original space $E(z, \bar{z}, \delta, C)$ under the sole condition of positive-definiteness of the isounit $\hat{I}$ [15]. In turn, this evidently sets the foundation for the local isomorphism of the corresponding symmetries.

Note that separation (2.3) is the most general possible nonlinear, nonlocal and noncanonical generalization of the original separation $z \dagger z$ under the sole condition of remaining positive-definite, i.e., of preserving the topology $\text{sig } \delta = \text{sig } \hat{\delta} = (+, +)$. The symmetries of invariant (2.3) are then expected to be nonlinear, nonlocal and noncanonical, as desired.

The preceding isotopies imply, for consistency, the isotopies of Hilbert spaces $\mathcal{H}: \langle \psi | \phi \rangle \in C$ into the so-called *iso-Hilbert space* $\hat{\mathcal{H}}_Q$ with *isoproduct* and *isonormalization*

$$\hat{\mathcal{H}}_Q: \langle \hat{\psi} | \hat{\phi} \rangle = \langle \hat{\psi} | Q | \hat{\phi} \rangle \hat{I} \in \hat{C}_Q; \quad \langle \hat{\psi} | \hat{\psi} \rangle = \hat{I}. \quad (2.4)$$

Then, operators which are Hermitian (observable) for QM remain Hermitian (observable) for HM [16].

The liftings of the Hilbert space require corresponding isotopies of all operations in $\mathcal{H}$ [13, 14]. We here mention isounitariness $\hat{U} * \hat{U}^\dagger = \hat{U}^\dagger * \hat{U} = \hat{I}$; isoeigenvalue equations $H * |\hat{\psi}\rangle = HQ|\hat{\psi}\rangle = \hat{E} * |\hat{\psi}\rangle \equiv E|\hat{\psi}\rangle$; isoexpectation values $\langle \hat{A} \rangle = \langle \hat{\psi} | QAQ | \hat{\psi} \rangle / \langle \hat{\psi} | Q | \hat{\psi} \rangle$; etc.

The lifting of the unit, base field and carrier space then require, for mathematical consistency, the lifting of the entire structure of Lie's theory first submitted in [10]. We are here referring to the isotopies of enveloping associative algebras $\hat{\xi}$, Lie algebras L , Lie groups G , representation theory, etc., today called Lie-Santilli theory. Here we mention the *isoassociative enveloping operator algebras* $\hat{\xi}_Q$ with isoproduct (2.2), $A * B \equiv AQB$; the *Lie-isotopic algebras* $\hat{L}_Q$ with isoproduct

$$[A, B]_{\hat{\xi}_Q} = [A, \hat{B}] = A * B - B * A \equiv AQB - BQA; \quad (2.5)$$

the (connected) *Lie-isotopic groups* $\hat{G}_Q$ of *isolinear isounitary transforms* on $\hat{E}_Q(z, \bar{z}, \hat{\delta}, \hat{O})$

$$z' = \hat{U}(w) * z = \hat{U}(w)Qz = \hat{U}(w)Q(z, \bar{z}, \hat{\delta}, \hat{O}, \hat{\psi}, \hat{\psi}^\dagger, \dots)z, \quad (2.6a)$$

$$\begin{aligned} \hat{U}(w) &= e_{\hat{\xi}_Q}^{iX * \hat{w}} = \hat{I} + (iXw)/1! + (iXw) * (iXw)/2! + \dots \\ &= \{e^{iXQw}\} \hat{I}, \end{aligned} \quad (2.6b)$$

$$\begin{aligned} \hat{U}(w) * \hat{U}(w') &= \hat{U}(w') * \hat{U}(w) = \hat{U}(w + w'), \\ \hat{U}(w) * \hat{U}(-w) &= \hat{U}(0) = \hat{I}, \end{aligned} \quad (2.6c)$$

where the reformulation in terms of the conventional exponentiation has been done for simplicity of calculations.

The *isounitary* $\hat{U}_Q(2)$ symmetry is the most general possible, nonlinear, nonlocal and noncanonical, simple, Lie-isotopic invariance group of separation (2.3b) with realization in terms of isounitary operators on $\hat{\mathcal{H}}_Q$

$$\hat{U} * \hat{U}^\dagger = \hat{U}^\dagger * \hat{U} = \hat{I} = Q^{-1}, \quad (2.7)$$

verifying isotopic laws (2.6). $\hat{U}(2)$ can be decomposed into the *connected, special isounitary symmetry* $S\hat{U}_Q(2)$ for

$$\det(\hat{U}Q) = +1, \quad (2.8)$$

plus a discrete part which is similar to that for $\hat{O}(3)$ [10] and is here ignored for brevity.

The connected $S\hat{U}_Q(2)$ components admit the realization in terms of new generators $\hat{J}_k$ and the same parameters $\theta_k \in R(n, +, \times)$ of $SU(2)$ although re-expressed in the isofield $\hat{R}(\hat{n}, +, *)$

$$\hat{U} = \prod_k e_{\hat{\xi}}^{i\hat{J}_k * \hat{\theta}_k} = \left\{ \prod_k e^{i\hat{J}_k Q \theta_k} \right\} \hat{I}, \quad (2.9)$$

under the conditions (necessary for isounitariness)

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$$\text{tr}(\hat{J}_k Q) \equiv 0, \quad k = 1, 2, 3. \quad (2.10)$$

The isorepresentations of the isotopic algebras $S\hat{U}_Q(2)$ can be studied by imposing that the isocommutation rules have the same structure constants of $SU(2)$, i.e., for the rules

$$[\hat{J}_i, \hat{J}_j] = \hat{J}_i Q \hat{J}_j - \hat{J}_j Q \hat{J}_i = i\epsilon_{ijk} \hat{J}_k. \quad (2.11)$$

with iso-Casimir

$$\hat{J}^2 = \sum_k \hat{J}_k * \hat{J}_k. \quad (2.12)$$

The maximal isocommuting set is then given by $\hat{J}^2$ and $\hat{J}_3$ as in the conventional case. These assumptions ensure the local isomorphism $S\hat{U}(2) \approx SU(2)$ by construction.

Let $|\hat{b}_k^d\rangle$ be the d -dimensional isobasis of $S\hat{U}_Q(2)$ with iso-orthogonality conditions

$$\langle \hat{b}_i^d | * | \hat{b}_j^d \rangle := \langle \hat{b}_i^d | Q | \hat{b}_j^d \rangle = \delta_{ij}, \quad i, j = 1, 2, \dots, n. \quad (2.13)$$

By putting as in the conventional case $\hat{J}_\pm = \hat{J}_1 \pm i\hat{J}_2$, and by repeating the same procedure as the familiar one [1], we have

$$\hat{J}_3 * |\hat{b}_k^d\rangle = b_k^d |\hat{b}_k^d\rangle, \quad \hat{J}^2 * |\hat{b}_k^d\rangle = b_k^d (b_k^d - 1) |\hat{b}_k^d\rangle, \quad (2.14a)$$

$$d = 1, 2, \dots, \quad k = 1, 2, \dots, d,$$

$$b_1^d \equiv -b_d^d, \quad b_1^d (b_1^d - 1) \equiv b_d^d (b_d^d + 1). \quad (2.14b)$$

A consequence is that the dimensions of the isorepresentations of $S\hat{U}_Q(2)$ remain the conventional ones, i.e., they can be characterized by the familiar expression $n = 2j + 1$, $j = 0, \frac{1}{2}, 1, \dots$ as expected from the isomorphism $S\hat{U}_Q(2) \approx SU(2)$.

However, the explicit form of the matrix representations are different than the conventional ones, as expressed by the rules

$$(\hat{J}_1)_{ij} = \frac{1}{2} i \langle \hat{b}_i^d | * (\hat{J}_- - \hat{J}_+) * | \hat{b}_j^d \rangle, \quad (2.15a)$$

$$(\hat{J}_2)_{ij} = \frac{1}{2} i \langle \hat{b}_i^d | * (\hat{J}_- + \hat{J}_+) * | \hat{b}_j^d \rangle, \quad (2.15b)$$

$$(\hat{J}_3)_{ij} = \langle \hat{b}_i^d | * \hat{J}_3 * | \hat{b}_j^d \rangle, \quad (2.15c)$$

under condition (2.10).

The isorepresentations of the desired dimension can then be constructed accordingly. In the next section we shall compute the two-dimensional isorepresentations, while those of higher dimensions will be studied in a subsequent paper.

A new image of the conventional $SU(2)$ symmetry is characterized by our isotopic methods via the antiautomorphic map $I = \text{diag}(1, 1) \Rightarrow I^d = -I$ called

isoduality ([12, 16]), which provides a novel and intriguing characterization of antiparticles. The corresponding *isodual isosymmetry* $S\hat{U}_Q^d(2)$ will be studied in a separate work.

In summary, our isotopic methods permit the identification of four physically relevant isotopies and isodualities of $SU(2)$ which, for the case of isospin, are given by the broken conventional $SU(2)$ for the usual treatment of $p - n$; the exact isotopic $S\hat{U}_Q(2)$ for the characterization of $p - n$ (see next section); the broken isodual $SU^d(2)$ symmetry for the characterization of the antiparticles $\bar{p} - \bar{n}$ in isodual spaces; and the exact, isodual, isotopic $S\hat{U}_Q^d(2)$ for the characterization of antiparticles $\bar{p} - \bar{n}$ in isodual isospace.

The reader may be interested in knowing that, when the positive- (or negative-) definiteness of the isotopic element Q is relaxed, the isotopes $S\hat{U}(2)$ unifies all three-dimensional simple Lie groups of Cartan classification over a complex field (of characteristic zero). In fact, we have the compact isotopes $S\hat{U}_Q(2) \approx SU(2)$ for $g_{11} > 0$, $g_{22} > 0$, and the noncompact isotopes $S\hat{U}_Q(2) \approx SU(1, 1)$ for $g_{11} > 0$ and $g_{22} < 0$ (see [8] for the corresponding unification of orthogonal groups over the reals). In this note we consider only positive-definite isotopic elements Q .

3. Isotopies of Pauli Matrices

Recall that the conventional Pauli matrices σ_k (see, e.g., [2, 6]) verify the rules $\sigma_i \sigma_j = i\epsilon_{ijk} \sigma_k$, $i, j, k = 1, 2, 3$. In this section we identify and classify the generalizations of these familiar matrices implied by the isoalgebra $S\hat{U}_Q(2)$.

To have a guiding principle, we recall that ([8, 15]), in general, *Lie-isotopic algebras are the image of Lie algebras under nonunitary transformations*. In fact, under a transformation $UU^\dagger = \hat{I} \neq I$, a Lie commutator among generic matrices A, B , acquires the Lie-isotopic form

$$\begin{aligned} U(AB - BA)U^\dagger &= A'QB' - B'QA', \\ A' &= UAU^\dagger, \quad B' = UBU^\dagger, \quad Q = (UU^\dagger)^{-1} = Q^\dagger. \end{aligned} \quad (3.1)$$

We therefore expect a first class of fundamental (adjoint) isorepresentations, here called *regular adjoint isorepresentations*, which are characterized by the maps $J_k = \frac{1}{2}\sigma_k \rightarrow \hat{J}_k = UJ_kU^\dagger$, $UU^\dagger \hat{I} \neq I$ with isotopic contributions that are factorizable in the spectra, $\pm \frac{1}{2} \rightarrow +\frac{1}{2}f(\Delta)$, $3/4 \rightarrow (3/4)f^2(\Delta)$, where $\Delta = \det Q$ and $f(\Delta)$ is a smooth nowhere-null function such that $f(1) = 1$.

An example is readily constructed via Equations (2.15) resulting in the following generalization of Pauli's matrices here called *regular iso-Pauli matrices*

$$\begin{aligned} \hat{\sigma}_1 &= \Delta^{-\frac{1}{2}} \begin{pmatrix} 0 & g_{11} \\ g_{22} & 0 \end{pmatrix}, & \hat{\sigma}_2 &= \Delta^{-\frac{1}{2}} \begin{pmatrix} 0 & -ig_{11} \\ +ig_{22} & 0 \end{pmatrix}, \\ \hat{\sigma}_3 &= \Delta^{-\frac{1}{2}} \begin{pmatrix} g_{22} & 0 \\ 0 & -g_{11} \end{pmatrix}, \end{aligned} \quad (3.2)$$

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where $\Delta = \det Q = g_{11}g_{22} > 0$. The above isorepresentation verifies the isotopic rules $\hat{\sigma}_i Q \hat{\sigma}_j = i\Delta^{\frac{1}{2}} \epsilon_{ijk} \hat{\sigma}_k$ and, consequently, the following isocommutator rules and generalized isoeigenvalues for $f(\Delta) = \Delta^{\frac{1}{2}}$

$$[\hat{\sigma}_i, \hat{\sigma}_j] = \hat{\sigma}_i Q \hat{\sigma}_j - \hat{\sigma}_j Q \hat{\sigma}_i = 2i\Delta^{\frac{1}{2}} \hat{\sigma}_k, \quad (3.3a)$$

$$\hat{\sigma}_3 * |\hat{b}_i^2\rangle = \pm \Delta^{\frac{1}{2}} |\hat{b}_i^2\rangle, \quad (3.3b)$$

$$\hat{\sigma}^2 * |\hat{b}_i^2\rangle = 3\Delta |\hat{b}_i^2\rangle, \quad i = 1, 2, \quad (3.3c)$$

which confirm the 'regular' character of the generalization here considered (that is, the factorizability of the isotopic contribution in the spectrum of eigenvalues). The isonormalized isobasis is then given by a trivial extension of the conventional basis $|\hat{b}\rangle = Q^{-\frac{1}{2}}|b\rangle$.

Recall that Pauli's matrices are essentially unique in the sense that their transformations under unitary equivalence do not yield significant changes in their structure, as well known ([1, 4]). The situation is different for the iso-Pauli matrices, because isorepresentations are based on various degrees of freedom which are absent in the conventional $SU(2)$ theory, such as: (1) infinitely possible isotopic elements Q ; (2) formulation of the isoalgebra in terms of structure functions [7, 9]; (3) use of an isotopic element for the iso-Hilbert space different than that of the isoalgebra [13, 14]; and others.

In fact, we can identified a second class of isorepresentations, here called *irregular adjoint isorepresentations*, in which the isotopic contributions is no longer factorizable in the entirety of the spectra of eigenvalues. A first example is given by the following *irregular iso-Pauli matrices*

$$\begin{aligned} \hat{\sigma}'_1 &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \sigma_1, & \hat{\sigma}'_2 &= \begin{pmatrix} 0 & -i \\ +i & 0 \end{pmatrix} = \sigma_2, \\ \hat{\sigma}'_3 &= \begin{pmatrix} g_{22} & 0 \\ 0 & -g_{11} \end{pmatrix} = \Delta \hat{\sigma}_3, \end{aligned} \quad (3.4)$$

which verify the isocommutation rules

$$[\hat{\sigma}'_1, \hat{\sigma}'_2] = 2i\hat{\sigma}'_3, \quad [\hat{\sigma}'_2, \hat{\sigma}'_3] = 2i\Delta \hat{\sigma}'_1, \quad [\hat{\sigma}'_1, \hat{\sigma}'_3] = 2i\Delta \hat{\sigma}'_2, \quad (3.5)$$

without evidently altering the local isomorphism $S\hat{U}_Q(2) \approx SU(2)$. The new isoeigenvalue equations are given by

$$\hat{\sigma}'_3 * |\hat{b}_i^2\rangle = \pm \Delta |\hat{b}_i^2\rangle, \quad \hat{\sigma}'^2 * |\hat{b}_i^2\rangle = \Delta(\Delta + 2) |\hat{b}_i^2\rangle, \quad (3.6)$$

which confirm the 'irregular' character consideration (that is, the lack of factorizability of the isotopic contributions in the entirety of the spectrum of eigenvalues). Isorepresentation (3.4) also provide an illustration of Equations (1.1) with the nontrivial lifting of the spin $s = \frac{1}{2} \rightarrow \hat{s} = \frac{1}{2}\Delta$.

The 'degrees of freedom' of isorepresentations are then illustrated via the following second example of irregular Pauli matrices

$$\begin{aligned}\hat{\sigma}_1'' &= \begin{pmatrix} 0 & g_{22}^{-\frac{1}{2}} \\ g_{11}^{-\frac{1}{2}} & 0 \end{pmatrix}, & \hat{\sigma}_2'' &= \begin{pmatrix} 0 & -ig_{22}^{-\frac{1}{2}} \\ ig_{11}^{-\frac{1}{2}} & 0 \end{pmatrix}, \\ \hat{\sigma}_3'' &= \begin{pmatrix} g_{11}^{-1} & 0 \\ 0 & -g_{22}^{-1} \end{pmatrix},\end{aligned}\quad (3.7)$$

with isocommutation rules and isoeigenvalues for $\hat{J}_k = \frac{1}{2}\hat{\sigma}_k''$

$$[\hat{J}_1, \hat{J}_2] = i\Delta\hat{J}_3, \quad [\hat{J}_2, \hat{J}_3] = i\hat{J}_1, \quad [\hat{J}_3, \hat{J}_1] = i\hat{J}_2, \quad (3.8a)$$

$$\hat{J}_3 * |\hat{b}_i^2\rangle = \pm \frac{1}{2}|\hat{b}_i^2\rangle, \quad \hat{J}^2 * |\hat{b}_i^2\rangle = \frac{1}{2}\left(\frac{1}{2} + \Delta\right)|\hat{b}_i^2\rangle, \quad (3.8b)$$

where, as one can see, the eigenvalue of the third component is conventional, but that of the magnitude is generalized with a nonfactorizable isotopic contribution.

Intriguingly, the isorepresentations generally occurring in physical applications are the irregular ones ([15, 16]) because the generators represent physical quantities and, as such, are not changed under isotopies [7-9]. Their embedding in an isotopic algebra then generally implies the appearance of the structure functions and irregular isorepresentations.

By no mean do the above two classes exhaust all possible, physically significant isorepresentations. We therefore introduce a third class of isorepresentations without any claim of completeness (in fact, we do not study here for brevity the isorepresentations with different isotopic elements for the isoenvelopes and iso-Hilbert space which characterize yet more general isorepresentations).

We here define as *standard adjoint isorepresentations* those occurring when the spectra of eigenvalues are conventional, but the representations are nontrivially generalized, i.e., remain nonunitarily equivalent to the conventional representations. In fact, regular iso-Pauli matrices (3.2) admit the conventional eigenvalues $1/2$ and $3/4$ for $\Delta = 1$. This condition can be verified by putting $g_{11} = g_{22}^{-1} = \lambda$. We discover in this way the existence of the *standard iso-Pauli matrices* first presented in [13]

$$\begin{aligned}\hat{\sigma}_1 &= \begin{pmatrix} 0 & g_{22}^{-1} \\ g_{11}^{-1} & 0 \end{pmatrix}, & \hat{\sigma}_2 &= \begin{pmatrix} 0 & -ig_{22}^{-1} \\ ig_{11}^{-1} & 0 \end{pmatrix}, \\ \hat{\sigma}_3 &= \begin{pmatrix} g_{11}^{-1} & 0 \\ 0 & -g_{22}^{-1} \end{pmatrix},\end{aligned}\quad (3.9)$$

which admit all conventional structure constants and eigenvalues for $\hat{J}_k = \frac{1}{2}\hat{\sigma}_k$,

$$[\hat{J}_i, \hat{J}_j] = i\epsilon_{ijk}\hat{J}_k, \quad \hat{J}_3 * |\hat{b}\rangle = \pm \frac{1}{2}|\hat{b}\rangle, \quad \hat{J}^2 * |\hat{b}\rangle = \frac{3}{4}|\hat{b}\rangle \quad (3.10)$$

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yet exhibit the 'hidden functions' g_{kk} in their structure.

Needless to say, isorepresentation (3.9) remains standard under the physically significant condition

$$\det Q = g_{11}g_{22} = 1,$$

which is realized for

$$g_{11} = g_{22} = \lambda \neq 0,$$

where λ is a sufficiently smooth, real-valued and nowhere-null function of the local variables. In this case, isorepresentation (3.9) assumes the form used in physical applications (see the next sections)

$$\begin{aligned} \hat{\sigma}_1 &= \begin{pmatrix} 0 & \lambda \\ \lambda^{-1} & 0 \end{pmatrix}, & \hat{\sigma}_2 &= \begin{pmatrix} 0 & -i\lambda \\ i\lambda^{-1} & 0 \end{pmatrix}, \\ \hat{\sigma}_3 &= \begin{pmatrix} \lambda^{-1} & 0 \\ 0 & -\lambda \end{pmatrix}. \end{aligned} \quad (3.11)$$

Similarly, irregular isorepresentations also become standard under condition (3.9) and realization (3.10). We therefore have the following additional standard iso-Pauli matrices

$$\begin{aligned} \hat{\sigma}'_1 &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \sigma_1, & \hat{\sigma}'_2 &= \begin{pmatrix} 0 & -i \\ +i & 0 \end{pmatrix} = \sigma_2, \\ \hat{\sigma}'_3 &= \begin{pmatrix} \lambda^{-1} & 0 \\ 0 & -\lambda \end{pmatrix}, \end{aligned} \quad (3.12a)$$

$$\begin{aligned} \hat{\sigma}''_1 &= \begin{pmatrix} 0 & \lambda^{\frac{1}{2}} \\ \lambda^{-\frac{1}{2}} & 0 \end{pmatrix}, & \hat{\sigma}''_2 &= \begin{pmatrix} 0 & -i\lambda^{\frac{1}{2}} \\ i\lambda^{-\frac{1}{2}} & 0 \end{pmatrix}, \\ \hat{\sigma}''_3 &= \begin{pmatrix} \lambda^{-1} & 0 \\ 0 & -\lambda \end{pmatrix}. \end{aligned} \quad (3.12b)$$

Iso-Pauli matrices with generalized eigenvalues are useful for interior structural problems, i.e., the description of a neutron in the core of a neutron star or, along the same lines, for a hadron constituent. As such, the applications of the general case of the $S\hat{U}_Q(2)$ isosymmetry is studied elsewhere [16].

When studying conventional particles, e.g., those of nuclear physics, the physically relevant subclass of $S\hat{U}_Q(2)$ is the special one with conventional eigenvalues, which is studied in the next sections. The image $\hat{\sigma}_k^d$ under isoduality, called *isodual Pauli matrices*, will be studied elsewhere.

4. Application to Isospin

As is well known (see, e.g., [2, 6]), the conventional $SU(2)$ -isospin symmetry is broken by electromagnetic and weak interactions. One of the first applications

of our isotopic/ Q -operator-deformation theory is to show that the $SU(2)$ -isospin symmetry can be reconstructed as exact at the isotopic level, namely, there exists a realization of the underlying isospace $\hat{E}_Q(z, \bar{z}, \hat{\delta}, \hat{O})$ in which protons and neutrons have the same mass, although the conventional values of mass are recovered under isoexpectation values.

The main idea is that the $SU(2)$ -isospin symmetry is broken when realized via the simplest conceivable Lie product $AB - BA$. However, when the same symmetry is realized via a lesser trivial product, such as our Lie-isotopic product $AQB - BQA$ [7], it can be proved to be exact even under electromagnetic and weak interactions. In this case, the elements of the Q -matrix are constants and acquires the meaning of average of these interactions.

The reader should be aware that this is an isolated occurrence, because it represents a rather general capabilities of the Lie-isotopic theory. In fact, it is referred to as the *isotopic reconstruction of exact spacetime and internal symmetries when conventionally broken*. For example, the rotational symmetry has been reconstructed as exact for all infinitely possible ellipsoidal deformations of the sphere; the Lorentz symmetry has been reconstructed as exact at the isotopic level for all possible signature preserving deformations $\hat{\eta} = Q\eta$ of the Minkowski metric, etc. [15].

The reconstruction of the exact $S\hat{U}_Q(2)$ -isospin symmetry is so simple to appear trivial. Consider a 2-component isostate

$$\hat{\psi}(x) = \begin{pmatrix} \hat{\psi}_p(x) \\ \hat{\psi}_n(x) \end{pmatrix}, \quad (4.1)$$

where $\hat{\psi}_p(x)$ and $\hat{\psi}_n(x)$ are solutions of the isodirac equation [19] which transforms isocovariantly under a standard isorepresentation of $\hat{P}_Q(3.1) \times S\hat{U}_Q(2)$. In this note we study only the $S\hat{U}_Q(2)$ part without any iso-Minkowskian coordinates, thus restricting our attention to the isonormalized isostates

$$\begin{aligned} |\hat{\psi}_p\rangle &= \begin{pmatrix} \lambda^{-1/2} \\ 0 \end{pmatrix}, & |\hat{\psi}_n\rangle &= \begin{pmatrix} 0 \\ \lambda^{1/2} \end{pmatrix}, \\ \langle \hat{\psi}_k | Q | \hat{\psi}_k \rangle &= 1, \quad k = p, n, \end{aligned} \quad (4.2)$$

where $Q = \text{diag}(\lambda, \lambda^{-1})$, $\hat{I} = Q^{-1} = \text{diag}(\lambda^{-1}, \lambda)$.

We then introduce the $S\hat{U}_Q(2)$ -isospin with isorepresentation (3.11) admitting conventional eigenvalues $\pm 1/2$ and $3/4$, defined over the isospace $\hat{E}_Q(z, \bar{z}, \hat{\delta}, \hat{O})$, $\hat{\delta} = Q\delta$.

We now select such an isospace to admit the same masses for the proton and the neutron. This is readily permitted by the 'hidden variable' λ when selected in such a way that

$$m_p \lambda^{-1} = m_n \lambda, \quad \text{i.e.,} \quad \lambda^2 = m_p / m_n = 0.99862. \quad (4.3)$$

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The mass operator can then be defined by

$$\begin{aligned}\widehat{M} &= \left\{ \frac{1}{2}\lambda(m_p + m_n)\widehat{I} + \frac{1}{2}\lambda^{-1}(m_p - m_n)\widehat{\sigma}_3 \right\} \widehat{I} \\ &= \begin{pmatrix} m_p\lambda^{-1} & 0 \\ 0 & m_n\lambda \end{pmatrix},\end{aligned}\quad (4.4)$$

and manifestly represents equal masses $\widehat{m} = m_p\lambda^{-1} = m_n\lambda$ in isospace.

The recovering of conventional masses in our physical space is readily achieved via the isoeigenvalue expression on an arbitrary isostate

$$\widehat{M} * |\widehat{\psi}\rangle = M\widehat{I}Q|\psi\rangle = M|\widehat{\psi}\rangle = \begin{pmatrix} m_p & 0 \\ 0 & m_n \end{pmatrix} |\widehat{\psi}\rangle \quad (4.5)$$

or, equivalently, via the isoexpectation values

$$\langle \widehat{\psi}_p | Q\widehat{M}Q | \widehat{\psi}_p \rangle = m_p, \quad \langle \widehat{\psi}_n | Q\widehat{M}Q | \widehat{\psi}_n \rangle = m_n. \quad (4.6)$$

Similarly, the charge operator can be defined by

$$q = \frac{1}{2}e(\widehat{I} + \widehat{\sigma}_3) = \begin{pmatrix} e\lambda^{-1} & 0 \\ 0 & 0 \end{pmatrix}. \quad (4.7)$$

Thus, the $S\widehat{U}_Q(2)$ charges on isospace are $q_p = e\lambda^{-1}$ and $q_n = 0$. However, the charges in our physical space are the conventional ones

$$\langle \widehat{\psi}_p | QqQ | \widehat{\psi}_p \rangle = e, \quad \langle \widehat{\psi}_n | QqQ | \widehat{\psi}_n \rangle = 0. \quad (4.8)$$

The *isodual* $S\widehat{U}_Q^Q(2)$ -isospin characterizing the antiparticle $\bar{p}$ and $\bar{n}$ will be studied elsewhere. The entire theory of isospin and its application can then be lifted in an isotopic form which remains exact under all interactions. This is not a mere mathematical curiosity, because it carries a corresponding isotopy of the nuclear force, e.g., via $S\widehat{U}_Q(2)$ -isotopic exchange mechanism, essentially representing the old legacy of a (generally small) nonlocal component in the nuclear structure. These dynamical implications are studied elsewhere.

5. Applications to Local Realism

The $S\widehat{U}_Q(2)$ theory studied above is based on a structural generalization of QM of nonlinear-nonlocal-non-Hamiltonian, although axiom-preserving type. However, in the so-called literature of *local realism* (see, e.g., [7]) there exist certain arguments, most notably Bell's inequality and von Neumann's theorem, *prohibiting* a generalization of quantum mechanics.

This note would therefore not be completed without an inspection of these issue and the proof that both Bell's inequality and von Neumann's theorem are *inapplicable* (and not 'violated') under isotopies. This then sets the foundations

for the isotopic completion of QM studied in this note. The study also serves as an application of the $S\hat{U}_Q(2)$ symmetry to spin.

The lack of applicability of Bell's inequality and von Neumann's theorem under regular and irregular isotopies is transparent from the alteration of the spectra of eigenvalues and, as such, deserves no additional comment.

In the following we show that the above inapplicability persists not only for standard isorepresentations (3.9) but also for the particular case of $\det Q = 1$, isorepresentations (3.11).

Consider two *standard isoparticles* with spin $\frac{1}{2}$, i.e., particles characterized by standard iso-Pauli matrices (3.9). Even though their spin is the same, there is no necessary reason to restrict their isotopic degrees of freedom λ to be the same outside isospin treatments (e.g., because their density may be different). We can therefore assume

$$\text{Particle 1: } Q = \text{diag}(\lambda, \lambda^{-1}), \quad \Delta = \det Q = 1, \quad \text{spin } \frac{1}{2}, \quad (5.1a)$$

$$\text{Particle 2: } Q' = \text{diag}(\lambda', \lambda'^{-1}), \quad \Delta' = \det Q' = 1, \quad \text{spin } \frac{1}{2} \quad (5.1b)$$

Next, consider the composite system of the two isoparticles 1 and 2 which is characterized by the isounit

$$\hat{I}_{\text{tot}} = \hat{I}_1 \times \hat{I}_2 = Q_{\text{Tot}}^{-1} = (Q \times Q')^{-1}. \quad (5.2)$$

To properly recompute the isotopies of Bell's inequality (see, e.g., [13] for the conventional case), it is necessary to identify the isonormalized basis $|\hat{S}_{1-2}\rangle$, that is, the basis of the total spin of the particles 1 and 2 normalized to $\hat{I}_{\text{tot}}$,

$$\langle \hat{S}_{1-2} | \hat{S}_{1-2} \rangle = \langle \hat{S}_{1-2} | Q_{\text{Tot}} | \hat{S}_{1-2} \rangle \hat{I}_{\text{tot}} = \hat{I}_{\text{tot}}. \quad (5.3)$$

A simple isotopy of the conventional case (see, e.g., [3], Sect. 17.9) then leads to the *isobasis for the singlet state*

$$|\hat{S}_{1-2}\rangle = \frac{1}{\sqrt{2}} \left\{ \begin{pmatrix} \lambda^{-\frac{1}{2}} \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ \lambda'^{\frac{1}{2}} \end{pmatrix} - \begin{pmatrix} 0 \\ \lambda^{\frac{1}{2}} \end{pmatrix} \begin{pmatrix} \lambda'^{-\frac{1}{2}} \\ 0 \end{pmatrix} \right\}. \quad (5.4)$$

It is a tedious but instructive exercise for the interested reader to verify isonormalization condition (5.3) by constructing the adjoint of basis (5.4), by sandwiching the quantity $T_{\text{tot}} = Q \times Q'$, by contracting only quantities of the same particle, and then multiplying the scalar results of the two different particles.

Next, recall that the conventional scalar product $\sigma \cdot a$, where a is a three-vector, has no mathematical or physical meaning in isospace $\hat{E}_Q(z, \bar{z}, \hat{\delta}, \mathbb{R})$ and must be replaced by the isoscalar product for isorepresentation (3.11)

$$\hat{\sigma} Q a = \begin{pmatrix} a_z & (a_x - i a_y) \\ (a_x + i a_y) & -a_z \end{pmatrix}. \quad (5.5)$$

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The tedious but straightforward repetition of the conventional procedure [7] under isotopy then leads to the expression

$$\begin{aligned} \langle \tilde{S}_{1-2} | (Q \times Q') \{ (\tilde{\sigma} * a) \times (\tilde{\sigma}' * b) \} (Q \times Q) | \tilde{S}_{1-2} \rangle \\ = -a_x b_x - a_y b_y - \frac{1}{2} (\lambda \lambda'^{-1} + \lambda^{-1} \lambda') a_z b_z. \end{aligned} \quad (5.6)$$

Consider now unit vectors a, b, a', b' along the z -axis. Then the Bell's inequality under the conventional $SU(2)$ symmetry [7]

$$D_{\text{Bell}} = \text{Max} |P(a, b) - P(a, b')| + |P(a', b) + P(a', b')| \leq 2, \quad (5.7a)$$

$$P(a, b) = \langle S_{1-2} | (\sigma_1 \cdot a) \times (\sigma_2 \cdot b) | S_{1-2} \rangle = -a \cdot b \quad (5.7b)$$

admits the following isotopic image under the covering $S\tilde{U}_Q(2)$ symmetry

$$D_{\text{Bell}} \leq D_{\text{Max}}^{\text{HM}} = \frac{1}{2} (\lambda \lambda'^{-1} + \lambda^{-1} \lambda') D_{\text{Bell}}. \quad (5.8)$$

But, the factor $\frac{1}{2} (\lambda \lambda'^{-1} + \lambda^{-1} \lambda')$ can be easily proved to admit values bigger than one. This establishes the statement of Section 1, to the effect that Bell's inequality is not universally valid, but holds, specifically, for the conventional, linear, local and canonical realization of the $SU(2)$ symmetry. The proof for arbitrary orientations of the unit vectors follows the conventional one [3] and it is here omitted for brevity.

Similarly, von Neumann theorem [7] is inapplicable under isotopies because based on the uniqueness of the spectrum of eigenvalues of Hermitian operators. In fact, isotopic theories establish that the *same* Hermitian operator H admits *an infinite variety of different spectra of eigenvalues*, trivially, because of the infinitely possible isotopic elements Q , $H * |\psi\rangle = H Q |\psi\rangle = E_Q |\psi\rangle$ [13].

Similar obstacles to the completion of QM into a covering theory are removed under isotopies as shown elsewhere [16]. We here merely mention the reason why HM is indeed a completion of QM much along the EPR argument [3]. Recall that

$$D_{\text{Max}}^{\text{Class.}} = \text{Max} |a \cdot b - a \cdot b'| + |a' \cdot b + a' \cdot b'| = 2\sqrt{2} > 2. \quad (5.9)$$

and that $D_{\text{Bell}} < D_{\text{Max}}^{\text{Class.}}$, thus preventing the completion of quantum mechanics.

However, under isotopic liftings, one can assume a classical iso-Euclidean space $\tilde{E}(r, \tilde{\delta}, \tilde{\mathbb{R}})$ (representing motion of extended objects within physical media [11]) with isotopic scalar product

$$a * b = a^i Q_i b = a_x g_{11} b_x + a_y g_{22} b_y + a_z g_{33} b_z. \quad (5.10)$$

Then, there *always exists a realization of $\tilde{E}(r, \tilde{\delta}, \tilde{\mathbb{R}})$ under which we have the identity of the maximal operator and classical values*, $\tilde{D}_{\text{Max}}^{\text{HM}} \equiv \tilde{D}_{\text{Max}}^{\text{Classical}}$, as it is the case for the orientation of the unit vectors as above, and values

$$g_{11} = g_{22} = 1, \quad g_{33} = \frac{1}{2} (\lambda \lambda'^{-1} + \lambda^{-1} \lambda') = \sqrt{2}. \quad (5.11)$$

A number of additional, intriguing completions of QM are provided by HM along the EPR argument, such as the recovering of classical determinism for a particle in the interior of a gravitational singularity and others [16].

In closing, it is hoped that systematic studies on the isorepresentations of Lie-isotopic algebras, such as the isotopic $\hat{O}(3)$, $\hat{O}(3.1)$, $S\hat{L}(2,\hat{\mathbb{C}})$, $\hat{P}(3.1)$, $S\hat{U}(3)$, etc., are conducted by interested colleagues because of their capabilities of novel applications, that is, results beyond the capacity of the conventional Lie theory.

Acknowledgement

The author would like to thank the JINR for kind hospitality during the Summer 1993.

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Studies on the classical determinism predicted by A. Einstein, B. Podolsky and N. Rosen

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Abstract

In this paper, we continue the study initiated in preceding works of the argument by A. Einstein, B. Podolsky and N. Rosen according to which quantum mechanics could be "completed" into a broader theory recovering classical determinism. By using the previously achieved isotopic lifting of applied mathematics into isomathematics and that of quantum mechanics into the isotopic branch of hadronic mechanics, we show that extended particles appear to progressively approach classical determinism in the interior of hadrons, nuclei and stars, and appear to recover classical determinism at the limit conditions in the interior of gravitational collapse.

Keywords: EPR argument, isomathematics, isomechanics.

2010 AMS subject classifications: 05C15, 05C60. ¹

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¹Received on September 21st, 2019. Accepted on December 20rd, 2019. Published on December 31st, 2019. doi:10.23755/rm.v36i1.471. ISSN: 1592-7415. eISSN: 2282-8214. doi: 10.23755/rm.v37i0.477 ©Ruggero Maria Santilli

1. Introduction

1.1. The EPR argument

As it is well known, Albert Einstein did not consider quantum mechanical uncertainties to be final, for which reason he made his famous quote "God does not play dice with the universe."

More particularly, Einstein accepted quantum mechanics for atomic structures and other systems, but believed that quantum mechanics is an "incomplete theory," in the sense that it could be broadened into such a form to recover classical determinism at least under limit conditions.

Einstein communicated his views to B. Podolsky and N. Rosen and they jointly published in 1935 the historical paper [1] that became known as the *EPR argument*.

Soon after the appearance of paper [1], N. N. Bohr published paper [2] expressing a negative judgment on the possibility of "completing" quantum mechanics along the lines of the EPR argument.

Bohr's paper was followed by a variety of papers essentially supporting Bohr's rejection of the EPR argument, among which we recall *Bell's inequality* [3] establishing that the $SU(2)$ spin algebra does not admit limit values with an identical classical counterpart.

We should also recall *von Neumann theorem* [4] achieving a rejection of the EPR argument via the uniqueness of the eigenvalues of quantum mechanical Hermitean operators under unitary transforms.

The field became known as *local realism* and was centered on the rejection of the EPR argument via additional claims that hidden variables [5] are not admitted by quantum axioms (see the review [6]).

1.2. The 1998 apparent proof of the EPR argument

In 1998, the author published paper [7] presenting an apparent proof of the EPR argument based on the following main steps that we here outline to render this paper minimally self-sufficient:

Step 1: The proof that Bell's inequality, von Neumann's theorem and other similar objections against the EPR argument [6] are indeed correct, but under the generally tacit assumptions of point-like particles moving in vacuum under sole potential/Hamiltonian interactions (*exterior dynamical systems*) when the systems are treated via quantum mechanics and its underlying 20th century mathematics, including Lie's theory and the

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Newton-Leibnitz differential calculus;

Step 2: The proof that the above treatments are not applicable for extended, therefore deformable and hyperdense particles under conditions of mutual penetration or entanglement occurring in the structure of hadrons, nuclei, stars, and gravitational collapse such as for black holes, with novel non-linear, non-local, and non-potential/non-Hamiltonian interactions (*interior dynamical systems*);

Step 3: The treatment of interior systems via the axiom-preserving lifting of 20th century applied mathematics known as *isomathematics*, whose study was initiated by the author in the late 1970's when he was at Harvard University under DOE support, Refs. [8] to [12] and then continued by various mathematicians. Isomathematics is based on:

3-A) The axiom-preserving isotopy of the conventional associative product between generic quantities a, b (numbers, functions, operators, etc.) first introduced in Eq. (5), p. 71 of Ref. [11]

$$ab \rightarrow a \star b = a\hat{T}b, \quad (1)$$

where $\hat{T}$ is a positive-definite quantity called the *isotopic element* providing a representation of the dimension, deformability and density of particles and physical media in which they are immersed via realizations of the type

$$\hat{T} = \text{Diag.}(\frac{1}{n_1^2}, \frac{1}{n_2^2}, \frac{1}{n_3^2}, \frac{1}{n_4^2})e^{-\Gamma}, \quad (2)$$

where: n_4^2 represents the density; n_k^2 , $k = 1, 2, 3$ represents the deformable share of particles; n_μ^2 , $\mu = 1, 2, 3, 4$, and Γ are solely restricted to be positive-definite but otherwise admit a functional dependence on any needed local variables, such as time t , coordinates r , momenta p , energy E , density d , temperature τ , pressure π , wavefunctions ψ , their derivatives $\partial\psi$, etc.

$$n_\mu = n_\mu(t, r, p, E, d, \tau, \pi, \psi, \partial\psi, \dots) > 0, \quad \mu = 1, 2, 3, 4, \quad (3)$$

$$\Gamma = \Gamma(t, r, p, E, d, \tau, \pi, \psi, \partial\psi, \dots) \gg 0. \quad (4)$$

$$e^{-\Gamma(t, r, p, E, d, \tau, \pi, \psi, \partial\psi, \dots)} \ll 1. \quad (5)$$

3-B) The formulation of isoassociative algebras on an *isofield* $\hat{F}(\hat{n}, \star, \hat{I})$ first introduced in Ref. [13] (see also independent work [14]), with *isounit*

$$\hat{I} = 1\hat{T}, \quad (6)$$

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and *isoreal, isocomplex and isoquaternionic isonumbers* $\hat{n} = n\hat{I}$ under isoproduct (1), with ensuing isooperations such as the isosquare

$$\hat{n}^2 = \hat{n} \star \hat{n}. \quad (7)$$

Isofields also imply the lifting of functions into *isofunctions* [11] [20]

$$\hat{f}(\hat{r}) = [f(r\hat{I})]\hat{I}, \quad (8)$$

among which we quote the *isoexponentiation*

$$\hat{e}^X = (e^{X\hat{T}})\hat{I} = \hat{I} (e^{\hat{T}X}), \quad (9)$$

where X is a Hermitean operator.

3-C) The ensuing axiom-preserving lifting of Lie's theory into a non-linear, non-local and non-Hamiltonian form first introduced in Ref. [11] (see also the recent paper [15] and independent work [16]), which theory is today known as the *Lie-Santilli isothory*, with isobrackets at the foundation of Ref. [7]

$$[X;Y] = X \star Y - Y \star X = X\hat{T}Y - Y\hat{T}X. \quad (10)$$

3-D) The isotopic lifting of the Newton-Leibnitz differential calculus, from its historical definition at isolated points, into a form defined on volumes, first introduced in Ref. [17] (see Refs. [18] for vast independent works) with *isodifferential*

$$\begin{aligned} \hat{d}\hat{r} &= \hat{T}(r, \dots)d\hat{r} = \\ &= \hat{T}(r, \dots)d[r\hat{I}(r, \dots)] = dr + r\hat{T}d\hat{I}(r, \dots), \end{aligned} \quad (11)$$

and corresponding *isoderivatives*

$$\frac{\hat{\partial}\hat{f}(\hat{r})}{\hat{\partial}\hat{r}} = \hat{I}\frac{\partial\hat{f}(\hat{r})}{\partial\hat{r}}. \quad (12)$$

Step 4: The axiom-preserving lifting of quantum mechanics into the *isotopic branch of hadronic mechanics*, or *isomechanics* for short, whose study was initiated in Refs. [8] to [12] (see the 1995 monographs [19] [20] [21] with 2008 upgrade [22] and independent studies [23][24]).

Isomechanics is formulated on a *Hilbert-Myung-Santilli (HMS) isospace* [25] $\hat{\mathcal{H}}$ over the isofield of isocomplex isonumbers $\hat{\mathcal{C}}$, and it is based on the

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iso-Heisenberg isoequations for the time evolution of a Hermitean operator $\hat{Q}$ in the infinitesimal form

$$\begin{aligned} \hat{i} \star \frac{d\hat{Q}}{dt} &= [\hat{Q}, \hat{H}] = \hat{Q} \star \hat{H} - \hat{H} \star \hat{Q} = \\ &= \hat{Q} \hat{T} \hat{H} - \hat{H} \hat{T} \hat{Q}, \end{aligned} \quad (13)$$

and the finite form

$$\begin{aligned} \hat{Q}(\hat{t}) &= \hat{U}(\hat{t})^\dagger \star \hat{Q}(0) \star \hat{U}(\hat{t}) = \\ &= \hat{e}^{\hat{H} \star \hat{t} \star \hat{H}} \star \hat{Q}(0) \star \hat{e}^{-\hat{H} \star \hat{t} \star \hat{H}} = \\ &= e^{\hat{H} \hat{T} \hat{t}} \hat{Q}(0) e^{-\hat{H} \hat{T} \hat{t}}, \end{aligned} \quad (14)$$

with the following rules for the basic *isounitary isotransforms*

$$\hat{U}(\hat{t})^\dagger \star \hat{U}(\hat{t}) = \hat{U}(\hat{t}) \star \hat{U}(\hat{t})^\dagger = \hat{I}, \quad (15)$$

where $\hat{t} = t\hat{I}_t$ is the *isotime* which is assumed hereon to coincide with conventional time, $\hat{I}_t = 1$. Dynamical equations (13) to (15) were first presented in Eq. (4.16.49), page 752 of Ref. [9] over conventional fields and reformulated via the full use of isomathematics in Ref. [17]).

Isomechanics is also based on the *iso-Schrödinger isorepresentation* characterized by the fundamental representation of the *isomomentum* permitted by the isodifferential isocalculus, Eq. (12),

$$\begin{aligned} \hat{p}|\psi(\hat{t}, \hat{r})\rangle &= -\hat{i} \star \hat{\partial}_{\hat{t}, \hat{r}}|\psi(\hat{t}, \hat{r})\rangle = \\ &= -i\hat{I}\hat{\partial}_{\hat{r}}|\psi(\hat{t}, \hat{r})\rangle, \end{aligned} \quad (16)$$

from which one can derive the *iso-Schrödinger isoequation*, [12] [17] [20]

$$\begin{aligned} \hat{i} \star \hat{\partial}_{\hat{t}}|\psi(\hat{t}, \hat{r})\rangle &= \hat{H} \star |\psi(\hat{t}, \hat{r})\rangle = \\ &= \hat{H}(r, p)\hat{T}(t, r, p, E, d, \tau, \pi, \psi, \partial\psi, \dots)|\psi(\hat{t}, \hat{r})\rangle = \\ &= \hat{E} \star |\psi(\hat{t}, \hat{r})\rangle = E|\psi(\hat{t}, \hat{r})\rangle \end{aligned} \quad (17)$$

and the *isocanonical isocommutation rules*,

$$[\hat{r}_i, \hat{p}_j]|\psi\rangle = \hat{i} \star \hat{\delta}_{i,j} \star |\psi\rangle = i\delta_{ij}|\psi\rangle \quad (18)$$

$$[\hat{r}_i, \hat{r}_j]|\hat{\psi}\rangle = [\hat{p}_i, \hat{p}_j]|\hat{\psi}\rangle = 0. \quad (19)$$

Note that the characterization of extended particles at mutual distances smaller than their size requires the knowledge of *two quantities*, the conventional Hamiltonian H for the representation of potential interactions, and the isotopic element $\hat{T}$ for the representation of dimension, shape, density as well as of non-linear, non-local and non-potential interactions.

Step 5: The proof in Ref. [7] that the isotopic $\hat{S}U(2)$ -spin symmetry for extended particles immersed within a dense hadronic medium admits an explicit and concrete realization of *hidden variables* [5], e.g., of the type

$$\hat{T} = \text{Diag.}(\lambda, 1/\lambda), \quad \text{Det}\hat{T} = 1. \quad (20)$$

In particular, the isotopic $\hat{S}U(2)$ -spin isosymmetry admits limit conditions with identical classical counterpart, Eq. (5.4) page 189 Ref. [7].

One aspect of isomathematics and isomechanics which is crucial for this paper is that *in all applications to date, the isotopic element $\hat{T}$ has values much smaller than 1*, Eqs. (4) (5), as it has been the case for: the synthesis of the neutron from the hydrogen in the core of stars; the representation of nuclear magnetic moments and spin; new clean energies; and other applications [21].

It should be also noted that thanks to the new interactions represented by $\hat{T}$, isomathematics and isomechanics have permitted the first known identification of the *attractive force between identical valence electron pairs* in molecular structures [26]. A significant confirmation of values $|\hat{T}| \ll 1$ is provided by the fact that exact representations of binding energies for the hydrogen and water molecules have been achieved with *isoserries based on isoproduct (1) that are at least one thousand times faster than conventional quantum chemical series* [27] [28].

We should finally indicate that the numerical invariance of the isotopic element $\hat{T}$ and therefore, of the isounit $\hat{I} = 1/\hat{T}$, under isounitary time evolutions (14) (15) was proved in Ref. [29]. Detailed reviews and upgrades of isomathematics, isomechanics, and their applications to interior problems which are specifically written for the EPR argument should soon be available in Refs. [30] [31].

1.3. Aim of the paper

In this work, we shall attempt to complete the proof of the EPR argument of Ref. [7] by showing that extended particles in interior dynamical

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conditions appear to progressively recover classical determinism in interior dynamical conditions with the increase of the density and other characteristics, as indicated at the end of Ref. [7].

It should be stressed that a technical understanding of this work requires technical knowledge of hadronic mechanics, e.g., from Refs. [19] [20] [21] or from the forthcoming reviews and upgrades [30] [31].

We should indicate that the words "completion of quantum mechanics" is used in Einstein's sense for the intent of honoring his memory. For instance, the conventional associative product ab of Eq. (1), which is at the foundation of quantum mechanics, admits a "completion" into the equally associative, yet more general isoproduct $a\hat{T}b$. Under no conditions Einstein's word "completed theory" should be confused with a 'final theory,' that is a theory admitting no additional Einstein's "completions." In fact, the time-reversal invariant, Lie-isotopic isomathematics and isomechanics studied in this work admit the "completion" into the covering, irreversible *Lie-admissible genomathematics and genomechanics* (in which $\hat{T}$ is no longer Hermitean) which, in turn, admit a covering via the most general mathematics and mechanics conceived by the human mind, the multi-valued *hypermathematics and hypermechanics* [32] [33], with additional "coverings" remaining possible in due time [19] [20] [21].

The reader should be finally aware that the isotopic element $\hat{T}$ and isounit $\hat{I} = 1/\hat{T}$ are inverted in some of the early quoted literature not dealing with determinism without affecting their consistency. An important aim of this paper has been that of achieving the final selection of isotopic element and isounit which is compatible with studies on determinism.

2. Recovering of determinism in interior conditions?

2.1. Heisenberg uncertainty principle

Consider an electron in empty space represented with the 3-dimensional Euclidean space $E(r, \delta, I)$, where r represents coordinates, $\delta = \text{Diag.}(1, 1, 1)$ represents the Euclidean metric and $I = \text{Dian}(1, 1, 1,)$ is the space unit.

Let the operator representation of said electron be done in a Hilbert space $\mathcal{H}$ over the field of complex numbers $\mathcal{C}$ with states $\Psi(r)$ and familiar normalization

$$\langle \Psi(r) | \Psi(r) \rangle = \int_{-\infty}^{+\infty} \Psi(r)^\dagger \Psi(r) dr = 1. \quad (21)$$

As it is well known, the primary objections against the EPR argument

[2] [3] [4] were based on the *uncertainty principle* formulated by Werner Heisenberg in 1927, according to which *the position r and the momentum p of said electron cannot both be measured exactly at the same time.*

By introducing the *standard deviations* Δr and Δp , the uncertainty principle is generally written in the form (see, e.g., [5])

$$\Delta r \Delta p \geq \frac{1}{2} \hbar, \quad (22)$$

easily derivable via the vacuum expectation value of the canonical commutation rule

$$\Delta r \Delta p \geq \left| \frac{1}{2i} \langle \Psi | [r, p] | \Psi \rangle \right| = \frac{1}{2} \hbar. \quad (23)$$

The standard deviations have the known form [34] (with $\hbar = 1$)

$$\begin{aligned} \Delta r &= \sqrt{\langle \Psi(r) | [r - (\langle \Psi(r) | r | \Psi(r) \rangle)]^2 | \Psi(r) \rangle}, \\ \Delta p &= \sqrt{\langle \Psi(p) | [p - (\langle \Psi(p) | p | \Psi(p) \rangle)]^2 | \Psi(p) \rangle}, \end{aligned} \quad (24)$$

where $\Psi(r)$ and $\Psi(p)$ are the wavefunctions in coordinate and momentum spaces, respectively.

2.2. Particle in interior conditions

We consider now the electron, this time, in the core of a star classically represented with the *iso-Euclidean isospace* $\hat{E}(\hat{r}, \hat{\delta}, \hat{I})$ [17] with basic isounit $\hat{I} = 1/\hat{T} > 0$, isocoordinates $\hat{r} = r\hat{I}$, isometric

$$\hat{\delta} = \hat{T}\delta, \quad (25)$$

and isotopic element of type (2) under conditions (3) to (5).

Besides being immersed in the core of a star, the electron has no Hamiltonian interactions. Consequently, we can represent the electron in the *HMS isospace* $\hat{\mathcal{H}}$ [25] over the isofield of isocomplex isonumbers $\hat{\mathcal{C}}$ [13], and introduce the time independent *isoplanewave* [20]

$$\begin{aligned} \hat{\Psi}(\hat{r}) &= \hat{\psi}(\hat{r})\hat{I} = \\ &= \hat{N} \star (\hat{e}^{i\hat{k}\hat{r}})\hat{I} = N(e^{ik\hat{T}\hat{r}})\hat{I}, \end{aligned} \quad (26)$$

where $\hat{N} = N\hat{I}$ is an *isonormalization isoscalar*, $\hat{k} = k\hat{I}$ is the *isowavenumber*, and the isoexponentiation is given by Eq. (9).

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The corresponding representation in isomomentum isospace is given by

$$\hat{\Psi}(\hat{p}) = \hat{M} \star \hat{e}^{i\hat{n}\star\hat{p}}, \quad (27)$$

where $\hat{M} = M\hat{I}$ is an isonormalization isoscalar and $\hat{n} = n\hat{I}$ is the isowavenumber in isomomentum isospace.

2.3. Isodeterministic isoprinciple

The *isoprobability isofunction* is given by [20]

$$\begin{aligned} \hat{\mathcal{P}} &= \hat{<} \star | \star | \hat{>} = \langle \hat{\Psi}(\hat{r}) | T | \hat{\Psi}(\hat{r}) \rangle \hat{I} = \\ &= [\int_{-\infty}^{+\infty} \hat{\Psi}(\hat{r})^\dagger \star \hat{\Psi}(\hat{r}) \star \hat{d}\hat{r}] \hat{I} = \\ &= [\int_{-\infty}^{+\infty} \hat{\psi}(\hat{r})^\dagger \hat{\psi}(\hat{r}) \hat{d}\hat{r}] \hat{I}, \end{aligned} \quad (28)$$

where one should keep in mind that the isodifferential $\hat{d}\hat{r}$ is now given by Eqs. (11).

The *isoexpectation isovalues* of a Hermitean operator $\hat{Q}$ are then given by [20]

$$\begin{aligned} \hat{<} \star \hat{Q} \star | \hat{>} &= \langle \hat{\Psi}(\hat{r}) | \star \hat{Q} \star | \hat{\Psi}(\hat{r}) \rangle \hat{I} = \\ &= [\int_{-\infty}^{+\infty} \hat{\Psi}(\hat{r})^\dagger \star \hat{Q} \star \hat{\Psi}(\hat{r}) \hat{d}\hat{r}] \hat{I} = \\ &= [\int_{-\infty}^{+\infty} \hat{\psi}(\hat{r})^\dagger \hat{Q} \hat{\psi}(\hat{r}) \hat{d}\hat{r}] \hat{I}, \end{aligned} \quad (29)$$

with corresponding expressions for the isoexpectation isovalues in isomomentum isospace.

We now introduce, apparently for the first time in this paper, the *isotopic operator*

$$\hat{\mathcal{T}} = \hat{T}\hat{I} = \hat{I}, \quad (30)$$

that, despite its seemingly irrelevant value, is indeed the correct operator formulation of the isotopic element for the transition of the isoproduct from its scalar form (1) into the isoscalar form

$$\hat{n}^{\hat{2}} = \hat{n} \star \hat{n} = \hat{n} \star \hat{\mathcal{T}} \star \hat{n} = n^2 \hat{I}. \quad (31)$$

Since the identity $\hat{I}$ can be inserted anywhere in the expectation values of quantum mechanics without altering the results, realization (33) illustrates the central feature of the isotopies, namely, the property that the abstract axioms of quantum mechanics admit a “hidden” realization broader

than that of the Copenhagen School whose degrees of freedom have been used in Ref.[7] for the proof of the EPR argument [1].

We now introduce the isoexpectation isovalue of the isotopic operator

$$\begin{aligned} \hat{I} &= \langle \hat{\Psi}(\hat{r}) | \hat{T} | \hat{\Psi}(\hat{r}) \rangle = \\ &= \int_{-\infty}^{+\infty} \hat{\psi}(\hat{r})^\dagger \hat{T} \hat{\psi}(\hat{r}) d\hat{r}, \end{aligned} \quad (32)$$

and assume the isonormalization

$$\begin{aligned} \langle \hat{\Psi}(\hat{r}) | \hat{T} | \hat{\Psi}(\hat{r}) \rangle &= \\ &= \int_{-\infty}^{+\infty} \hat{\psi}(\hat{r})^\dagger \hat{T} \hat{\psi}(\hat{r}) d\hat{r} = \hat{T}. \end{aligned} \quad (33)$$

We then introduce, in this paper apparently for the first time, the *iso-standard isodeviation* for isocoordinates $\Delta\hat{r} = \Delta r \hat{I}$ and isomomenta $\Delta\hat{p} = \Delta p \hat{I}$, where Δr and Δp are the standard deviations in our space.

By using isocanonical isocommutation rules (18), we obtain the expression

$$\begin{aligned} \Delta\hat{r} \star \Delta\hat{p} &= \Delta r \Delta p \hat{I} \approx \frac{1}{2} | \langle \hat{\Psi}(\hat{r}) | \star [\hat{r}, \hat{p}] \star | \hat{\Psi}(\hat{r}) \rangle | = \\ &= \frac{1}{2} | \langle \hat{\Psi}(\hat{r}) | \hat{T} [\hat{r}, \hat{p}] \hat{T} | \hat{\Psi}(\hat{r}) \rangle |. \end{aligned} \quad (34)$$

By eliminating the common isounit $\hat{I}$, we then have the desired *isode-terministic isoprinciple* here proposed apparently for the first time

$$\begin{aligned} \Delta r \Delta p &\approx \frac{1}{2} | \langle \hat{\Psi}(\hat{r}) | \star [\hat{r}, \hat{p}] \star | \hat{\Psi}(\hat{r}) \rangle | = \\ &= \frac{1}{2} | \langle \hat{\Psi}(\hat{r}) | \hat{T} [\hat{r}, \hat{p}] \hat{T} | \hat{\Psi}(\hat{r}) \rangle | = \\ &= \int_{-\infty}^{+\infty} \hat{\psi}(\hat{r})^\dagger \hat{T} \hat{\psi}(\hat{r}) d\hat{r} = T \ll 1 \end{aligned} \quad (35)$$

where the property $\Delta r \Delta p \ll 1$ follows from the fact that *the isotopic element $\hat{T}$ has always a value smaller than 1* (Section 1.2).

It is now necessary to verify isoprinciple (35) by proving that the iso-standard isodeviations tend to null values when $\hat{T} \rightarrow 0$.

For this purpose, we introduce the following simple isotopy of Eqs. (24) (where we ignore the common multiplication by the isounit)

$$\begin{aligned} \Delta r &= \sqrt{\langle \hat{\Psi}(\hat{r}) | [\hat{r} - \langle \hat{\Psi}(\hat{r}) | \star \hat{r} \star | \hat{\Psi}(\hat{r}) \rangle]^2 | \hat{\Psi}(\hat{r}) \rangle}, \\ \Delta p &= \sqrt{\langle \hat{\Psi}(\hat{p}) | [\hat{p} - \langle \hat{\Psi}(\hat{p}) | \star \hat{p} \star | \hat{\Psi}(\hat{p}) \rangle]^2 | \hat{\Psi}(\hat{p}) \rangle}, \end{aligned} \quad (36)$$

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where the differentiation between the isotopic elements for isocoordinates and isomomenta is ignored for simplicity.

It is then easy to see that the isosquare (7) implies the covering forms of the isostandard isodeviations

$$\begin{aligned}\Delta r &= \sqrt{\hat{T} < \hat{\Psi}(\hat{r}) | [\hat{r} - < \hat{\Psi}(\hat{r}) | \star \hat{r} \star | \hat{\Psi}(\hat{r}) >]^2 | \hat{\Psi}(\hat{r}) >}, \\ \Delta p &= \sqrt{\hat{T} < \hat{\Psi}(\hat{p}) | [\hat{p} - < \hat{\Psi}(\hat{p}) | \star \hat{p} \star | \hat{\Psi}(\hat{p}) >]^2 | \hat{\Psi}(\hat{p}) >},\end{aligned}\tag{37}$$

that indeed approach null value under the limit conditions

$$\begin{aligned}\lim_{\hat{T}=0} \Delta r &= 0, \\ \lim_{\hat{T}=0} \Delta p &= 0,\end{aligned}\tag{38}$$

thus confirming isodeterministic isoprinciple (35).

2.4. Particles under pressure

To illustrate the above expressions, we consider an electron in the center of a star, thus being under extreme pressures π from the surrounding hadronic medium in all radial directions, while ignoring particle reactions in first approximation or under a sufficiently short period of time.

These conditions are here rudimentarily represented by assuming that the $\Gamma > 0$ function of the isotopic element (2) is a constant linearly dependent on the pressure π , resulting in a realization of the isotopic element of the type

$$\hat{T} = e^{-w\pi} \ll 1, \quad \hat{I} = e^{+w\pi} \gg 1,\tag{39}$$

where w is a positive constant.

The isodeterministic isoprinciple for the considered particle is then given by

$$\Delta r \Delta p \approx \frac{1}{2} e^{-w\pi} \ll 1,\tag{40}$$

and tends to null values for diverging pressures.

The above example illustrates the consistency of isorenormalization (33) because, a constant isotopic element implies the consistent expression

$$\begin{aligned}\hat{\Psi}(\hat{r}) | \hat{T} | \hat{\Psi}(\hat{r}) > \hat{I} &= \\ T < \hat{\Psi}(\hat{r}) | \hat{\Psi}(\hat{r}) > \hat{I} &= \\ < \hat{\Psi}(\hat{r}) | \hat{\Psi}(\hat{r}) >, &\end{aligned}\tag{41}$$

while, by contrast, the following alternative isonormalization

$$\hat{\psi}(\hat{r})|\hat{T}|\hat{\psi}(\hat{r}) > \hat{I} = \hat{I}, \quad (42)$$

would imply the expression

$$\langle \hat{\psi}(\hat{r}) | \hat{\psi}(\hat{r}) \rangle > \hat{I} = \hat{I}, \quad (43)$$

which is manifestly inconsistent since $\langle \hat{\psi}(\hat{r}) | \hat{\psi}(\hat{r}) \rangle$ is an ordinary number while $\hat{I}$ is a matrix with integro-differential elements.

Note that we have considered a free particle immersed in a hadronic medium, rather than a bound state of extended particles in condition of mutual penetration. Consequently, in our view, isotopic element (2) represents a *subsidiary constraint* caused by the pressure of the hadronic medium encompassing the particle considered, by therefore restricting the values of the isostandard isodeviations for isocoordinates and isomomenta.

Illustrations of the isodeterministic isoprinciple in specific structure models of hadrons and related aspects have been studied in Ref. [21] and their interpretation in terms of the isodeterministic isoprinciple will be studied in future works.

2.5. Gravitational example

To provide a gravitational illustration, recall that isotopic element (2) contains as particular cases all possible symmetric metrics in (3+1)-dimensions, thus including the Riemannian metric [20].

We then consider the 3-dimensional sub-case of isotopic element (2) and factorize the space component of the Schwartzchild metric $g_s(r)$ according to isotopic rule introduced in Refs. [35] [36]

$$g_s(r) = \hat{T}(r)\delta, \quad (44)$$

where δ is the Euclidean metric.

We reach in this way the following realization of the isotopic element

$$\hat{T} = \frac{1}{1 - \frac{2M}{r}} = \frac{r}{r - 2M}, \quad (45)$$

where M is the gravitational mass of the body considered, with ensuing isodeterministic isoprinciple

$$\Delta\hat{r}\Delta\hat{p} \approx \hat{T} = \frac{r}{r - 2M} \Rightarrow_{r \rightarrow 0} 0, \quad (46)$$

which confirms the statement in page 190 of Ref. [7], on the possible recovering of full classical determinism in the interior of gravitational collapse (see Ref. [37], Chapter 6 in particular, for a penetrating critical analysis of black holes).

It should perhaps be indicated that Refs. [35] [36] introduced the factorization of a full Riemannian metric $g(x)$, $x = (r, t)$ in $(3+1)$ -dimensions

$$g(x) = \hat{T}_{gr}(x)\eta, \quad (47)$$

where $\hat{T}_{gr}$ is the *gravitational isotopic element*, and η is the Minkowski metric $\eta = \text{Diag.}(1, 1, 1, -1)$.

Refs. [35] [36] then reformulated the Riemannian geometry via the transition from a formulation over the field of real numbers $\mathcal{R}$ to that over the isofield of isoreal isonumbers $\hat{\mathcal{R}}$ where the *gravitational isounit* is evidently given by

$$\hat{I}_{gr}(x) = 1/\hat{T}_{gr}(x). \quad (48)$$

The above reformulation turns the Riemannian geometry into a new geometry called *iso-Minkowskian isogeometry*, which is locally isomorphic to the *Minkowskian* geometry, while maintaining the mathematical machinery of the Riemannian geometry (covariant derivative, connection, geodesics, etc.) us fully maintained, although reformulated in terms of the isodifferential isocalculus [38].

The apparent advantages of the *identical* iso-Minkowskian reformulation of Riemannian metrics and Einstein's field equations (see, e.g., Eqs. (2.9), page 390 of Ref. [38]) are:

- 1) The achievement of a consistent operator form gravity in terms of *relativistic hadronic mechanics* [39] whose axioms are those of quantum mechanics, only subjected to a broader realization;
- 2) The achievement of a universal *symmetry* of *all* non-singular Riemannian metrics, which symmetry is locally isomorphic to the Lorentz-Poincaré symmetry, today known as the *Lorentz-Poincaré-Santilli (LPS) isosymmetry* [40], and it is notoriously impossible on a conventional Riemannian space over the reals;
- 3) The achievement of clear compatibility of Einstein's field equation with 20th century sciences, such as a clear compatibility of general relativity with special relativity via the simple limit $\hat{I}_{gr} = I$ implying the transition from the universal LPS isosymmetry to the Poincaré symmetry of special relativity with ensuing recovering of conservation and other special relativity laws [41] [42]; the achievement of axiomatic compatibility of gravitation with electroweak interactions thanks to the replacement of curvature into the new notion of isoflatness with the ensuing, currently

impossible, foundations for a grand unification [43]; and other intriguing advances.

3. Concluding remarks

In this paper, we have continued the study of the EPR argument [1] conducted in Ref. [7] and preceding works, with particular reference to the study of the uncertainties for extended particles immersed within hyperdense medias with ensuing linear and non-linear, local and non-local and Hamiltonian as well as non-Hamiltonian interactions.

This study has been conducted via the use of isomathematics and isomechanics characterized by the isotopic element $\hat{T}$ of Eq. (1) which represents the non-linear, non-local and non-Hamiltonian interactions of the particles with the medium [19] [20] [21].

The main result of this paper is that the standard deviations of coordinates and momenta for particles within hyperdense media are characterized by the isotopic element that, being always very small, $\hat{T} \ll 1$, reduces the uncertainties in a way inversely proportional to a non-linear increase of the density, pressure, temperature, and other characteristics of the medium, while admitting the value $\hat{T} = 0$ under extreme/limit conditions with ensuing recovering of full determinism as predicted by A. Einstein, B. Podolsky and N. Rosen [1].

We can, therefore, tentatively summarize the content of this paper with the following:

ISODETERMINISTIC ISOPRINCIPLE: The product of isostandard isodeviations for isocoordinates $\Delta\hat{r}$ and isomomenta $\Delta\hat{p}$, as well as the individual isodeviations, progressively approach classical determinism for extended particles in the interior of hadrons, nuclei, and stars, and achieve classical determinism at the extreme densities in the interior of gravitational collapse.

Acknowledgments

Sincere thanks are due to Thomas Vougiouklis, Svetlin Georgiev and Jeremy Dunning Davies for penetrating critical comments. Additional thanks are also due to Mrs. Sherri Stone for an accurate proofreading of the manuscript.

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Studies on A. Einstein , B. Podolsky and N. Rosen argument that "quantum mechanics is not a complete theory," I: Basic methods

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Abstract

In 1935, A. Einstein expressed his view, jointly with B. Podolsky and N. Rosen, that "quantum mechanics is not a complete theory" (*EPR argument*). Following decades of preparatory studies, R. M. Santilli published in 1998 a paper showing that the objections against the EPR argument are valid for point-like particles in vacuum (*exterior dynamical systems*), but the same objections are inapplicable (rather than being violated) for extended particles within hyperdense physical media (*interior dynamical systems*) because the latter systems appear to admit an identical classical counterpart when treated with the isotopic branch of hadronic mathematics and mechanics. In a more recent paper, Santilli has shown that quantum uncertainties of extended particles appear to progressively tend to zero when in the interior of hadrons, nuclei and stars, and appear to be identically null at the limit of gravitational collapse, essentially along the EPR argument. In this first paper, we review, upgrade and specialize the basic mathematical, physical and chemical methods for the study of the EPR argument. In two subsequent papers, we review the above results and provide specific illustrations and applications.

Keywords: EPR argument, isomathematics, isomechanics.

2010 AMS subject classifications: 05C15, 05C60. ¹

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¹Received 27th of January, 2020, accepted 19th of June, 2020, published 30th of June, 2020
doi: doi 10.23755/rm.v38i0.516, ISSN 1592-7415; e-ISSN 2282-8214. ©Ruggero Maria Santilli

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5. CONCLUDING REMARKS.

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1. INTRODUCTION.

1.1. The EPR argument.

As it is well known, quantum mechanics does not admit classical precision in the measurement of the position or the mutual distance of particles (Figure 1) in view of Heisenberg's uncertainty principle and other physical laws.

Albert Einstein did not accept this uncertainty as being final for all possible conditions existing in the universe and made his famous quote "*God does not play dice with the universe.*"

More specifically, Einstein accepted quantum mechanics for atomic structures and other systems of point-like particles in vacuum (conditions known as *exterior dynamical problems*), but believed that quantum mechanics is an "incomplete theory," in the sense that it could admit a "completion" into



Figure 1: In this figure, we present a conceptual rendering of the sole representation of particles permitted by the differential calculus underlying quantum mechanics, namely, the representation as isolated points in empty space which particles, being dimensionless, can only be at a distance, with ensuing EPR argument on the need for superluminal interactions to explain quantum entanglement [1].

such a form to recover classical determinism at least under limit conditions.

Einstein communicated his view to the post doctoral associates, B. Podolsky and N. Rosen at the Institute for Advanced Study, Princeton, NJ, and all three together published in the 1935, May 15th issue of the Physical Review, the paper entitled "*Can Quantum Mechanical Description of Physical reality be Considered Complete?*" which paper became known as the *EPR argument* [1].

Soon after the appearance of paper [1], N. Bohr published paper [2] expressing a negative judgment on the possibility of "completing" quantum mechanics along the EPR argument.

Bohr's paper was followed by a variety of papers essentially supporting Bohr's rejection of the EPR argument, among which we recall *Bell's inequality* [3] establishing that the $SU(2)$ spin algebra does not admit limit values with an identical classical counterpart.

We should also recall *von Neumann theorem* [4] achieving a rejection of the EPR argument via the uniqueness of the eigenvalues of quantum mechanical Hermitean operators under unitary transforms.

The field became known as *local realism* and was centered on the rejection of the EPR argument on claims of *lack of existence of hidden variables* λ [5] in quantum mechanics (see the review [6] with a comprehensive literature).

Nowadays, the EPR argument is generally ignored in view of the widespread belief that quantum mechanics is universally valid for whatever conditions may exist in the universe without any scrutiny of the limitations and/or insufficiencies of quantum mechanics in various fields reviewed in this section.

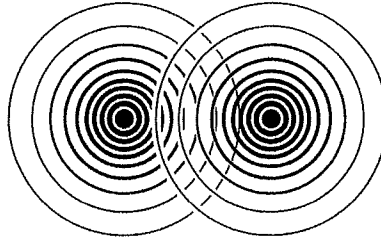


Figure 2: A conceptual rendering of the main assumption of the apparent proofs [7] [8] of the EPR argument [1], consisting in the representation of particles as extended, deformable and hyperdense in conditions of mutual overlapping with ensuing continuous contact at a distance eliminating the need for superluminal interactions to explain quantum entanglement. Despite its simplicity, the quantitative treatment of said view required decades of studies due to the need for a “completion” of the mathematics underlying quantum mechanics. Intriguingly, the “completion” here considered turned out to be of isotopic/axiom-preserving type, thus being fully admitted by quantum mechanical axioms, merely subjected to a realization broader than that of the Copenhagen school.

1.2. Apparent proofs of the EPR argument.

In Vol. 50, pages 177-190, 1998, of *Acta Applicandae Mathematica*, R. M. Santilli published paper [7] entitled “Isorepresentation of the Lie-isotopic $SU(2)$ Algebra with Application to Nuclear Physics and Local Realism,” which paper appears to confirm Einstein’s view on the existence of a “completion” of quantum mechanics into the *isotopic branch of hadronic mechanics*, or *isomechanics* for short and a “completion” of quantum chemistry into a form known as *isochemistry*. These “completions” are based on a broadening of applied mathematics known as *isomathematics* and admit progressive conditions of particles in the interior of hadrons, nuclei, stars and black holes that appear to recover classical determinism.

The proof presented in paper [7] was done via the following three main steps:

1.2.1. The proof that Bell’s inequality, von Neumann’s theorems and other similar objections of the EPR argument [6] are indeed correct, but under the generally tacit assumptions:

A) The point-like approximation of particles moving in vacuum (Figure 1);

B) The sole admission of Hamiltonian interactions [18];

C) The treatment of assumptions A and B via 20th century applied mathematics, including Lie’s theory and the Newton-Leibnitz differential

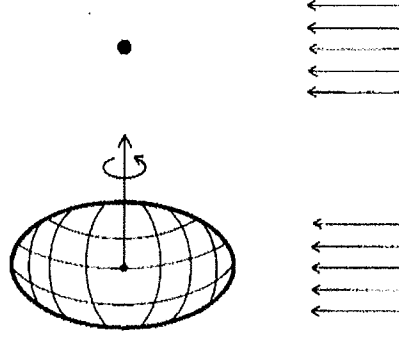


Figure 3: An illustration of the central objective for the proof of the EPR argument consisting in the transition from the quantum mechanical representation of the Newtonian notion of massive points moving in vacuum under linear, local and potential interactions (top view), to the time invariant representation of extended particles moving within physical media under linear and non-linear, local and non-local and potential as well as non-potential interactions (bottom view).

calculus;

1.2.2. The proof that the above treatments are not applicable to Einstein's vision on the existence of a "completion" of quantum mechanics based on the following assumptions:

A') The representation of extended, therefore deformable and hyperdense particles under conditions of mutual penetration/entanglement known (Figure 2) as occurring in the structure of hadrons, nuclei, stars and black holes (systems known as *interior dynamical problems*);

B') The emergence under condition A' of Hamiltonian as well as contact non-Hamiltonian interactions of non-linear, non-local and non-potential character;

C') The treatment of assumptions A' and B' via isomathematics that, as we shall see in Section 2, is based on:

i) The axiom-preserving isotopy $ab = ab \rightarrow a \star b = a \hat{T} b$ of the associative product ab between generic quantities a, b (numbers, functions, operators, etc.), where $\hat{T}$ is a positive-definite quantity called the *isotopic element* representing the dimension, deformability and density of particles;

ii) The ensuing axiom-preserving "completion" of Lie's theory with isotopic product $[x, y] = x \star y - y \star x$ between Hermitean operators x, y ;

iii) The reconstruction of the 20th century applied mathematics into a form compatible with isoproduct $a \star b$, including most importantly the isotopic lifting of the Newton-Leibnitz differential calculus from its centuries



Figure 4: A first illustration of the lack of “completion” of quantum mechanics beyond scientific doubt is the time reversal invariance of the theory with equal probability for events forward and backward in time. Such a time reversibility is acceptable for atomic structures, particles in accelerators, crystals and other reversible systems, but it does not allow a consistent physical or chemical representation of energy-releasing process, such as the coal burning depicted in this figure. In fact, the time reversal image of coal burning implies that smoke must reconstruct coal with evident violation of causality.

old definition at isolated points to its definition in the volumes of particles represented by $\hat{T}$.

1.2.3. The proof that the Lie-isotopic $\hat{S}U(2)$ algebra with isoproduct $[x\hat{y}]$ admits limit conditions with an identical classical counterpart.

More recently, R. M. Santilli completed the above proof in paper [8] by showing that, under the above indicated conditions, the standard deviations for coordinates Δr and momenta Δp appear to progressively tend to zero for extended particles within hadrons, nuclei and stars, and appear to be identically null for extended particles within the limit conditions in the interior of gravitational collapse, essentially along Einstein’s vision.

It should be noted that the above proofs of the EPR argument are centered in the *preservation* of the basic axioms of quantum mechanics, only submitted to their broadest possible *realization*.

It should be also noted that, under said broadest possible realization, quantum axioms do admit an *explicit and concrete realization of hidden variables* embedded in the structure of the Lie-isotopic product $[x\hat{y}] = x\hat{T}y - y\hat{T}x$, for instance, via realization $\hat{T} = \text{Diag.}(\lambda, 1/\lambda)$, $\text{Det}\hat{T} = 1$ [7].

1.3. Insufficiencies of quantum mechanics for irreversible processes.

One of the insufficiencies of quantum mechanics well known since the 1930's is the inability to represent physical or chemical energy releasing processes, such as nuclear fusions, fuel combustion and other processes.

This is due to the fact that energy releasing processes are *irreversible over time* (namely, the time reversal image of the processes violates causality), while quantum axioms were conceived for and remain solely applicable to *systems reversible over time* (namely, the time reversal image of the systems verifies causality), such as atomic structures, particles in accelerators, crystals and other systems (Figure 4).

It is hoped that the ongoing alarming deterioration of our environment, with the consequential need for *new* clean energies, illustrate the need for a "completion" of quantum mechanics for irreversible processes outlined for completeness in Section 2.

It should be indicated that, except for the short presentation in Section 2, this and the following paper are solely devoted to the apparent proofs of the EPR argument for reversible interior systems, while the study for the broader irreversible interior systems is done elsewhere (see Refs. [9] to [83]).

This is due to the fact that the objections against the EPR argument were formulated for reversible exterior systems. Consequently, proofs [7] and [8] studied in these papers were formulated for reversible interior systems.

1.4. Insufficiencies of quantum mechanics in particle physics.

Quantum mechanics is justly considered to be *exactly valid* for the structure of the hydrogen atom because it achieved a numerically exact representation of *all* experimental data for the system considered.

It is hoped serious scholars will admit that quantum mechanics cannot be considered as being exactly valid for particle physics because of the known inability to achieve an exact representation of *all* experimental data of any given family of particles, despite the admission of a number of hypothetical neutrinos and other *ad hoc* conjectures.

Recall that, with the exception of electrons, protons and the hypothetical neutrinos, *all particles produced by contemporary accelerators are structurally irreversible because unstable, thus being outside the capability of a full representation via quantum mechanics (Section 1.3), e.g., because they require a covering Lie-admissible (rather the Lie) treatment.* [22] [23].

Additionally, quantum mechanics is completely inapplicable (rather than violated) for the most fundamental synthesis in nature, that of the

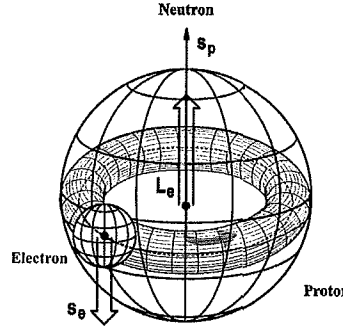


Figure 5: A conceptual rendering of the synthesis of the neutron as a “compressed hydrogen atom” in the core of stars according to H. Rutherford [84], which synthesis cannot be represented by quantum mechanics due to a mass excess and other reasons (see Section 1.4). By contrast, all characteristics of the neutron in said synthesis have been represented at the non-relativistic and relativistic levels by the “completion” of 20th century applied mathematics and physics studied in this paper (Refs. [85] [95]).

neutron from a proton and an electron as occurring in the core of stars [84], with ensuing inapplicability to the synthesis of other particles, such as that of the π^0 meson from the positronium [17].

This is due to the fact that quantum mechanical axioms have been conceived for the synthesis of particles in which the mass of the final state is *smaller* than the sum of the masses of the original constituents, resulting in the well known *mass defect* caused by *negative potentials*.

By contrast, the mass of the neutron $E_n = 939.565 \text{ MeV}$ is 0.782 MeV bigger than the sum of the masses of the proton $E_p = 938.272 \text{ MeV}$ and of the electron $E_e = 0.511 \text{ MeV}$, resulting in a *mass excess* requiring a *positive potential* for which Schrödinger, Dirac and other quantum mechanical equations admit no physically meaningful solutions, with similar cases occurring for the synthesis of other particles [17].

The inability by quantum mechanics to represent the fundamental synthesis of the neutron in a star is ultimately due to the *point-like characterization of particles* since it is mathematically and physically impossible to fuse together two point-like particles (the proton and the electron) into a third point-like particle (the neutron).

In turn, this insufficiency identified the need for the representation of hadrons as extended, deformable and hyperdense, which representation is at the foundation of the EPR proof [7].

Another insufficiency of quantum mechanics is its sole capability of representing *linear interactions* (i.e., interactions linear in the wave function). By contrast, the sole known possibility of achieving a bound state with excess mass is that via the admission of non-linear, generally non-Hamiltonian interactions for extended particles in conditions of mutual penetration/entanglement, as it is the case for the electron when totally compressed inside the proton [16] [17].

In turn, non-linear interactions are crucial for the “completion of the wavefunction” advocated by Einstein, Podolsky and Rosen [1] as we shall see in Paper III of this series.

It is nowadays known that the above insufficiencies originate from the theory at the foundation of quantum mechanics, Lie’s theory, because said theory solely admits Hamiltonian linear interactions [19].

It is hoped that the above insufficiencies illustrate the significance of the “completion” of Lie’s theory used for the proof of the EPR argument [7].

In fact, only following the achievement of a “completion” of 20th century mathematical and physical methods for extended, deformable and hyperdense particles in interior dynamical conditions, Santilli achieved a numerically exact representation of *all*— characteristic of the neutron in its synthesis from a proton and an electron at the non-relativistic (Refs. [85] to [87]), relativistic (Refs. [88] to [89]) and experimental (Refs. [90] to [95]) levels (see Sections 2, 3 and Paper II of this series).

1.5. Insufficiencies of quantum mechanics in nuclear physics.

There is no doubt that quantum mechanics has permitted historical achievements in nuclear physics.

However, quantum mechanics is only *approximately valid* in nuclear physics because of the inability to achieve over the last century a representation of the characteristics of the simplest nucleus, the deuteron, with embarrassing deviations of the prediction of the theory from experimental data for heavier nuclei, such as the Zirconium [59].

In Santilli’s view, the primary reason for the indicated insufficiency is that the *mathematics* underlying quantum mechanics, with particular reference to the *Newton-Leibnitz differential calculus*, imply the conception of nuclei as ideal spheres with isolated points in its interior (Figure 6) while in the physical reality, nuclei are composed by extended and hyperdense protons and neutrons in conditions of partial mutual penetration established by the comparison of nuclear volumes with the constituent volumes [59] (Figure 7).

The inability to represent nuclei as they are in the physical reality im-

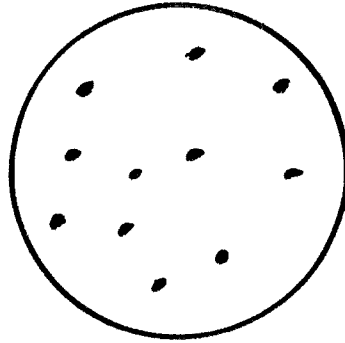


Figure 6: *The mathematics underlying quantum mechanics is local-differential, thus solely admitting a point-like approximation of particles. This figure illustrates the consequential conception of nuclei as ideal spheres with isolated points in their interior with ensuing insufficiencies beyond scientific doubt.*

plies the inability to achieve an exact representation of nuclear magnetic moments. In fact, the quantum mechanical representation of the anomalous magnetic moment of the deuteron still misses 1 % despite all possible relativistic or quark-based corrections.

Additionally, quantum mechanics misses much bigger percentages of nuclear magnetic moments for heavier nuclei (as illustrated in Figure 8).

In turn, the inability to represent protons and neutrons as extended charge distributions implies the inability to represent deformations under strong nuclear forces, with related deformation of their angular momenta.

In fact, J. M. Blatt and V. F. Weisskopf state on page 31 of their treatise in nuclear physics [96]: *It is possible that the intrinsic magnetism of a nucleon is different when it is in close proximity to another nucleon* (Figure 9).

The representation of nucleons as extended, thus deformable and hyperdense charge distributions via the "completion of 20th century mathematics (Section 2) and physics (Section 3) has permitted the exact representation of the anomalous magnetic moment of the deuteron [97], as well as of heavier nuclei [98].

It should be noted that the representation of nuclear magnetic moments is presented in Ref. [7] as an illustration of the implications of the proof of the EPR arguments for extended particles in interior conditions.

A second insufficiency of quantum mechanics in nuclear physics is the lack of a consistent representation of nuclear spins despite efforts also conducted for about one century.

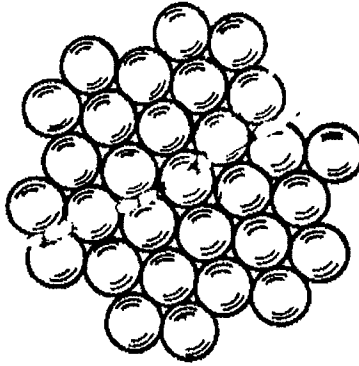


Figure 7: A conceptual rendering of nuclei as they occur in the physical reality, i.e., a collection of extended and hyperdense protons and neutrons in conditions of partial mutual penetration, with ensuing non-linear, non-local and non-potential interactions beyond any dream of quantitative treatment via quantum mechanics.

Recall that the proton and the neutron have both spin $1/2$ and that the only stable bound state predicted by quantum mechanics between two particles with spin $1/2$ is the singlet with spin 0.

Therefore, quantum mechanics predicts that the deuteron in its ground state must have spin 0, while experimental data establish that the deuteron has spin 1.

In an attempt of salvaging quantum mechanics, the spin of the deuteron is generally represented via a combination of *excited orbital states* which, even though significant, does not represent the spin 1 of the deuteron *in its ground state*.

The achievement of the synthesis of the neutron (Refs. [85] to [95]) has permitted a resolution of the above impasse because *the deuteron emerges as being a three-body state composed by two protons and one exchange electron, with ensuing spin 1 in the ground state* [59].

Subsequent studies by A. A. Bhalekar and R. M. Santilli [100] based on the "completion" of 20th century mathematical and physical methods have achieved a representation of the spin of stable nuclei in their bound state.

It should be finally noted that the biggest insufficiency of quantum mechanics in nuclear physics is given by the inability to achieve a consistent representation of nuclear forces in one century of efforts.

This is due to the fact that the sole forces permitted by quantum mechanics are of *potential*, thus of action- at-a-distance type, which is solely

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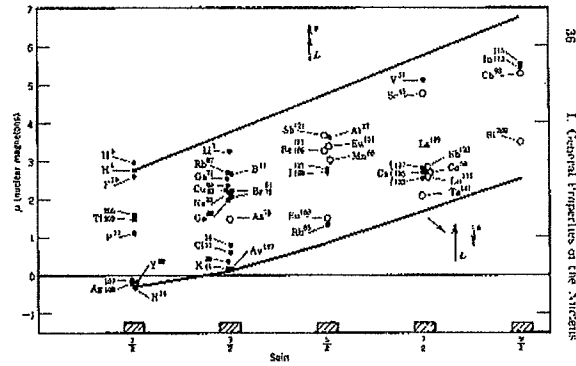


Figure 8: A view of the experimental data of nuclear magnetic moments that cannot be exactly represented by quantum mechanics. Similar insufficiencies exist for nuclear spins.

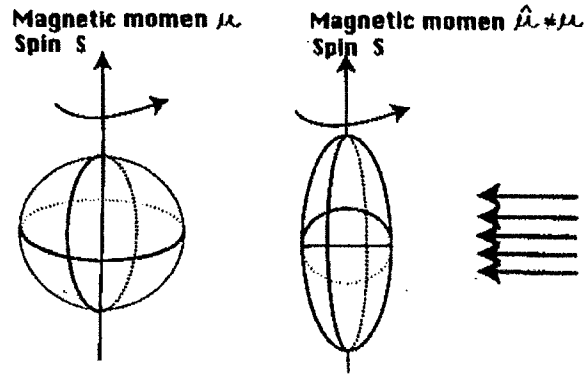


Figure 9: A conceptual rendering of the deformability of protons and neutrons under strong nuclear interactions predicted by J. M. Blatt and V. F. Weisskopf [96] as being the origin of the inability by quantum mechanics to represent nuclear magnetic moments.

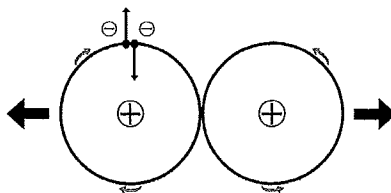


Figure 10: This picture depicts the hydrogen molecule at absolute zero degree temperature to illustrate the fact that, despite historical achievements, quantum mechanics and chemistry have been unable to identify the force attracting identical electrons in valence bonds, because the sole admitted forces are of potential-Coulomb type, thus implying a repulsion in valence bonds with ensuing lack of a “complete” representation of molecular structures [60].

possible, mathematical and physical conception for nuclei as ideal spheres with point-like particles in their interior (Figure 6).

One of the most important applications of the new methods studied in this work is that of representing nuclear forces as being non-linear, non-local and non-potential forces due to the mutual penetration/entanglement of the charge distribution of the hyperdense nucleons. The latter forces have emerged as being strongly attractive thus allowing the first known initiation of the understanding of the charge independence of nuclear forces [59].

1.6. Insufficiencies of quantum mechanics in chemistry.

Without doubt, quantum mechanics and chemistry have permitted chemical discoveries of historical proportions. Hence, the historical and scientific value of quantum chemistry is out of question.

Yet, it is the fate of all theories to admit, with the advancement of scientific knowledge, suitable coverings and this is the fate of quantum chemistry as well.

In fact, on strict scientific grounds quantum chemistry is only *approximately valid* in chemistry, thus admitting a suitable “completion,” because of the inability in one century of efforts to achieve an exact representation of molecular experimental data from first axiomatic principles without *ad hoc* form factors and other adaptations.

Recall that quantum mechanics has achieved a numerically exact representation of all experimental data of the hydrogen atom.

By contrast, when two hydrogen atoms are bonded into the hydrogen molecule H_2 , quantum mechanics and chemistry still miss the represen-

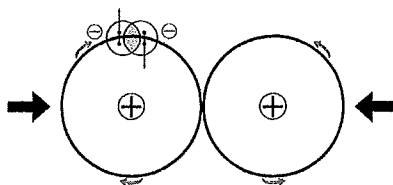


Figure 11: *This picture depicts the hydrogen molecule at absolute zero degree temperature with the strongly attractive force between identical valence electrons permitted by the representation of valence pairs as being composed by electrons with extended wavepackets in conditions of mutual penetration/entanglement with ensuing non-linear, non-local and non-potential interactions that result to be so strongly attractive [17] to overcome repulsive Coulomb forces [60]. The mutual distance d of said valence pair approaches Einstein's classical determinism and achieves it fully when it is in the interior of black holes [7].*

tation of 1 % of the H_2 binding energy, which is not insignificant since it corresponds to about 940 kcal/mole.

An in depth study of the impasse has shown that the above insufficiency is due to the inability by quantum mechanics and chemistry to represent the attractive force between identical valence electrons in molecular bonds (Figure 10) because according to basic axioms, two identical valence electrons must repel each other according to quantum mechanics and chemistry due to their equal charge [60].

As it had been the case for other problems, the primary difficulty to achieve an attractive force between identical valence electrons was of mathematical rather than of physical or chemical character, because quantum mechanics and chemistry solely admit potential forces that, in this case, can only be of repulsive Coulomb type.

By contrast, the sole possibility of resolving the impasse was the representation of electrons as extended wavepackets, that when in conditions of mutual penetration at 1 fm mutual distance, admit contact, non-linear, non-local and non-potential interactions of Hulthen type.

These new interactions result to be so strong to "absorb" repulsive Coulomb forces resulting in the needed attraction [60] (Figure 11).

The new valence force was first identified in Table 5 of the 1978 paper [17] as responsible for the birth of strong interactions in the synthesis of the π^0 meson from an electron and a positron.

In 1995, A. O. E. Animalu and R. M. Santilli published paper [101] establishing that the Hulthen force of paper [17] is so strong to account for the

bond of the two identical electrons in the Cooper pair of superconductivity.

A generalization of superconductivity based on the new methods was then developed and it is today known as *Animalu isosuperconductivity* [102].

In 2000, R. M. Santilli and D. D. Shillady showed that valence electron pairs with a strongly attractive force, called *isoelectronia*, permit numerically exact representations from first axiomatic principles of experimental data on the hydrogen [103] and water [104] molecule, which representation had escaped quantum chemistry for about one century (see also review [105]).

The achievement of a new model of molecular structures based on the isoelectronium valence bond has permitted novel advances in larger molecules whose study has been initiated by A. A. Bhalekar and R. M. Santilli [106] with intriguing implications, e.g., possible improvements in the combustion of fossil fuels based on a more accurate representation of their molecular structure [53].

Additionally, Santilli and Shillady showed that *perturbative series of the resulting "completion" of quantum chemistry converge at least one thousand times faster than the corresponding series of quantum chemistry* (see Section 4.13).

1.7. Implications of the EPR argument.

It is hoped that the preceding sections have indicated the truly vast implications for all quantitative sciences of Einstein's view that "quantum mechanics is not a complete theory" [1], thus warranting due scientific process.

In this paper we outline the main aspects of the new mathematical and physical methods underlying proof [7], with the understanding that a technical knowledge can be solely achieved via a study of the original literature.

The reader should be aware that the literature accumulated in half a century of research in the field by numerous scientists is rather vast. Consequently, in this paper we can only quote the most important original contributions and provide comprehensive references for interested readers.

2. LIE-ADMISSIBLE "COMPLETION" OF 20TH CENTURY APPLIED MATHEMATICS.

2.1. Foreword.

R. M. Santilli never accepted quantum mechanics as a "complete" theory

beginning with his graduate studies in physics at the University of Torino, Italy, in the mid 1960's because quantum axioms are invariant under time reversal, due to the invariance under anti-Hermiticity of the Lie product between Hermitean operators

$$[x, y] = -[x, y]^\dagger, \quad (1)$$

and other physical laws.

By recalling the fundamental character of Lie's theory, it follows that the *mathematics* (more than physical laws) underlying quantum mechanics does not allow a consistent representation of nuclear fusions and other physical or chemical energy releasing processes, due to their known irreversibility over time (Figure 4).

2.2. The historical teachings by Lagrange and Hamilton.

In view of the above lack of "completeness" of quantum mechanics, R. M. Santilli initiated his Ph. D. studies with the reading of the original works by J. L. Lagrange and studying his true analytic equations, those with external terms [9]

$$\frac{d}{dt} \frac{\partial L(r, v)}{\partial v} - \frac{\partial L(r, v)}{\partial r} = F_{ak}(t, r, v), \quad (2)$$

as well as the true Hamilton's equations, those with external terms [10]

$$\frac{dr}{dt} = \frac{\partial H(r, p)}{\partial p}, \quad (3)$$

$$\frac{dp}{dt} = -\frac{\partial H(r, p)}{\partial r} + F(t, r, p),$$

where the Lagrangian L and Hamiltonian H were used to represent conservative and potential, thus notoriously reversible forces, while the irreversibility of nature was represented with their *external forces* F .

2.3. The "No Reduction Theorem."

The external terms have been truncated in the 20th century sciences on claims that irreversible systems can be decomposed into their elementary particle constituents at which level the validity of quantum mechanics is fully recovered.

However, Santilli proved the following theorem as part of his Ph. D. thesis (see Refs. [56] [25]).

THEOREM 2.3.1 (No reduction Theorem): *A macroscopic time irreversible system cannot be consistently decomposed into a finite number of quantum mechanical particles and vice versa, a finite collection of quantum mechanical particles*

cannot reproduce a macroscopic irreversible system under the correspondence or other principle.

Consequently, a serious study of irreversible systems requires a return to the true Lagrange's and Hamilton's equations, those with external terms.

2.4. The inevitability of the Lie-admissible "completion" of 20th century science.

As it is well known, the true Lagrange and Hamilton equations cannot be assumed as a "completion" of 20th century sciences because they are not derivable from a potential.

Additionally, the brackets of the time evolution of an observable Q represented via Hamilton's equations with external terms

$$\begin{aligned} \frac{dA}{dt} &= (Q, H, F) = \\ &= \frac{\partial Q}{\partial r} \frac{\partial H}{\partial p} - \frac{\partial H}{\partial r} \frac{\partial Q}{\partial p} + \frac{\partial Q}{\partial r} F, \end{aligned} \quad (4)$$

characterizes the *triple system* (Q, H, F) that, in view of the external terms, violate the right scalar and associative axioms to characterize an algebra as currently understood in mathematics.

In the absence of a consistent algebra in the brackets of the time evolution, it was not possible to achieve a "completion" of quantum mechanics via a covering for irreversible systems.

Hence, Santilli was forced to seek the needed "completion" on algebraic grounds.

Following a year of research in the European mathematics libraries, Santilli did his Ph. D. thesis in 1965 on the "completion" of Lie algebras into A. A. Albert's *Lie-admissible and Jordan-admissible algebras* [11] with product [12]

$$(a, b) = pab - qba, \quad (5)$$

later on known as (p, q) -deformations [13], where $p, q, p \pm q$ are non-null scalars, and time irreversibility is assured for $p \neq q$ for which irreversibility is ensured for $p \neq q$ by the property $(x, y) \neq -(x, y)^\dagger$.

To achieve a first approximation of Hamilton's equations with external terms, Santilli introduced the following *parametric Lie-admissible generalization of Hamilton's equation* [14] [15]

$$\frac{dr}{dt} = p \frac{\partial H(r, p)}{\partial p}, \quad \frac{dp}{dt} = -q \frac{\partial H(r, p)}{\partial r}, \quad (6)$$

with corresponding *parametric Lie-admissible generalization of Heisenberg equation* for the time evolution of a Hermitean operator Q (for $\hbar = 1$)

$$i\frac{dQ}{dt} = (Q, H) = pQH - qHQ. \quad (7)$$

As one can see, all dynamical equations are manifestly irreversible over time, as desired.

2.5. Lie-admissible genomathematics and genomechanics.

In September 1977, Santilli joined the Department of Mathematics of Harvard University under DOE support during which stay he introduced the most general known realization of irreversible Lie-admissible algebras (see Refs. [16] to [23]) based on the generalization and differentiation of the ordinary product ab of arbitrary quantities (numbers, functions, operators, etc.) into the *ordered genomodular product to the right*

$$a > b = a\hat{R}b, \quad (8)$$

and that to the left

$$a < b = a\hat{S}b, \quad (9)$$

where $\hat{R}, \hat{S}$ and $R \pm S$ are positive-definite operators with an unrestricted functional dependence on wavefunctions $\psi(t, r)$ and any other needed variables.

The operators R and S were called *genotopic element to the right and to the left*, respectively, where the prefix "geno" was suggested by Carla Santilli in the Greek sense of "inducing a new structure" [16].

The new genomodular products permitted the construction of new mathematics known as *genomathematics to the right and to the left*, [19] with corresponding "completion" of quantum mechanics into an irreversible covering known as *genotopic branch of hadronic mechanics* or *genomechanics for short* [19] in which irreversible energy releasing processes are represented with ordered genomodular product to the right, as it is the case for the *geno-Schrödinger equation* or *Schrödinger-Santilli genoequation* [20]

$$H > \psi = H(r, p)R(\psi, \dots)\psi = E\psi, \quad (10)$$

Irreversibility is then assured whenever the genotopic elements R and S are not invariant under time reversal.

The time evolution of a Hermitean operator Q is given by the *Lie-admissible generalization of Heisenberg equations*, also known as *Heisenberg-Santilli genoequations* (see, e.g., Ref. [107]) first introduced in Eqs. (4.15.34),

page 746 of Ref. [17] (see the 2006 general treatment [24] and the 2016 update [25]) which can be written in the infinitesimal form

$$\begin{aligned} i\frac{dQ}{dt} &= (Q.H) = Q < H - H > Q = \\ &= QSH - HRQ, \end{aligned} \quad (11)$$

and the finite form

$$Q(t) = e^{HRt}Q(0)e^{-itSH}. \quad (12)$$

The representation of Lagrange's and Hamilton's external terms is provided by the difference $R - S$ at both classical and operator levels [24].

2.6. Universality of Lie-admissible formulations.

The following simple realization of the genotopic elements

$$\begin{aligned} S &= 1, \quad R = 1 - \frac{1}{H}K(\psi, \partial\psi, \dots), \\ i\frac{dQ}{dt} &= (Q.H) = [Q, H] + QK, \end{aligned} \quad (13)$$

where K is a positive-definite operator representing non-Hamiltonian interactions illustrates that the Lie-admissible generalization (11) of Heisenberg's equations constitute an operator image of Hamilton's equations with external terms (3) [56].

The double infinity of possible realizations of the genotopic elements R and S then allows Lie-admissible equations (11) to be "directly universal" for the representation of all possible (regular) non-linear, non-local and non-Hamiltonian interactions in the sense of representing all of them ("universality") directly in the frame of the experimentalism without the use of the transformation theory ("direct universality") (for details, see Ref. [24]).

It should be finally indicated that the original 1978 proposal [16] established the universality of Lie-admissible algebras because the product $(A, B) = A < B - B > A$ admit as particular case the product of all possible "algebras" as commonly understood in mathematics, including Associative, Lie, Jordan, Lie-isotopic, Jordan-isotopic, alternative, super-associative, super-Lie, super-Jordan, nilpotent, flexible and other possible algebras.

2.7. Prediction of new clean nuclear fusions.

Scientific and industrial applications to search for new clean energies were initiated in the late 1990's only following the achievement of maturity in

the mathematical and physical methods needed for the representation of irreversible processes.

The first application was the conception of the new *Intermediate Controlled Nuclear Fusion* (ICNF) of light, natural and stable elements into light, natural and stable elements with smaller mass which occur without the emission of harmful (e.g., neutron) radiations and without the release of radioactive waste (see Ref. [26] to [32] for originating papers and Refs. [33] to [40] for independent studies). Laboratory analyses [41] to [52] confirmed the *existence* of ICNF at the particle levels, with the understanding that the related industrial production of new clean energy require additional extensive research.

2.8. Prediction of a new clean combustion of fossil fuels.

To illustrate the implications of the lack of “completion” of quantum mechanics for energy releasing processes, we should note that the current combustion of fossil fuels is essentially that at the dawn of our civilization, because we essentially strike a spark and ignite the fuel with known alarming environmental deterioration of our planet.

The achievement of the Lie-admissible representation of energy releasing processes has permitted the first known conception and initiation of tests for a new principle of combustion called *HyperCombustion* which is based on the conventional combustion of carbon and oxygen dating back to the dawn of our civilization, plus the novel synthesis of a limited number of nuclei C-12 and O-16 into Si-28 to achieve full combustion of fossil fuels as well as a significant increase of energy output [53].

2.9. Literature on hadronic mathematics and mechanics.

Due to the prediction of new clean nuclear energies, the connection between irreversible mechanics and thermodynamics and other features, the literature on the foundations of hadronic mechanics is rather vast.

Ref. [54] provides a summary of the formalism of hadronic mechanics. Refs. [55] to [61] provide general presentations of hadronic mechanics. Vol. I of Refs. [61] contains a comprehensive literature up to 2008 with an upgrade to 2016 in Ref. [25].

Additional references are available in the reprint volumes [62] [63] and in the proceedings of five *Workshops on Lie-Admissible Algebras*, twenty five *Workshops on Hadronic Mechanics*, and three international conferences on the *Lie-admissible treatment of Irreversible Processes* whose references are available from Ref. [61]. Representative independent papers are available from Refs. [64] to [74] and independent monographs are available from Refs. [75] to [83].

3. LIE-ISOTOPIC “COMPLETION” OF 20TH CENTURY APPLIED MATHEMATICS.

3.1. Foreword.

As it is well known, the objections against the EPR argument (Section 1.1) were formulated for *isolated reversible systems of point particles in vacuum with linear, local and Hamiltonian interactions* called *reversible exterior dynamical problems*.

Hence, the proof of the EPR argument had to study *isolated reversible systems of extended, therefore deformable and hyperdense particles in conditions of mutual penetration/entanglement with ensuing linear and non-linear, local and non-local and Hamiltonian as well as non-Hamiltonian internal interactions*. The latter systems are called *reversible interior dynamical problems*, and they occur in the structure of hadrons, nuclei, stars and black holes.

Despite their reversible character, the latter systems could not be studied with 20th century applied mathematics, including Lie’s theory, due to its strictly Hamiltonian character. Reversible interior dynamical systems could not be studied with Lie-admissible formulations due to their irreversible character. Hence, the needed new mathematics had to be built.

In this section, we review the foundations of the new mathematics for the consistent representation of reversible interior dynamical systems which was essentially constructed as a reversible particular case of universal Lie-admissible formulations.

3.2. Isoproduct.

In order to achieve a representation of the latter systems, Santilli introduced in the original proposal [16] of 1978 the axiom-preserving particular case of genomathematics called *isomathematics*, which is characterized by the genoproduct to the right being equal to that to the left, $R = S = \hat{T}$, resulting in the time-reversal invariant *isoproduct*, (first introduced in classical realization in Eqs. (3.7.10), page 352 of Ref. [16], introduced in operator form in Eq. (4.15.46), page 751, Ref. [17] and then studied in details in Ref. [19], Eq. (2), page 71 on)

$$a \star b = a \hat{T} b, \quad (14)$$

where $\hat{T}$, called the *isotopic element*, is a function, matrix or operator solely restricted to be positive-definite, but possesses otherwise an unrestricted functional dependence on all needed local variables, such as: spacetime coordinates $x = (t, r)$; linear momentum p ; energy E ; frequency ν ; density of the medium α ; temperature τ ; pressure π ; wavefunctions ψ ; their

derivatives $\partial\psi$; and any other needed variable

$$\hat{T} = \hat{T}(x, p, E, \nu, \alpha, \tau, \pi, \psi, \partial\psi, \dots) > 0, \quad (15)$$

where the prefix “iso” was also suggested by Carla Santilli in its Greek meaning of preserving the axioms.

The *physical* significant of isoproduct (14) is illustrated by nothing that it allows “ab initio” a *direct representation of extended, thus deformable and hyperdense particles and their non-Hamiltonian interactions* illustrated in Figure 3 (Section 1.4). This important task is achieved via simple realizations of the isotopic element of the type (needed for the neutron synthesis from the hydrogen studied in Section 2.4) [55] [56]

$$\hat{T} = \text{Diag.}(\frac{1}{n_1^2}, \frac{1}{n_2^2}, \frac{1}{n_3^2}, \frac{1}{n_4^2})e^{-\Gamma}, \quad (16)$$

with subsidiary conditions

$$n_\mu = n_\mu(x, p, E, \nu, \alpha, \tau, \pi, \psi, \partial\psi, \dots) > 0, \quad \mu = 1, 2, 3, 4, \quad (17)$$

$$\Gamma(x, p, E, \nu, \alpha, \tau, \pi, \psi, \partial\psi, \dots) \geq 0.$$

The isoproduct also allows a direct representation of nuclei as a collection of extended nucleons in conditions of mutual penetration/entanglement as presented in Figure 7 and Section 1.5 with broader realizations of the type (needed to represent nuclear magnetic moments and spins, or for the achievement of an attractive force between identical valence electron bonds in molecular structures) [57]

$$\hat{T} = \Pi_{k=1, \dots, N} \text{Diag.}(\frac{1}{n_{1k}^2}, \frac{1}{n_{2k}^2}, \frac{1}{n_{3k}^2}, \frac{1}{n_{4k}^2})e^{-\Gamma}, \quad (18)$$

$$k = 1, 2, \dots, N, \quad \mu = 1, 2, 3, 4.$$

In the above realizations of the isotopic element, n_1^2, n_2^2, n_3^2 , (called *characteristic quantities*) represent the deformable semi-axes of the particle normalized to the values $n_1^2 = n_2^2 = n_3^2 = 1$, for the sphere; n_4^2 represents the *density* of the particle considered normalized to the value $n_4 = 1$ for the vacuum; and Γ represents non-linear, non-local and non-Hamiltonian interactions caused by mutual penetrations/entanglement of particles.

The *mathematical* significance of basic assumption (14) is that it requires, for consistency, a compatible “completion” of *all* aspects of 20th century applied mathematics without any known exception.

This program was initiated in the 1978 proposal [16], continued in the 1981 monograph [19] and completed in numerous works by various mathematicians (see the 1995 monograph [55] for a comprehensive presentation).

Regrettably we cannot provide a technical review of isomathematics to prevent excessive length. Nevertheless, a rudimentary outline of the main aspects of isomathematics appears to be recommendable for an understanding of proof [7] of the EPR argument.

3.3. Isonumbers.

As it is well known, physical theories are formulated over a field $F(n, \times, 1)$ of real, complex or quaternionic numbers n with product $nm = n \times m$ and multiplicative unit 1. Said field remains invariant under the unitary time evolution of quantum mechanics, thus allowing the prediction of the same numerical values under the same conditions at different times.

But the time evolutions of hadronic mechanics, such as Eqs. (11), are *non-unitary* when formulated on conventional space over a conventional field (not so for isomathematics as shown below).

This implies the loss over time of the multiplicative unit 1, and consequently, of the entire numeric field, with ensuing lack of consistent experimental verifications.

To resolve this impasse, Santilli had no other choice than that of re-inspecting the historical classification of numbers, by discovering in this way that *the abstract axioms of a numeric field do not necessarily restrict the multiplicative unit to be the number 1, and allow for unit an arbitrary positive-definite quantity $\hat{I}$ provided that the multiplication is redefined for $\hat{I}$ to verify the unit axiom* [107].

This lead to "completion" of numeric fields $F(n, \times, 1)$ into *isofields* $\hat{F}(\hat{n}, \star, \hat{I})$ of *isoreal*, *isocomplex*, and *isoquaternionic isonumbers* $\hat{n} = n\hat{I}$ with *isounit*

$$\hat{I} = 1/\hat{T} > 0, \quad (19)$$

isoproduct (14), $\hat{n} \star \hat{m} = (nm)\hat{I}$, and isounit isoaxiom

$$\hat{I} \star \hat{n} = \hat{n} \star \hat{I} = \hat{n}, \quad \forall \hat{n} \in \hat{F}. \quad (20)$$

Isofields are completed by compatible redefinitions of all numeric operations, such as *isoquotient*, *isosquare*, *isosquareroot*, etc. [107].

It should be indicated that *isofields verify all axioms of a field. Hence, isonumbers are fully acceptable for experimental verifications* [57].

Isofields are classified into those of the *first kind* (*second kind*) depending on whether the isounit $\hat{I}$ is (is not) an element of the original field.

In this paper, we shall solely consider isofields of the second kind since the representation of extended particles and their non-Hamiltonian interactions is achieved via the isounit or equivalently, the isotopic element.

Paper [107] has stimulated various studies in number theory, among which we mention the study by C-Xu, Jiang [79], A. K. Aringazin [108], C. Corda [109] and others.

3.4. Isofunctions.

As it is well known, a necessary condition for a variable to be measurable is that it is an element of the base field, and the same holds for functions of said variable.

The implementation of the same rule under isotopic “completion” stimulated the construction of the *isofunctional isoanalysis* initiated by: J. V. Kadeisvili [110][111]; A. K. Aringazin, D. A. Kirukhin and R. M. Santilli [112]; Raul M. Falcon Ganfornina and Juan Nunez Valdes [80]; and others.

We here limit ourselves to indicate: the *isotime* $\hat{t} = t\hat{I}_t$, *isospace isocoordinates* $\hat{r} = r\hat{I}_r$, and the *isofunctions of isovariable*,

$$\hat{f}(\hat{r}) = [f(r\hat{I})]\hat{I}, \quad (21)$$

such as the *isoexponentiation*

$$\hat{e}^{\hat{X}} = [e^{\hat{X}\hat{I}}]\hat{I} = \hat{I}[e^{\hat{I}\hat{X}}]. \quad (22)$$

Similar expressions hold for virtually all conventional functions used in applications [55].

3.5. Isospaces.

The initial construction of isomathematics [19] was formulated via conventional vector or metric spaces over conventional fields.

The consistent need to formulate spaces over isofields triggered the isotopic “completion” of metric spaces into *isospaces* whose study was initiated by the mathematician Gr. Tsagas and his school [113]. In turn, these studies triggered the construction of the *isotopology* by R. M. Falcon Ganfornina and J. Nunez Valdes [114], yielding the first known topology for the characterization of extended particles, known as the *Tsagas, Sourlas, Santilli, Ganfornina and Nunez (TSSGN) isotopology*.

Let $E(r, \delta, I)$ be the conventional Euclidean space with space coordinates $r = (x, y, z)$, metric $\delta = \text{Diag.}(1, 1, 1)$, unit $I = \text{Diag}(1, 1, 1)$ and invariant

$$r^2 = (x^2 + y^2 + z^2)1, \quad (23)$$

where one should note the trivial multiplication by 1 for compatibility with the isotopic image studied below.

The representation isospace of the non-relativistic proof of the EPR argument is given by the infinite family of *iso-Euclidean isospaces*, $\hat{E}(\hat{r}, \hat{\Delta}, \hat{I}_r)$, first formulated in the 1978 Ref. [16] and treated in detail in Ref. [19], finalized in 1996 Ref. [115] and extensively treated in monograph [55].

For the simple realization of the isotopic element

$$\hat{T} = \text{Diag.}(1/n_1^2, 1/n_2^2, 1/n_3^2), \quad (24)$$

iso-Euclidean isospaces $\hat{E}(\hat{r}, \hat{\Delta}, \hat{I}_r)$ are characterized by: the *isocoordinates* $\hat{r} = r\hat{I}$, the *iso-Euclidean isometric* $\hat{\Delta} = (\hat{T}\delta\hat{I}$, and the *isospace isounit* $\hat{I}_r = 1/\hat{T} > 0$ resulting in the *iso-Euclidean isoinvariant*

$$\begin{aligned} \hat{r}^{\hat{\Delta}} &= (\hat{r}^j \star \hat{\Delta}_{jm} \star \hat{r}^m) = (r^j \hat{\delta}_{jm} r^m) \hat{I}_r = \\ &= \left(\frac{r_1^2}{n_1^2} + \frac{r_2^2}{n_2^2} + \frac{r_3^2}{n_3^2} \right) \hat{I}_r, \end{aligned} \quad (25)$$

where we should recall that, for consistency, all scalar quantities have to be elements of an isofield $\hat{F}$.

The above conditions require that: squares must be isosquares $\hat{r}^{\hat{\Delta}} = \hat{r} \star \hat{r} = \hat{r}^2 \hat{I}_r$; coordinates have to be isocoordinates $\hat{r} = r\hat{I}_r$; to be isomatrices, isometrics must have the structure $\hat{\Delta} = \delta\hat{I}_r$; and the elaboration requires the use of the *isotrigonometric isofunctions* as well as of the *isospherical isocoordinates* (see Ref. [55] for details).

Let $M(x, \eta, I)$ be the conventional Minkowski space with spacetime coordinates $x = (x^1, x^2, x^3, x^4 = ct)$, metric $\eta = \text{Diag.}(1, 1, 1, -1)$, unit $I = \text{Diag.}(1, 1, 1, 1)$ and invariant

$$\begin{aligned} x^2 &= (x^\mu \eta_{\mu\nu} x^\nu) I = \\ &= (x_1^2 + x_2^2 + x_3^2 - c^2 t^2) I. \end{aligned} \quad (26)$$

The isospace for relativistic treatments of extended particles is given by the infinite family of *iso-Minkowski spaces* $\hat{M}(\hat{x}, \hat{\Omega}, \hat{I}_x)$ also known as *Minkowski-Santilli isospaces*, (see e.g., Ref. [107]) first introduced in Ref. [116] and then treated in details in Ref. [56].

Iso-Minkowskian isospaces are characterized by the *isospace-time isocoordinates* $\hat{x} = x\hat{I}$; isounit $\hat{I} = 1/\hat{T}$, and *isometric* $\hat{\Omega} = \hat{\eta}\hat{I}_x = (\hat{T}\eta)\hat{I}_x$ formulated on the isoreal isonumbers $\hat{\mathcal{R}}$.

For the simple realization of the isotopic element

$$\hat{T} = \text{Diag.}(1/n_1^2, 1/n_2^2, 1/n_3^2, 1/n_4^2), \quad (27)$$

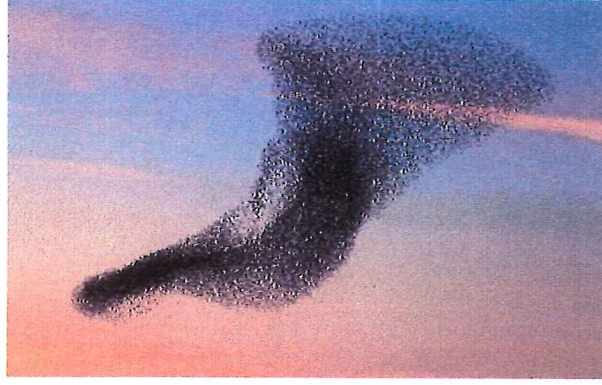


Figure 12: This picture illustrates the isodifferential calculus [115] via birds flying in close formation without wing interferences, which can be best understood by assuming that birds conceive themselves as a volume encompassing their wings, rather than a mass concentrated in their center of gravity as it would be requested by the Newton-Leibnitz differential calculus.

we have the infinite family of realizations of the *isospace-time isoinvariant*

$$\begin{aligned}\hat{x}^{\hat{2}} &= \hat{x}^{\mu} \star \hat{\Omega}_{\mu\nu} \star \hat{x}^{\nu} = (x^{\mu} \hat{\eta}_{\mu\nu} x^{\nu}) \hat{I} = \\ &= \left(\frac{x_1^2}{n_1^2} + \frac{x_2^2}{n_2^2} + \frac{x_3^2}{n_3^2} - t^2 \frac{c^2}{n_4^2} \right) \hat{I},\end{aligned}\tag{28}$$

where the final multiplication by the isounit is again necessary for the invariant to be an isoscalar.

It should be noted that, in addition to the use of the isospherical isocoordinates, data elaborations in the iso-Minkowskian isospace requires the use of *isohyperbolic isofunctions* (see Ref. [56], Chapters 5 and 6).

Note also that invariant (30) is the most general possible symmetric (non-singular) invariant in $(3 + 1)$ -dimensions, thus including as particular cases all possible Minkowskian, Riemannian, Fynslerian and all other geometries.

3.6. Isodifferential isocalculus.

Despite the above advances, numerical predictions of isomathematics lacked the crucial property of invariance over time.

In addition, isomechanics and hadronic mechanics at large, were incomplete due to the inability to formulate the isotopies and genotopies of the linear and angular momenta (see next section).

In order to resolve this impasse, Santilli had no other choice than that of reinspecting the Newton-Leibnitz differential calculus, by discovering in this way that, contrary to rather popular belief for four centuries, *the differential calculus depends on the multiplicative unit of the base field* because, when the unit depends on the variable of differentiation, said calculus has to be "completed" into the infinite family of *isodifferentials*, e.g., of the isocoordinates $\hat{r}$, first formulated in memoir [115] submitted in 1995 and published in 1996 and then treated in details in Refs. [55] [56]

$$\hat{d}\hat{r} = \hat{T}d[r\hat{I}(r, \dots)] = dr + r\hat{T}d\hat{I}(r, \dots), \quad (29)$$

with corresponding *isoderivative* [114]

$$\frac{\hat{\partial}\hat{f}(\hat{r})}{\hat{\partial}\hat{r}} = \hat{I}\frac{\partial\hat{f}(\hat{r})}{\partial\hat{r}}. \quad (30)$$

Following decades of searches, the discovery of the isodifferential isocalculus finally permitted the achievement of the invariance over time of numerical predictions, the formulation of isolinear and genolinear momenta and signaled the achievement of maturity for applications and experimental verifications (see Refs. [56] [57] for details).

All novel applications of isomathematics in physics, chemistry and other fields, including the proof of the EPR argument [7], originate from the extra term $r\hat{T}d\hat{I}(r, \dots)$ in isodifferential (29), which is absent in the mathematics for point particles.

The covering character of the isodifferential isocalculus over the conventional calculus is illustrated by the fact that whenever the isounit is independent from the differentiation variable or it is a constant, the conventional calculus is recovered uniquely and identically.

It should be indicated that the biggest difficulty in the use of the isodifferential isocalculus is of *conceptual*, rather than of mathematical character, because it requires the transition from the visualization of the calculus at individual points to volumes (or surfaces) represented by the isotopic element $\hat{T}$ (Figure 12).

By looking in retrospect, it appears nowadays evident that a "completion" of quantum mechanics for the representation of extended particles is fundamentally inconsistent when formulated via the conventional differential calculus, because of its sole possible characterization of particles as being point-like.

Consequently, the generalization of the differential calculus into a form defined on volumes represented by $\hat{T}$, rather than defined on coordinate points r , is essential for a consistent representation of extended particles.

Nineteen years following the discovery of the isodifferential isocalculus for extended particles, comprehensive studies in the field have been conducted by the mathematician S. Georgiev in the series of six monographs [83] which consider the broadest possible formulation of the isodifferential calculus, including the case of different isotopic elements for different isovariables.

3.7. Lie-Santilli isothory.

In Santilli's view, the physically most important part of isomathematics is given by the isotopic "completion" of the various branches of Lie's theory, today known as the *Lie-Santilli isothory*, which was first formulated in papers [16] [17] of 1978, systematically studied in monograph [19], finalized in Refs. [55] [56] of 1995 following the discovery of the isodifferential calculus [115] and recently studied in paper [117].

In this section, we follow the presentation of Ref. [19] of 1981 upgraded into a formulation on isospaces over isofields and elaborated via the isodifferential isocalculus.

Let L be a n -dimensional Lie algebra with Hermitean generators X_k , $k = 1, 2, \dots, n$ defined on a conventional space over a conventional numeric field. Then, the infinite family of isotopies $\hat{L}$ of L are characterized by the following main theorems:

THEOREM 3.7.1 [19]: (*Poincaré-Birkhoff-Witt-Santilli isothorem*): *The isocosets of the isounit and of the isostandard isomononials*

$$\hat{I}, \hat{X}_k, \hat{X}_i \star \hat{X}_j, i \leq j, \hat{X}_i \star \hat{X}_j \star \hat{X}_k, i \leq j \leq k, \dots, \quad (31)$$

form an infinite dimensional isobasis of the universal enveloping isoassociative isoalgebra $\hat{E}(\hat{L})$ (also called isoenvelope for short) of a Lie-Santilli isoalgebra $\hat{L}$.

The first illustration of the above theorem is given by isoexponential isofunction (22) whose correct derivation requires infinite basis (31).

The appearance of the non-linear, non-local and non-potential isotopic element $\hat{T}$ in the *exponent* illustrates the non-trivial character of the Lie-Santilli isothory.

THEOREM 3.7.2 [19]: (*Lie-Santilli isoalgebras*) *The antisymmetric isoalgebras $\hat{L}$ attached to the isoenveloping algebras $\hat{E}(\hat{L})$ verify the isocommutation rules*

$$\begin{aligned} [\hat{X}_i, \hat{X}_j] &= \hat{X}_i \star \hat{X}_j - \hat{X}_j \star \hat{X}_i = \\ &= \hat{C}_{ij}^k(t, r, p, E, \mu, \tau, \psi, \partial\psi, \dots) \star \hat{X}_k, \end{aligned} \quad (32)$$

where the quantities $\hat{C}$ are called the structure quantities.

The above isocommutation rules show the axiom-preserving character of the isotopies in view of their evident verification of the Lie axioms, although via a broader realization.

The Lie-Santilli isothory is called *regular* or *irregular* depending on whether the structure quantities $\hat{C}_{i,j}^k$ are isoscalars or isofunctions, respectively.

This classification is important because as shown in the next section, the regular Lie-Santilli isothory can be constructed via a well defined transform of corresponding Lie theory, while irregular Lie-Santilli isothories are truly new theories verifying Lie's axioms without a known map from the conventional formulation.

THEOREM 3.7.3 [19]: (Lie-Santilli isogroups) The isoexponentiated form $\hat{G}$ of isocommutation rules (32) defined on an isospace $\hat{S}$ with local isocoordinates $\hat{x}$ over an isofield $\hat{F}$ with isounit $\hat{I} = 1/\hat{T} > 0$ is a group mapping each element $\hat{x} \in \hat{S}$ into a new element $\hat{x}' \in \hat{S}$ via the isotransformations

$$\hat{x}' = \hat{g}(\hat{w}) \star \hat{x}, \quad \hat{x}, \hat{x}' \in \hat{S}, \quad \hat{w} \in \hat{F}, \quad (33)$$

verifying the following isomodular isoaction to the right:

- 1) The isomap of $\hat{g} \star \hat{S}$ into $\hat{S}$ is isodifferentiable $\forall \hat{g} \in \hat{G}$;
- 2) $\hat{I}$ is the left and right isounit of $\hat{G}$,

$$\hat{I} \star \hat{g} = \hat{g} \star \hat{I} \equiv \hat{g}, \quad \forall \hat{g} \in \hat{G}; \quad (34)$$

- 3) The isomodular isoaction is isoassociative,

$$\hat{g}_1 \star (\hat{g}_2 \star \hat{x}) = (\hat{g}_1 \star \hat{g}_2) \star \hat{x}, \quad \forall \hat{g}_1, \hat{g}_2 \in \hat{G}; \quad (35)$$

- 4) In correspondence with every element $\hat{g}(\hat{w}) \in \hat{G}$ with $\hat{w} \in \hat{F}$ there exists the inverse element $\hat{g}(-\hat{w})$ such that

$$\hat{g}(\hat{0}) = \hat{g}(\hat{w}) \star \hat{g}(-\hat{w}) = \hat{I}; \quad (36)$$

- 5) The following composition laws are verified

$$\hat{g}(\hat{w}) \star \hat{g}(\hat{w}') = \hat{g}(\hat{w}') \star \hat{g}(\hat{w}) = \hat{g}(\hat{w} + \hat{w}'), \quad \forall \hat{g} \in \hat{G}, \quad \hat{w} \in \hat{F}; \quad (37)$$

with corresponding isomodular action to the left, and general expression

$$\hat{g}(\hat{w}) = \prod_k \hat{e}^{\hat{i} \star \hat{w}_k \star \hat{X}_k} \star \hat{g}(0) \star \prod_k \hat{e}^{-\hat{i} \star \hat{w}_k \star \hat{X}_k}. \quad (38)$$

Nowadays, *isomathematics* is referred to as the infinite family of isotopies of 20th century applied mathematics, with particular reference to the isotopies of Lie's theory when formulated on isospaces over isofields and elaborated via the isodifferential isocalculus. Isomathematics is then classified into *regular and irregular isomathematics* depending on whether the structure quantities $\hat{C}_{ij}$ are isoscalars or isofunctions of local isovariables, respectively.

It should be indicated that, the proof [7] of the EPR argument uses both regular and irregular Lie-Santilli isoalgebras.

Since Lie's theory is at the foundation of the axiomatic structure and applications of quantum mechanics, the covering Lie-Santilli isothory predictably stimulated a number of independent contributions, such as the studies by the mathematicians: D. S. Sourlas and Gr. T. Tsagas [76], J. V. Kadeisvili [118], T. Vougiouklis [119] and papers quoted therein.

3.8. Simple construction of regular isomathematics.

A simple method for physicists has been identified in Ref. [120] of 1997 for the construction of regular isomathematics. The method consists of: 1) Selecting the desired representation of extended particles with non-Hamiltonian interactions via isotopic elements $\hat{T}$ of type (16) or (18); 2) Identifying a *non-unitary* transformation representing the selected isounit $\hat{I} = 1\hat{T}$

$$UU^\dagger = \hat{I}; \quad (39)$$

3) Subjecting the *totality* of conventional applied mathematics to the above nonunitary transform with no known exception, resulting in expressions of the type

$$I \rightarrow \hat{I} = UIU^\dagger = 1/\hat{T}, \quad (40)$$

$$n \rightarrow \hat{n} = UnU^\dagger = nUU^\dagger = n\hat{I}, \quad n \in F, \quad (41)$$

$$f(r) \rightarrow \hat{f}(\hat{r}) = Uf(r)U^\dagger, \quad (42)$$

$$e^A \rightarrow Ue^AU^\dagger = \hat{I}e^{\hat{T}\hat{A}} = (e^{\hat{A}\hat{T}})\hat{I}, \quad (43)$$

$$\begin{aligned} AB &\rightarrow U(AB)U^\dagger = \\ &= (UAU^\dagger)(UU^\dagger)^{-1}(UBU^\dagger) = \hat{A} \star \hat{B}. \end{aligned} \quad (44)$$

It should be indicated that the above transformations imply the possibility of constructing the infinite family of Lie-Santilli isoalgebras via non-unitary transforms of the considered Lie algebra. This is possible in view

of the transformation of commutation rules into their covering isocommutator forms,

$$[A, B] \rightarrow U[A, B]U^\dagger = [\hat{A}, \hat{B}]. \quad (45)$$

This property is evidently important for the construction of the isorepresentation of regular isoalgebras used for physical and chemical applications.

Note that *serious inconsistencies occur, at times without their detection by non-experts, in the event only one single quantity or operation of 20th century applied mathematics is not subjected to the above non-unitary map.*

We should finally indicate that *the proof of the EPR argument used in Ref. [7] is of 'non-unitary' character*, thus implying that the physical conditions for said proof are outside the class of equivalence of quantum mechanics.

3.9. Invariance of regular isomathematics.

An additional contribution of paper [120] is the proof that *the dimension, shape and density of extended particles and their non-Hamiltonian interactions are represented by isomathematics in a form invariant over time.*

Firstly, Ref. [120] showed that, following the construction of regular isomathematics via non-unitary transformations (Section 2.2.8), isomathematics is *not* invariant under additional non-unitary transforms, e.g., because of the lack of invariance of the basic isounit

$$\hat{I} \rightarrow \hat{I}' = W\hat{I}W^\dagger \neq \hat{I}, \quad WW^\dagger \neq I, \quad (46)$$

with consequential physical inconsistencies since any structural change of the isounit implies the transition to a different physical or chemical system.

However, non-unitary transforms can always be identically rewritten as *isounitary isotransforms* according to the rule [56]

$$WW^\dagger = \hat{I}, \quad W = \hat{W}\hat{T}^{1/2}, \quad (47)$$

$$WW^\dagger = \hat{W} \star \hat{W}^\dagger = \hat{W}^\dagger \star \hat{W} = \hat{I}, \quad (48)$$

under which reformulation we have the following *invariance of the isounit of the isotopic element and of the isoproduct of regular isomathematics* [120]

$$\hat{I} \rightarrow \hat{I}' = \hat{W} \star \hat{I} \star \hat{W}^\dagger \equiv \hat{I}, \quad (49)$$

$$\begin{aligned} \hat{A} \star \hat{B} &\rightarrow \hat{W} \star (\hat{A} \star \hat{B}) \star \hat{W}^\dagger = \\ &= \hat{A}' \star \hat{B}' = \hat{A}'\hat{T}\hat{B}', \end{aligned} \quad (50)$$

$$\hat{A}' = \hat{W} \star \hat{A} \star \hat{W}^\dagger, \quad \hat{B}' = \hat{W} \star \hat{B} \star \hat{W}^\dagger, \quad \hat{T} = (W^\dagger \star \hat{W})^{-1}.$$

The invariance of the entire isomathematics follows. Note that the invariance is ensured by the *invariant numeric values of the isounit and therefore, of the isotopic element under isounitary isotransforms*,

$$\hat{I} \rightarrow \hat{I}' \equiv \hat{I}, \quad (51)$$

$$A \star B = A\hat{T}B \rightarrow A' \star' B' = A'\hat{T}'B' \quad (52)$$

$$= \hat{A}' \star \hat{B}' = \hat{A}'\hat{T}'\hat{B}'$$

$$\hat{T} \rightarrow \hat{T}' \equiv \hat{T}. \quad (53)$$

By noting that, as shown in the next section, the time evolution of the isotopic “completion” of quantum mechanics is an isounitary isotransform, paper [120] established that *isomechanics is an axiom-preserving “completion” of quantum mechanics capable of representing extended particles under Hamiltonian as well as non-Hamiltonian interactions in a form invariant over time*.

In closing, we should recall other generalizations of 20th century mathematics and their applications, such as the so-called *deformations*. These generalizations are mathematically correct, but physically inconsistent because they violate causality laws (for brevity, see the *Theorems of Inconsistency of Non- Unitary Theories* in Vol. I of Refs. [61]). These inconsistencies arise from a structural generalization of Lie’s algebras and other quantum laws when formulated on conventional spaces over conventional fields, thus preventing their reformulation as isounitary theories.

Note that the invariance of isomathematics reviewed in this section implies the verification of causality on isospaces over isofields in the same way as quantum mechanics verify causality laws.

4. LIE-ISOTOPIC “COMPLETION” OF QUANTUM MECHANICS.

4.1. Foreword.

In this section, we review the foundation of the *isotopic branch of hadronic mechanics*, also known as *isomechanics*, which is used in the proof of the EPR argument [7].

An important difference between preceding works and the presentation in this section is that the previous works generally present isomechanics in its *projection* on conventional spaces over conventional fields.

By contrast, in this section we put the emphasis in the full formulation of isomechanics, that on isospaces over isofields because important to illustrate that, contrary to opposing view (Section 1.1), proof [7] of the EPR

argument is fully compatible with quantum axioms, only subjected to a broader realization.

Isomechanics was first introduced via isoproducts defined on conventional spaces over conventional fields in the 1978 papers [16] [17] and in the 1981 monograph [19]; it first achieved mathematical maturity in the 1996 memoir [115] thanks to the discovery of the isodifferential isocalculus; isomechanics was finally presented in a systematic way in monographs [55] [56] [57] with a 2016 upgrade in Section 2 of memoir [25].

Independent studies are available in Refs. [75] to [83]. A comprehensive list of references up to 2008 is available in Vol. I of Refs. [61] while a 2016 upgrade is available in Ref. [25]. We regret the inability of reviewing all important contributions on isomechanics to prevent an excessive length and are forced to outline only the most salient structural contributions.

4.2. Iso-Newton isoequations.

As it is well known, the fundamental equations of mechanics are the historical *Newton's equations*, representing systems of point-particles with Hamiltonian (that is, *variationally selfadjoint*, SA [18]) and non-Hamiltonian (*variationally non selfadjoint*, NSA [18]) forces [18] defined on a conventional Euclidean space, $E(r, \delta, I)$ over the field of real numbers $\mathcal{R}$

$$m \frac{dv_{ak}}{dt} - F_{ak}^{SA}(r, v) - F_{ak}^{NSA}(t, r, v) = 0, \quad (54)$$

$$k = 1, 2, 3, \quad a = 1, 2, \dots, N, \quad N \geq 2.$$

It is generally believed that Newton's equations with non-conservative forces can solely represent open, irreversible systems. In Ref. [19] Section 6.3, Page 236, Santilli introduced *closed non-self-adjoint systems*, which are given by systems (54) violating the integrability conditions for their representation via Lagrange's or Hamilton's equations, yet verifying all ten conservation laws of Galileo relativity under the conditions

$$\begin{aligned} \sum_{ak} F_{ak}^{NSA} &= 0, \\ \sum_{ak} \mathbf{r} \cdot \mathbf{F}^{NSA} &= 0, \\ \sum_{ak} \mathbf{r} \mathbf{F}^{NSA} &= 0. \end{aligned} \quad (55)$$

which conditions are evidently applicable only for $N \geq 2$, since the case of one particle $N = 1$ is trivial.

The fundamental equations of isomechanics are given by the isotopic "completion" of Eqs. (54) known as *iso-Newton isoequations*, which were

first introduced in Ref. [115] immediately following the discovery of the isodifferential isocalculus, and they are also known as *Newton-Santilli isoequations*, (see, e.g., Refs. [107] [75] [81]) defined on iso-Euclidean isospaces $\hat{E}(\hat{r}, \hat{\delta}, \hat{I})$ (Section 2.2.5) over isoreal isonumbers $\hat{\mathcal{R}}$ (Section 2.2.3)

$$\hat{m}_a \star \frac{\hat{d}\hat{v}_{ak}}{\hat{d}\hat{t}} - F_{ak}^{SA}(\hat{r}, \hat{v}) = 0. \quad (56)$$

A first important feature of Eqs. (56) is that of providing the first known consistent representation of the actual shapes and dimensions of the particles considered via the isodifferential calculus, with realization of the isotopic element of type (16).

A second important feature of Eqs. (56) is that of representing all potential-(SA) forces via conventional Newtonian forces $F^{SA}(r, v)$ while representing all non-potential-(NSA) forces via the isodifferential calculus.

This feature is treated in details in Ref. [115] and can be summarized as follows.

Note that the basic isounit of Eqs. (56) is the *isovelocity isounit*, $\hat{I} = \hat{I}_v$. Assume for simplicity that the isotime is equal to the conventional time,

$$\hat{t} = t \quad \hat{I}_t = 1. \quad (57)$$

Consequently, from isoderivative (30), we have

$$\frac{\hat{d}\hat{v}}{\hat{d}\hat{t}} = I_t d\hat{v}/d\hat{t} = d\hat{v}/dt. \quad (58)$$

Consider then the projection of Eqs. (56) in the Euclidean space $E(v, \delta, I)$ and use the various rules of isomathematics (Section 2.2). Then Eqs. (56) can be written in the projected form (where all multiplications are conventional),

$$\begin{aligned} (m\hat{I})\hat{I}\frac{\hat{d}(v\hat{I})}{dt} - F^{SA}\hat{I} &= \\ = \hat{I}[m\frac{dv}{dt} - F^{SA}] + mv\hat{I}\frac{d\hat{I}}{dt} &= 0. \end{aligned} \quad (59)$$

By dividing the above equation with $\hat{I} > 0$, one obtains Newton's equations with the following realizations of the NSA forces

$$F^{NSA}(t, r, v) = mv\hat{I}\frac{d\hat{I}}{dt}. \quad (60)$$

In conclusion, the *iso-Newton isoequations* (56) embed all NSA forces into the isoderivatives by therefore allowing the first known consistent operator image of non-Hamiltonian forces studied in subsequent sections.

4.3. Iso-Lagrangian and iso-Hamiltonian isomechanics.

Iso-Newton isoequations (56) admit a representation in terms of the *iso-Lagrange isoequations* first formulated in memoir [115] via the isodifferential calculus, thus being defined on an iso-Euclidean isospace over the isoreal isofield

$$\frac{\hat{d}}{\hat{d}\hat{t}} \frac{\partial \hat{L}(\hat{r}, \hat{v})}{\partial \hat{v}_{ak}} - \frac{\partial \hat{L}(\hat{r}, \hat{v})}{\partial \hat{r}_{ak}} = 0, \quad (61)$$

where $\hat{L} = L\hat{I}$ is an *iso-Lagrangian*, namely, a conventional Lagrangian formulated on isospaces over isofields thus being multiplied by the isounit to be an isoscalar.

Eqs. (56) also admit the isocanonically isoequivalent isorepresentation in terms of the *iso-Hamilton equations* first formulated in memoir [115] on an isophase isospaces over an isoreal isofield and also known as *Hamilton-Santilli isoequations* (see, e.g., Ref. [107])

$$\begin{aligned} \frac{\hat{d}\hat{r}_{ak}}{\hat{d}\hat{t}} &= \frac{\partial \hat{H}(\hat{r}, \hat{p})}{\partial \hat{p}_{ak}}, \\ \frac{\hat{d}\hat{p}_{ak}}{\hat{d}\hat{t}} &= -\frac{\partial \hat{H}(\hat{r}, \hat{p})}{\partial \hat{r}_{ak}}, \end{aligned} \quad (62)$$

where $\hat{H} = H\hat{I}$ is the *iso-Hamiltonian*, that is, a conventional Hamiltonian formulated on an isophase isospace over an isofield.

Note that iso-Hamiltonian isomechanics admits the following time evolution for a quantity $\hat{Q}$

$$\frac{\hat{d}\hat{Q}}{\hat{d}\hat{t}} = [\hat{Q}, \hat{H}] = \frac{\partial \hat{A}}{\partial \hat{r}_{ak}} \frac{\partial \hat{H}}{\partial \hat{p}_{ak}} - \frac{\partial \hat{H}}{\partial \hat{p}_{ak}} \frac{\partial \hat{Q}}{\partial \hat{r}_{ak}}, \quad (63)$$

where the brackets $[\hat{Q}, \hat{H}]$ constitute a classical realization of Lie-Santilli isoalgebras.

4.4. Isovariational isoprinciple.

Another important feature of Eqs. (56) is that of permitting the first known representation of variationally non-selfadjoint/non-Hamiltonian systems via an *isovariational isoprinciple* [115],

$$\hat{\delta} \hat{A} = \hat{\delta} \int (\hat{p}_{ak} \star \hat{d}\hat{r}_{ak} - \hat{H} \star \hat{d}\hat{t}) = 0, \quad (64)$$

which representation is notoriously impossible for NSA Newton's equations, with the consequential lack of achievement of consistent operator forms of nonconservative forces.

In view of the universality of Eqs. (56), the above isovariational isoprinciple is *directly universal*, that is, capable of representing all infinitely possible, regular, time reversal invariant Newtonian systems (54) ("universality") directly in the coordinates of the experimenter ("direct universality").

4.5. Iso-Hamilton-Jacobi isoequations.

Much along conventional analytic procedures, it is easy to prove that isovariational isoprinciple (64) implies the following *iso-Hamilton-Jacobi isoequations* also called *Hamilton-Jacobi-Santilli isoequations* [115] [25] that are at the foundation of the *isoquantization* reviewed in the next section

$$\frac{\hat{\partial}\hat{A}}{\hat{\partial}\hat{t}} + \hat{H} = 0, \quad (65)$$

$$\frac{\hat{\partial}\hat{A}}{\hat{\partial}\hat{r}_{ak}} - \hat{p}_{ak} = 0, \quad (66)$$

$$\frac{\hat{\partial}\hat{A}}{\hat{\partial}\hat{p}_{ak}} = 0. \quad (67)$$

We should recall from Section 2.2.5 that isodynamical isoequations of classical isomechanics require two different isofields, the first being the *isotime isofield* with isounits $\hat{I}_t$ and the second being the *isovelocity isofield* with isounits $\hat{I}_v$.

However, the direct universality is already achieved with the sole use of the isovelocity isounit. Hence, the isotime isounit can be assumed to be 1 without any loss of direct universality.

For non-relativistic formulations, we shall use isotime in the isodynamical equations for completeness, with the tacit understanding that, unless otherwise specified, isotime will be assumed to be equal to the conventional time.

As it will soon be evident, the Hamilton-Jacobi-Santilli isoequations (65)-(67) are truly fundamental for the construction of operator isomechanics, as well as for the proof of the EPR argument because said equations have permitted:

1) The achievement from Eqs. (65) of a unique and unambiguous map of classical into operator isomechanics;

2) The achievement from Eqs. (66) of the first known operator form of the isolinear isomomentum;

3) The achievement, from Eqs. (67) of an operator isomechanics whose *isowave Isofunctions* solely depend on local isocoordinates $\hat{\psi}(\hat{r})$. This feature is necessary for a consistent isotopic "completion" of quantum mechanics since conventional wave functions $\psi(t, r)$ do not depend on linear momenta p . Hence, any "completion" of quantum mechanics whose wavefunctions also depend on linear momenta would not be an axiom-preserving map.

It should be noted that, by comparison, the use of the Birkhoffian mechanics would imply broader Hamilton-Jacobi-Santilli isoequations (see page 205 of Ref. [19]) with 'wave functions' depending also on isomomenta, $\hat{\psi}(\hat{t}, \hat{r}, \hat{p})$, resulting in an operator mechanics beyond our current knowledge for quantitative treatments.

4.6. Naive isoquantization.

As it is well known, the conventional "naive quantization" of Hamiltonian mechanics into quantum mechanics is based on the following map generated by the conventional Hamilton-Jacobi equations

$$\begin{aligned} A = \int (p_k dx^k - H dt) &\rightarrow \\ &\rightarrow -i\hbar \log \psi(t, r), \end{aligned} \quad (68)$$

that identifies Planck's constant $\hbar = 1$ as the fundamental unit of the theory.

The isotopic lifting of the naive quantization, called *naive isoquantization* (first identified by A. E. O. Animalu and R. M. Santilli in Ref. [121]), characterizes the following map of (classical) iso-Hamiltonian mechanics into (operator) isomechanics via Hamilton-Jacobi-Santilli isoequation (65) (where the sum over the indices ak is omitted for simplicity)

$$\begin{aligned} \hat{A} = \hat{\int} (\hat{p} \star \hat{d}\hat{r} - \hat{H} \star \hat{d}\hat{t}) &\rightarrow \\ &\rightarrow -i\hat{I} \text{Log} \hat{\psi}, \end{aligned} \quad (69)$$

with the following fundamental identification of the *isolinear isomomentum* from Eq. (66)

$$\hat{p} \star \psi = -\hat{i} \star \hat{\partial}_r \hat{\psi}, \quad (70)$$

and the equally fundamental, independence of the isowavefunction from the isolinear isomomentum from Eq. (67)

$$\hat{\partial}_{\hat{p}} \hat{\psi} = 0, \quad (71)$$

where we use the notion of *isolog* $\hat{\log}\hat{\psi} = \hat{I} \log \psi$ (see [55]).

The above naive isoquantization identifies the central assumption of isomechanics, namely, *the map of Planck's constant $\hbar$ into an integro-differential operator, the isounit $\hat{I}$,*

$$\hbar \rightarrow \hat{I}(t, r, p, E, \mu, \tau, \psi, \partial\psi, \dots) > 0. \quad (72)$$

that, from Section 2.2.8, can be achieved via a non-unitary transform of Planck's constant selected in such a way to represent the desired systems, e.g., as in model (16),

$$\hbar = 1 \rightarrow \hat{\hbar} = U \hbar U^\dagger = U U^\dagger = \hat{I}. \quad (73)$$

The above transform is then restricted by the subsidiary condition that isomechanics must recover quantum mechanics at mutual distances of particles bigger than their size d

$$\lim_{r > d/2} \hat{I} = 1. \quad (74)$$

Therefore, the studies herein reported assume that isomechanics is solely valid within the volume occupied by hadrons, nuclei or stars, while quantum mechanics is assumed to be exactly valid everywhere else (Figure 13).

Note that the structure of the isotopic element (16) permits a smooth transition from isomechanics to quantum mechanics.

The above condition means that, for the case of the structure of a hadron, isomechanics is solely valid within a sphere with diameter $d \approx 1 fm = 10^{-15} cn$. For the case of the structure of the deuteron, isomechanics is solely valid within a volume with diameter $d \approx 2.50 fm$; and the same applies for nuclei, stars and black holes.

The main objective of the "completion" of Planck's constant $\hbar$ into the integro-differential isounit $\hat{I}$ is to represent the expected, generalized, energy exchanges of particles in interior dynamical conditions (as expected for an electron in the core of a star), which exchanges cannot be the same as those occurring when particles moves in vacuum due to the surrounding pressures and other factors (see paper II for details).

4.7. Iso-Hilbert isospaces.

Another basic notion of isomechanics is its formulation on the *iso-Hilbert isospaces* $\hat{\mathcal{H}}$, also called the *Hilbert-Myung-Santilli isospace* (HMS isospace) because first introduced by H.C. Myung and R.M. Santilli in Ref. [122] of 1982 over a conventional field of complex numbers $\mathcal{C}$ and then formulated on an isocomplex isofield $\hat{\mathcal{C}}$ in Ref. [115].

Quantum Mechanics

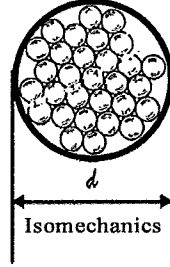


Figure 13: An illustration of the central assumption according to which isomechanics is solely valid within the volume occupied by hadrons, nuclei, stars or black holes, while quantum mechanics is valid everywhere else thanks to the rapid convergence of isotopic elements, such as Eq. (16), to the unit value 1.

HMS isospaces are characterized by (see Ref. [55] for details): *isostates* $\hat{\psi}$, with *isonormalization*

$$\langle \hat{\psi} | \star | \hat{\psi} \rangle = \hat{I}, \quad (75)$$

isoexpectation isovalues of an iso-Hermitian operator $\hat{A}$,

$$\hat{\langle A \rangle} = \langle \hat{\psi} | \star \hat{A} \star | \hat{\psi} \rangle, \quad (76)$$

and basic isoidentity

$$\hat{\langle \hat{I} \rangle} = \hat{I}, \quad (77)$$

where the "hat" denotes definition on isospace over isofields.

It should be recalled from Ref. [122] that the condition of iso-Hermiticity coincides with that of Hermiticity. Therefore, *all quantities that are observable in quantum mechanics remain observable in isomechanics* (see monograph [56] for details).

4.8. Iso-Schrödinger isorepresentation.

Recall that the Schrödinger representation is crucially dependent on the realization of the linear momentum in term of the differential calculus,

$$p\psi(t, r) = -i\hbar\partial_r\psi(t, r). \quad (78)$$

Consequently, the "completion" of quantum mechanics mandated the search for the "completion" of the differential calculus [115] to achieve a consistent formulation of the linear momentum such as that of Eq. (70),

that we rewrite in the more detailed form on the Hilbert-Myung-Santilli isospace $\hat{\mathcal{H}}$ over the isofield of isocomplex isonumbers $1calC$ (with $\hbar = 1$)

$$\begin{aligned} \hat{p}_k \star |\hat{\psi}(\hat{t}, \hat{r})\rangle &= -\hat{i} \star \hat{\partial}_{\hat{r}k} |\hat{\psi}(\hat{t}, \hat{r})\rangle = \\ &= -i\hat{I}\partial_{\hat{r}k} |\hat{\psi}(\hat{t}, \hat{r})\rangle, \end{aligned} \quad (79)$$

where: $\hat{r} = r\hat{I}$ are the isocoordinates on an iso-Euclidean isospace over an isofield, from which one can derive the *iso-Schrödinger isoequation*, also called *Schrödinger-Santilli isoequation* [17] [115]

$$\begin{aligned} \hat{i} \star \hat{\partial}_{\hat{t}} |\hat{\psi}(\hat{t}, \hat{r})\rangle &= \hat{H} \star |\hat{\psi}(\hat{t}, \hat{r})\rangle = \\ &= \hat{H}(r, p)\hat{T}(t, r, p, \psi, \partial\psi, \dots) |\hat{\psi}(\hat{t}, \hat{r})\rangle = \\ &= \hat{E} \star |\hat{\psi}(\hat{t}, \hat{r})\rangle = E |\hat{\psi}(\hat{t}, \hat{r})\rangle, \end{aligned} \quad (80)$$

where $\hat{E} = E\hat{I}$ is an isoeigenvalue defined on the isoreal isofield $\hat{\mathcal{R}}$, and E is an ordinary eigenvalue defined on the field of real numbers.

The iso-Schrödinger isorepresentation is completed by the *isocanonical isocommutation rules*, solely definable thanks to the isodifferential realization (79) of the isolinear isomomentum [115]

$$[\hat{r}_i, \hat{p}_j] |\hat{\psi}\rangle = \hat{i} \star \hat{\delta}_{i,j} |\hat{\psi}\rangle = i\delta_{i,j} |\hat{\psi}\rangle, [\hat{r}_i, \hat{r}_j] |\hat{\psi}\rangle = [\hat{p}_i, \hat{p}_j] |\hat{\psi}\rangle = 0. \quad (81)$$

Note that the characterization of extended particles at short mutual distances requires the knowledge of *two isoobservables*, the conventional Hamiltonian H for the representation of SA interactions and the isotopic element $\hat{T}$ for the representation of dimension, shape, density and NSA interactions.

On more technical grounds, Eqs. (80) are referred to as *regular iso-Schrödinger equations* to emphasize, in the sense of Theorem 3.7.2, the fact that they can be derived from the conventional Schrödinger equation via non-unitary transformations. The broader *irregular iso-Schrödinger equation* which cannot be derived via non-unitary transformations due to the addition of strong interactions, are studied in Paper II, Section 4.3., Eqs. (89).

4.9. Iso-Heisenberg isorepresentation.

Non-relativistic isomechanics is additionally based on the *iso-Heisenberg isoequations*, also called *Heisenberg-Santilli isoequations* (first formulated in

Eq. (4.15.49), page 752 of the 1978 paper [17] over conventional fields and formulated via the full use of isomathematics in the 1996 memoir [115]), here written for the iso infinitesimal isotime evolution of an iso-Hermitean operator $\hat{Q}$

$$\begin{aligned} \hat{i} \star \frac{d\hat{Q}}{dt} &= [\hat{Q}, \hat{H}] = \hat{Q} \star \hat{H} - \hat{H} \star \hat{Q} = \\ &= \hat{Q}\hat{T}(\psi, \dots)\hat{H}(r, p) - \hat{H}(r, p)\hat{T}(\psi, \dots)\hat{Q}, \end{aligned} \quad (82)$$

and then in their isoexponentiated form

$$\begin{aligned} \hat{Q}(\hat{t}) &= \hat{e}^{\hat{H} \star \hat{t} \star \hat{i}} \star \hat{Q}(0) \star \hat{e}^{-\hat{i} \star \hat{t} \star \hat{H}} = \\ &= e^{\hat{H}\hat{T}\hat{t}} Q(0) e^{-\hat{i}\hat{T}\hat{H}}, \end{aligned} \quad (83)$$

where we have used isoexponentiation (22).

Note the characterization of isoinfinitesimal isoequations (82) via the Lie-Santilli isomathematics (Theorem 3.7.2.) and the characterization of their finite form (83) via the Lie-Santilli isogroups (Theorem 3.7.3).

Note also that the Lie-isotopic equations (82) (83) are a particular case of the broader Lie-admissible equations (11) (12), respectively.

4.10. Iso-Klein-Gordon isoequation.

The relativistic isoequations of hadronic mechanics are characterized by the iso-Casimir isoinvariants of the basic symmetry of the iso-Minkowski isospace-time, the *Lorentz-Poincaré-Santilli isosymmetry* studied in paper II.

At this stage of our analysis, we merely consider the following isotopic "completion" of the second order invariant of the *Lorentz-Poincaré symmetry* formulated on iso-Minkowskian isospace $\hat{M}(\hat{x}, \hat{\Omega}, \hat{I}$ over the isoreals $\hat{\mathcal{R}}$ (Section 2.5)

$$\hat{p}^{\hat{2}} = \hat{p}_\mu \star \hat{p}^\mu = (\hat{M} \star \hat{C})^{\hat{2}} = (mC)^2 \hat{I}, \quad (84)$$

where

$$\hat{M} = m\hat{I}, \quad (85)$$

is the *isomass*, and

$$\hat{C} = C\hat{I} = \frac{c}{n_4} \hat{I}, \quad (86)$$

is the *light isospeed* from isoinvariant (26).

By using the isolinear isomomentum (79) isoinvariant (84) characterizes the second order isoequation of isomechanics known as *iso-Klein-Gordon*

isoequation [123]

$$\begin{aligned}
 \hat{p}_\mu \star \hat{p}^\mu |\hat{\psi}(\hat{x})\rangle &= \hat{\Omega}^{\mu\nu} \star \hat{p}_\mu \star \hat{p}_\nu |\hat{\psi}(\hat{x})\rangle = \\
 &= \hat{\eta}^{\mu\nu} (-i\hat{I}\partial_\mu) \hat{I} (-i\hat{I}\partial_\nu) |\hat{\psi}(\hat{x})\rangle = \\
 &= -\hat{I}\hat{\eta}^{\mu\nu} \partial_\mu \partial_\nu |\hat{\psi}(\hat{x})\rangle = \hat{I}(mC)^2 |\hat{\psi}(\hat{x})\rangle,
 \end{aligned} \tag{87}$$

also called *Klein-Gordon-Santilli isoequation*, first introduced with the isodifferential isocalculus in Chapter 9 of Refs. [56] and in papers [123] [124], (see also review [25]).

4.11. Iso-Dirac isoequation.

The first-order relativistic isoequation of hadronic mechanics is given by the isolinearization [124] of isoinvariant (84) and it is called the *iso-Dirac isoequation*, or *Dirac-Santilli isoequation* (see Refs. [115] [56] [123] [124])

$$\begin{aligned}
 [\hat{\Omega}^{\mu\nu} \star \hat{\Gamma}_\mu \star \hat{\partial}_\nu + \hat{M} \star \hat{C}] |\hat{\psi}(\hat{x})\rangle &= \\
 = (-i\hat{I}\hat{\eta}^{\mu\nu} \hat{\gamma}_\mu \partial_\nu + mC) |\hat{\psi}(\hat{x})\rangle &= 0,
 \end{aligned} \tag{88}$$

where the *Dirac-Santilli isogamma isomatrices* $\hat{\Gamma} = \hat{\gamma}\hat{I}$ are given by

$$\begin{aligned}
 \hat{\gamma}_k &= \frac{1}{n_k} \begin{pmatrix} 0 & \hat{\sigma}_k \\ -\hat{\sigma}_k & 0 \end{pmatrix}, \\
 \hat{\gamma}_4 &= \frac{i}{n_4} \begin{pmatrix} I_{2 \times 2} & 0 \\ 0 & -I_{2 \times 2} \end{pmatrix},
 \end{aligned} \tag{89}$$

where $\hat{\sigma}_k$ are the *regular iso-Pauli isomatrices* studied in Section 3.3 of paper II, with *anti-isocommutation rules*

$$\begin{aligned}
 \{\hat{\gamma}_\mu, \hat{\gamma}_\nu\} &= \hat{\gamma}_\mu \hat{I} \hat{\gamma}_\nu + \hat{\gamma}_\nu \hat{I} \hat{\gamma}_\mu = \\
 &= 2\hat{\eta}_{\mu\nu}.
 \end{aligned} \tag{90}$$

One should note that the anti isoanticommutators of the Dirac-Santilli isogamma isomatrices yield the isometric $\hat{\eta}_{\mu\nu}$ of the iso-Minkowski isospace-time (Section 2.5).

Recall that the iso-Minkowski isospace-time includes as particular cases all possible, non-singular, symmetric space-times, thus including the Riemannian space-time reformulated with isomathematics. In fact, the iso-Minkowskian isometric $\eta^{\mu\nu}$ admits as a particular case the Schwartzchild

metric via the following simple realization of the isotopic element

$$\begin{aligned}\hat{T}_{kk} &= 1/(1 - 2M/r), \\ \hat{T}_{44} &= 1 - 2M/r.\end{aligned}\tag{91}$$

Additionally, the iso-Minkowskian isometric admits the following combination of the Schwartzchild metric for exterior gravitational problems and that for interior problems (see, Eqs. (9.5.18) page 448, Ref. [56])

$$\begin{aligned}\hat{T}_{kk} &= 1/(1 - 2M/r)n_k^2, \\ \hat{T}_{44} &= (1 - 2M/r)/n_4^2.\end{aligned}\tag{92}$$

Consequently, the Dirac-Santilli isoequation with realization (91) of the isotopic element permits the study of an electron in an *exterior gravitational field*, while realization (92) permits the study of *electrons in interior gravitational fields*.

Relativistic isoequations are far from being mere academic curiosities because, as we shall see in paper II, they have provided the first and only known relativistic representation of *all* characteristics of the neutron in its synthesis from a proton and an electron [124], *none* of which characteristics are representable via quantum mechanics. Additional advances permitted by relativistic isomechanics will be indicated in paper II.

More technically, Eq. (88) is referred to as *regular iso-Dirac equations* to emphasize, in the sense of Theorem 3.7.2, the fact that they can be derived from the conventional Dirac equation via non-unitary transformations. The broader *irregular iso-Dirac equation* which cannot be derived via non-unitary transformations due to the addition of strong interactions, are studied in Paper II, Section 4.4., Eqs. (97).

4.12. Representation of non-linear interactions.

An important insufficiency of quantum mechanics is the inability to characterize individual constituents under non-linear internal forces in view of the inapplicability of the superposition principle.

In fact, the sole possible, quantum mechanical representation of non-linear interactions is that via the Hamiltonian

$$H(r, p, \psi, \dots)|\psi(t, r) \rangle = E|\psi(t, r) \rangle,\tag{93}$$

under which the total state $|\psi(t, r) \rangle$ does not admit a consistent decomposition into the individual states.

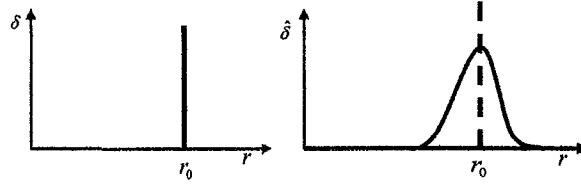


Figure 14: An illustration on the left of the divergence of the Dirac delta distribution at its origin caused by the point-like approximation of particles, with ensuing divergencies in quantum mechanics. An illustration on the right of the removal of said divergencies by the Dirac-Myung-Santilli isodelta isofunction thanks to the representation of particles as extended, with ensuing lack of divergencies in isomechanics.

It is easy to see that this insufficiency is resolved by isomechanics thanks to the embedding of all non-linear forces in the basic invariant of the theory, the isounit (or isotopic element).

In fact, the Schrödinger-Santilli isoequation (80) can be explicitly written

$$\hat{H} \star |\hat{\psi}\rangle = \hat{H}(\hat{r}, \hat{p}) \hat{T}(\hat{\psi}, \dots) |\hat{\psi}\rangle = E |\hat{\psi}\rangle, \quad (94)$$

and its total isostate verifies the factorization

$$\hat{\psi} = \Pi_k \hat{\psi}_k, \quad k = 1, 2, \dots, N, \quad (95)$$

called *isosuperposition isoprinciple* [56].

It is evident that factorization (94) allows the characterization of individual constituents under non-linear internal interactions, thus permitting new structural models of hadrons, nuclei, stars and black holes.

4.13. Isostrong isoconvergence.

In all applications to date, the basic isotopic element (16) resulted to have a numeric value smaller than one

$$\|\hat{T}\| \ll 1. \quad (96)$$

This feature has the important consequence that *perturbative and other series that are slowly convergent or divergent in quantum mechanics become strongly convergent under their isotopic "completion"* [56].

To illustrate this important feature, consider a divergent quantum mechanical series, such as the canonical series

$$\begin{aligned} A(w) &= A(0) + (AH - HA)/1! + \dots \Rightarrow \\ &\rightarrow \infty, \quad w > 1. \end{aligned} \quad (97)$$

But the value of the isotopic element is much smaller than the parameter w . Therefore, the isotopic "completion" of the above series

$$\begin{aligned} A(w) &= A(0) + (A\hat{T}H - H\hat{T}A)/1! + \dots \rightarrow \\ &\rightarrow N < \infty, \end{aligned} \quad (98)$$

is strongly convergent.

Specific examples of convergence of isoperturbative isoserries much more rapid than corresponding quantum mechanical series have been provided in Refs. [103] [104] (see Section 1.6 for additional comments).

4.14. Removal of quantum divergencies.

As it is well known, the divergencies of quantum mechanics originate from the singularity existing at the origin of the *Dirac delta distribution* (Figure 14) which divergence originates from the point-like approximation of particles.

Another important feature of isomechanics is that of avoiding these singularities as illustrated by the isotopic image of Dirac's delta "distribution", known as *Dirac-Myung-Santilli isodelta isofunction* first introduced in Ref. [122] (see also Nishioka's studies [125] to [128])

$$\hat{\delta}(\hat{r}) = \int \hat{e}^{\hat{k} \star \hat{r}} \star \hat{d}\hat{k} = \int e^{\hat{k} \hat{T} \hat{r}} d\hat{k}, \quad (99)$$

where we have used isoexponentials and isointegrals [56].

As illustrated in Figure 14, the appearance of the isotopic element in the *exponent* of the integrant changes a sharp singularity at the origin $r = 0$ into a bell-shaped function.

In summary, *the singularities of quantum mechanics are ultimately due to the point-like abstraction of particles or equivalently, to the formulation of the differential calculus at isolated points. Whenever particles are represented with their actual extended size, and the differential calculus is extended to formulations over volumes, quantum singularities no longer hold.*

4.15. Isoscattering isothory.

Another important application of isomechanics is the isotopic "completion" of the conventional, potential, scattering theory into the covering *isoscattering isothory* studied by R. Mignani [129] to [131], A. K. Aringazin and D. A. Kirukhin [132], A. O. E. Animalu and R. M. Santilli [133] and others (see Chapter 12 of Ref. [56]).

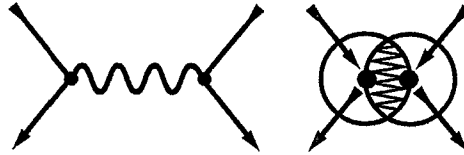


Figure 15: *The left view illustrates the scattering of point-particles, with ensuing realization of interactions via point-particle exchanges, that have been experimentally established for electromagnetic interactions. The right view illustrates the scattering of extended hadrons at high energy implying the presence of contact, non-linear, non-local and non-potential interactions in the scattering region. These conditions imply the impossibility of conventional particle exchanges evidently because of the extremely big density of the scattering regions that may approach the density of black holes, thus requiring broader scattering theories.*

Regrettably, we cannot review these studies to avoid a prohibitive length. We limit ourselves to recall that the potential scattering theory was originally conceived by Feynman and others for the electromagnetic interactions of *point-like* particles in vacuum.

Its dominant notion is the characterization of interactions via the *exchange of point-like particles*.

The historical experimental verifications of the potential scattering theory under the indicated conditions triggered its use for the scattering of *extended* hadrons all the way to their recent very high energies.

The studies herein reported on the covering isoscattering isothory have indicated that the consistent representation of hadrons as extended, therefore deformable and hyperdense, implies necessary revisions of hadron physics beginning with a “completion” of its mathematical foundations, then passing to the “completion” of quantum laws. These advances then require, for consistency, the “completion” of the potential scattering theory into a covering theory in which the hyperdense character of the scattering region implies the presence of non-linear, non-local and non-potential effects preventing any consistent representation of interactions via the sole exchanges of extended hadrons in favor of covering vistas (Figure 15).

It should be stressed that the isoscattering isothory has a Lie-Santilli algebraic structure, thus being solely applicable to time reversal invariant collisions generally given by elastic scattering. The study of inelastic scattering of extended hadrons at high energy requires the broader *genoscattering genothory* with the covering Lie-admissible algebraic structure, which broader theory cannot be considered here for brevity (see Refs. [56] [133]).

To avoid major misconceptions, it should be indicated that *the isoscattering isothory cannot change the numeric values of scattering angles, cross sections and other experimentally measured quantities*. However, the isoscattering isothory does require serious revisions of the *theoretical interpretation* of measured quantities.

In closing, we should recall that rather vast studies have been conducted on *non-unitary scattering theories* (see, e.g., Ref. [134] and papers quoted therein).

As it is well known, these studies had to be abandoned because non-unitary theories violate causality laws. By contrast, *the isoscattering isothory is isounitary on the iso-Hilbert isospace over an isofield*, thus restoring causality (Section 2.9).

This occurrence illustrates again the importance of isomathematics for the verification of the EPR argument and related applications.

4.16. Geno- and Iso-chemistry.

The Lie-admissible mathematical and physical methods of Section 2 have allowed the "completion" of quantum chemistry into *Lie-admissible hadronic chemistry*, also known as *genochemistry* [60], which is the first known chemical formulation specifically built for the consistent treatment of chemical reactions at large and energy releasing chemical processes in particular.

As it is well known but generally ignored, chemical reactions are generally *irreversible over time*, while quantum chemistry is strictly reversible. Hence, the EPR argument on the "lack of completion of quantum mechanics" does indeed apply to quantum chemistry.

In addition to a "completion" for chemical reactions, quantum chemistry needs an additional "completion," this time, for the achievement of an *attractive* force between the *identical* electrons of valence couplings in molecular structures (Section 1.6).

Since isolated molecules existing in nature are stable, thus being *reversible— over time*, and so are their valence electron bonds, there was the need of building the *Lie-isotopic particularization of the Lie-admissible hadronic chemistry* which became known as *isochemistry* because based on the isomathematics of Section 3.

Isochemistry did indeed achieve the first known attractive force between identical electrons in valence couplings [60] in a form permitting the exact representation of experimental data on the hydrogen [103] and the water [104] molecules.

As reviewed in details in Paper II, these studies essentially established that *the contact, non-potential interactions occurring in deep mutual penetration/entanglement of the wavepackets of particles (Figure 2) are strongly attrac-*

tive, thus being responsible for the neutron and other hadron syntheses (Section 1.4), as well as for the attractive force between identical electrons in valence pairs (Section 1.6), thus illustrating the truly fundamental character of short range, contact, non-potential interactions in the ultimate structure of nature.

5. CONCLUDING REMARKS.

In 1935, A. Einstein, B. Podolsky and N. Rosen presented the view that *quantum mechanics is not a complete theory* [1].

Following decades of preparatory works, R. M. Santilli published in 1998 paper [7] that:

- 1) Confirmed the validity of the objections against the EPR argument by Bohr [2], Bell [3], von Neumann [4] and others for point-particles in vacuum under linear, local and potential/Hamiltonian interactions *exterior dynamical problems*;
- 2) Proved the inapplicability (rather than the violation) of said objections for extended, thus deformable particles within hyperdense physical media with ensuing linear and non-linear, local and non-local and potential as well as non-potential/non-Hamiltonian interactions expected in the structure of hadrons, nuclei, stars and black holes (*interior dynamical systems*);
- 3) Proved the existence of *hidden variables* [5] for interior dynamical systems when represented via the isotopic branch of hadronic mathematics and mechanics;
- 4) Achieved a consequential exact representation of nuclear magnetic moments; and
- 5) Showed the apparent existence of identical classical counterparts for extended particles in interior dynamical conditions.

More recently, Santilli completed the above proof in paper [8] by showing that the standard deviations for coordinates Δr and momenta Δp appear to progressively tend to zero for extended particles within hadrons, nuclei and stars, and appear to be identically null for extended particles at the limit conditions in the interior of gravitational collapse, essentially along Einstein's vision.

In this paper, we have reviewed and upgraded the mathematical, physical and chemical methods used for proofs [7] [8], with particular reference to the following aspects:

- 1) Review and upgrade the Lie-admissible and Lie-isotopic "completions" of 20th century applied mathematics for the representation of time irreversible and reversible interior systems, respectively,, which "completions" were initiated by Santilli in 1978 [16] when at Harvard University

with DOE support under the names of *geno- and iso-mathematics*, respectively, and were subsequently studied by various authors [55];

2) Review and upgrade the "completions" of quantum mechanics and chemistry into the Lie-admissible and Lie-isotopic branches of hadronic mechanics and chemistry that were also initiated by Santilli in 1978 [17] under the names of *genotopic and isotopic branches of hadronic mechanics and chemistry*, which "completions" were then studied by various authors [56];

3) Review and upgrade the main aspect of the studies herein considered, namely, the "completions" of the Newton-Leibnitz differential calculus into forms applicable to irreversible and reversible interior systems of extended particles, which "completions" were initiated by Santilli in the 1996 paper [115] under the name of *geno- and iso-differential isocalculus* and studied in details by S. Georgiev [83] and other mathematicians [55].

In a second paper of this series, we review and upgrade *geno- and iso-symmetries* for interior systems and then review the apparent proofs [7] [8] of the EPR argument.

In the third paper of this series, we study specific cases of interior dynamical systems in particle physics, nuclear physics and chemistry that progressively approach classical determinism, and present applications to new clean energies that are prohibited by quantum mechanics, yet fully admitted by its "completion."

Acknowledgments. The writing of these papers has been possible thanks to pioneering contributions by numerous scientists we regret not having been able to quote in this paper (see Vol. I, Refs. [61] for comprehensive literature), including:

The mathematicians: H. C. Myung, for the initiation of iso-Hilbert spaces; S. Okubo, for pioneering work in non-associative algebras; N. Kamiya, for basic advances on isofields; M. Tomber, for an important bibliography on non-associative algebras; R. Ohemke, for advances in Lie-admissible algebras; J. V. Kadeisvili, for the initiation of isofunctional iso-analysis; Chun-Xuan Jiang, for advances in isofield theory; Gr. Tsagas, for advances in the Lie-Santilli theory; A. U. Klimyk, for the initiation of Lie-admissible deformations; Raul M. Falcon Ganfornina and Juan Nunez Valdes, for advances in isomanifolds and isotopology; Svetlin Georgiev, for advances in the isodifferential isocalculus; A. S. Muktibodh, for advances in the isorepresentations of Lie-Santilli isoalgebras; Thomas Vougiouklis, for advances on Lie-admissible hypermathematics; and numer-

ous other mathematicians.

The physicists and chemist: A. Jannussis, for the initiation of isocreation and isoannihilation operators; A. O. E. Animalu, for advances in the Lie-admissible formulation of hadronic mechanics; J. Fronteau, for advances in the connection between Lie-admissible mechanics and statistical mechanics; A. K. Aringazin for basic physical and chemical advances; R. Mignani for the initiation of the isoscattering theory; P. Caldirola, for advances in isotime; T. Gill, for advances in the isorelativity; J. Dunning Davies, for the initiation of the connection between irreversible Lie-admissible mechanics and thermodynamics; A. Kalnay, for the connection between Lie-admissible mechanics and Nambu mechanics; Yu. Arestov, for the experimental verification of the iso-Minkowskian structure of hadrons; A. Ahmar, for experimental verification of the iso-Minkowskian character of Earth's atmosphere; L. Ying, for the experimental verification of nuclear fusions of light elements without harmful radiation or waste; Y. Yang, for the experimental confirmation of magnecules; D. D. Shillady, for the isorepresentation of molecular binding energies; R. L. Norman, for advances in the laboratory synthesis of the neutron from the hydrogen; I. Gandzha for important advances on isorelativities; A. A. Bhalekar, for the exact representation of nuclear spins; Simone Beghella Bartoli important contributions in experimental verifications of hadronic mechanics; and numerous other physicists.

I would like to express my deepest appreciation for all the above courageous contributions written at a point in time of the history of science dominated by organized oppositions against the pursuit of new scientific knowledge due to the trillions of dollars of public funds released by governmental agencies for research on pre-existing theories.

Special thanks are due to A. A. Bhalekar, J. Dunning-Davies, S. Georgiev, R. Norman, T. Vougiouklis and other colleague for deep critical comments.

Additional special thanks are due to Simone Beghella Bartoli for a detailed critical inspection of this Paper I.

Special thanks and appreciation are due to Arun Muktibodh for a very detailed inspection and control of all three papers, although I am solely responsible for their content due to numerous revisions implemented in their final version.

Thanks are also due to Mrs. Sherri Stone for an accurate linguistic control of the manuscript.

Finally, I would like to express my deepest appreciation and gratitude to my wife Carla Gandiglio Santilli for decades of continued support of my research amidst a documented international chorus of voices opposing the study of the most important vision by Albert Einstein.

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Studies on A. Einstein, B. Podolsky and N. Rosen argument that “quantum mechanics is not a complete theory,” II: Apparent confirmation of the EPR argument

Ruggero Maria Santilli*

Abstract

In 1935, A. Einstein expressed his historical view, jointly with B. Podolsky and N. Rosen, that quantum mechanics could be “completed” into a form recovering classical determinism at least under limit conditions (*EPR argument*). In the preceding Paper I, we have outlined the novel methods underlying the “completion” of quantum mechanics into hadronic mechanics for the representation of extended, thus deformable particles within physical media. In this Paper II, we study the *isosymmetries* for interior dynamical systems; we confirm the 1998 apparent proof that interior dynamical systems admit a classical counterpart; we confirm the 2019 apparent proof that Einstein’s determinism is progressively approached for extended particles in the interior of hadrons, nuclei and stars, while being fully verified in the interior of gravitational collapse; and we show for the first time that the recovering of Einstein’s determinism in interior systems implies the apparent removal of quantum mechanical divergencies. In the subsequent Paper III, we present a number of illustrative examples and novel applications in mathematics, physics and chemistry.

Keywords: EPR argument, isomathematics, isomechanics.

2010 AMS subject classifications: 05C15, 05C60. ¹

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¹Received 26th of April, 2020, accepted 19th of June, 2020, published 30th of June, 2020.
doi 10.23755/rm.v38i0.518, ISSN 1592-7415; e-ISSN 2282-8214., ©Ruggero Maria Santilli

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1. INTRODUCTION

1.1. The EPR argument.

As it is well known, Albert Einstein did not accept quantum mechanical uncertainties as being final, for which reason he made his famous quote "God does not play dice with the universe."

More particularly, Einstein believed that "quantum mechanics is not a complete theory," in the sense that it could be broadened into such a form to recover classical determinism at least under limit conditions.

Einstein communicated his views to B. Podolsky and N. Rosen and they jointly published in 1935 the historical paper [1] that became known as the *EPR argument*.

In view of the rather widespread belief that quantum mechanics is a final theory valid for all conceivable conditions existing in the universe, objections against the EPR argument have been voiced by numerous scholars, including by N. Bohr [2], J. S. Bell [3] [4], J. von Neumann [5] and others (see Ref. [6] for a review and comprehensive literature). The field became known as *local realism* and included the dismissal of the EPR argument based on claims that quantum axioms do not admit *hidden variables* λ [7] [8].

1.2. Outline of Paper I.

This paper, and the preceding Ref. [9] (hereinafter referred to as Paper I), are dedicated to the review and upgrade of decades of studies by mathematicians, physicists, and chemists (see Refs. [10] to [71] and papers quoted therein) on the apparent proof of the EPR argument via the "completion," also called *isotopic lifting*, of quantum mechanics into the axiom-preserving *hadronic mechanics* (see the 1995 monographs [29] [30] [31] and literature quoted therein).

More specifically, in Section I-1.1, we have outlined the EPR argument [1] jointly with representative objections [2] to [6].

In Section I-1.2, we have outlined the apparent proof by R. M. Santilli [10] (See also the detailed study in monograph [30], particularly Chapter 4 and Appendix 4C, page 166) that interior dynamical systems represented



Figure 1: *In this figure, we present a conceptual rendering of the tacit assumption underlying the objections against the EPR argument [2] - [6], namely, the representation of particles as being point-like because mandated by the Newton-Leibnitz differential calculus underlying quantum mechanics, namely, the representation of particles as isolated points in empty space. A first consequence is that, being dimensionless, particles can only be at a distance, with ensuing Einstein's argument on the need for superluminal interactions to explain quantum entanglement [1]. A second consequence is that, being at a distance, the sole possible interactions are of linear, local and potential type, under which assumptions the objections against the EPR argument are indeed valid.*

with hadronic mathematics and mechanics admit classical counterparts.

In the same Section I-1.2, we have outlined the apparent second proof by Santilli [11] that classical determinism is progressively approached in the interior of hadrons, nuclei, stars and gravitational collapse as predicted by Einstein.

In support of the plausibility of the EPR argument, in the subsequent Sections I-1.3 to I-1.7, we have outlined insufficiencies of quantum mechanics for time-irreversible processes, particle physics, nuclear physics, chemistry, and other fields. We have also provided various references indicating the apparent resolution of said insufficiencies by hadronic mechanics.

In Section I-2, we have outlined the *Lie-admissible covering of Lie's theory* [12] [13], with ensuing time-irreversible *Lie-admissible branch of hadronic mechanics*, also known as *genomechanics*, [12] [14] allowing studies on the compatibility of mechanics with thermodynamics, said compatibility being notoriously impossible for quantum mechanics.

Quantum mechanics and the objections against the EPR argument are formulated for time-reversal invariant systems of exterior dynamical systems. Therefore, in preparation for the proof of the EPR argument studied in Section 3, we have outlined and upgraded in Section I-3 the time-reversal invariant *Lie-isotopic subclass of Lie-admissible mathematics*, also known as *isomathematics*, [15] [18] which is used for the representation of time-reversible invariant interior dynamical systems.

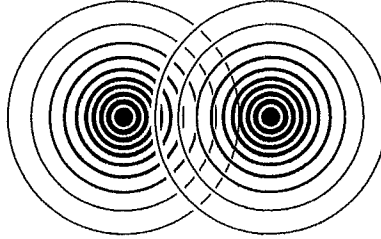


Figure 2: A conceptual rendering of the main assumption of the apparent proofs [10] [11] of the EPR argument [1], is the representation of particles as extended, deformable and hyperdense in conditions of mutual overlapping/entanglement with ensuing continuous contact at a distance which eliminates the need for superluminal interactions to explain quantum entanglement. A first implication is the need, for consistency, of generalizing Newton-Leibnitz differential calculus from its historical form solely definable at isolated points, to a covering form definable in volumes [21]. Another implication is the emergence of contact, non-linear, non-local and non-potential interactions that, being not representable by Hamiltonians are Lagrangians, require a structural lifting of the Lie algebra of quantum mechanics under which the objections against the EPR argument are inapplicable (Section 3). Intriguingly, the “completions” here considered turned out to be of isotopic/axiom-preserving type, thus being fully admitted by quantum mechanical axioms, merely subjected to a realization broader than that of the Copenhagen school. The apparent proofs of the EPR argument [10] [11] become an unavoidable consequence of the indicated “completions” (Section 3).

In the same Section I-3, we have devoted particular attention to the “completion” of conventional Hilbert spaces [19], numeric fields [20] and Newton-Leibnitz differential calculus [21] into forms defined on *volumes*, rather than points.

In the same Section I-1.3, we have provided particular attention to the main methods for the proofs of the EPR argument, namely, the *axiom-preserving, isotopic lifting of Lie’s theory* [26], today known as the *Lie-Santilli isothory* [38].

Finally, in Section I-4, we have outlined and upgraded the time-reversal invariant *isotopic branch of hadronic mechanics*, also known as *isomechanics* [30] which provides the dynamical foundations of the proofs of the EPR argument [10] [11].

1.3. Basic assumptions.

The most dominant aspects underlying the studies here considered are:

1) The validity of quantum mechanics for point-like particles in vacuum with ensuing linear, local and action-at-a-distance/potential interactions (*exterior dynamical problems*) occurring in atomic structures, particles in accelerators, crystals and numerous other systems in nature (Figure 1);

2) The "completion" of quantum mechanics into hadronic mechanics for the representation of extended, therefore deformable and hyperdense particles within physical media with ensuing, additional, non-linear, non-local and contact/non-potential interactions (*interior dynamical problems*), occurring in the structure of hadrons, nuclei and stars, with limit conditions occurring in the interior of gravitational collapse where the inapplicability (rather than the violation) of quantum mechanics is already accepted by the majority of serious scholars (Figure 2, 3).

The central assumption of these studies is the axiom-preserving lifting of the conventional associative product $ab = a \times b$ between *all* possible quantum mechanical quantities (numbers, functions, matrices, etc.) into the *isoproduct* [14] [26] (Section 3)

$$a \star b = a \hat{T} b, \quad (1)$$

where $\hat{T}$, called the *isotopic element*, is restricted to be positive-definite, $\hat{T} > 0$, but possesses otherwise an unrestricted functional dependence on all needed local variables.

Refs. [14] [26] constructed an axiom-preserving isotopy of the various branches of Lie's theory, resulting in a theory today known as the *Lie-Santilli isothory* [38] (Section I-3.7) with isotopic lifting of lie algebras of the type [10]

$$[X_i, \hat{X}_j] = X_i \star X_j - X_j \star X_i = C_{ij}^k X_k, \quad i, j = 1, 2, \dots, N. \quad (2)$$

Following laborious efforts for the achievement of mathematical maturity, Ref. [10] applied the Lie-Santilli isothory to the isotopy $\hat{S}\hat{U}(2)$ of the $SU(2)$ spin with three-dimensional isoalgebras of type (2) and introduced the realization of hidden variables [7] [8] of the type

$$\hat{T} = \text{Diag.}(1/\lambda, \lambda), \quad \text{Det}\hat{T} = 1. \quad (3)$$

Ref. [10], therefore establishing that, contrary to objections [2] to [6], *the abstract axioms of quantum mechanics do indeed admit explicit and concrete realizations of hidden variables.*

The proof in Ref. [10] that interior systems admit identical classical counterparts was consequential (Section 3).

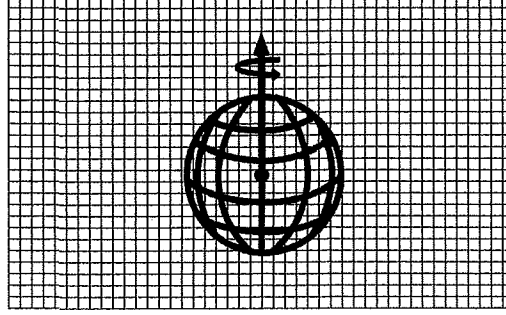


Figure 3: A conceptual rendering of the central notion used for the study of the EPR argument, namely, a mathematically consistent representation invariant over time of extended, deformable and hyperdense particles in interior conditions, such as protons in the interior of a star, thus being under the most general known (non-singular) non-linear, non-local and non-potential interactions fully representable via the isotopic element of isoproduct (1).

Isoproduct (1) also allows a direct and immediate representation of extended particles in conditions of mutual penetration with realizations of the type (Figure 3) [33]

$$\hat{T} = \Pi_{k=1,\dots,N} \text{Diag.} \left(\frac{1}{n_{1k}^2}, \frac{1}{n_{2k}^2}, \frac{1}{n_{3k}^2}, \frac{1}{n_{4k}^2} \right) e^{-\Gamma}, \quad (4)$$

$$k = 1, 2, \dots, N, \quad \mu = 1, 2, 3, 4,$$

where n_1^2, n_2^2, n_3^2 , (called *characteristic quantities*) represent the deformable semi-axes of the particle normalized to the values $n_k^2 = 1, k = 1, 2, 3$ for the sphere; n_4^2 represents the *density* of the particle considered normalized to the value $n_4 = 1$ for the vacuum; and Γ represents non-linear, non-local and non-Hamiltonian interactions caused by mutual penetrations/entanglement of particles.

The smaller than 1 absolute value of the isotopic element $\hat{T}$ occurring in all known applications [26]-[36]

$$|\hat{T}| \leq 1, \quad (5)$$

permitted Ref. [?] to show that the standard deviations Δr and Δp appear to progressively tend to zero with the increase of the density of the medium, and appear to achieve full classical determinism in the interior of gravitational collapse, as originally conceived by Einstein.

The initial construction of the isotopies of 20th century applied mathematics with isoproduct (1) defined over conventional numeric fields $F(n, \times, 1)$ [26] turned out to be inconsistent because the underlying time evolution is *non-unitary*, thus causing the lack of invariance over time of the traditional basic unit 1, with ensuing inapplicability over time of the entire field $F(n, \times, 1)$.

The above occurrence mandated the construction of *isofields* $\hat{F}(\hat{n}, \star, \hat{I})$ [20] [41](Section I-3.3) with basic *isounit*

$$\hat{I} = 1/\hat{T} > 0, \quad (6)$$

and *isonumbers* $\hat{n} = n\hat{I}$ equipped with isoproduct (1).

Ref. [20] essentially established that *the abstract axioms of a numeric field do not require that the multiplicative unit of the field be the trivial number 1, since said unit can be an arbitrary quantity with an unrestricted functional dependence on local variables, provided that said multiplicative unit is positive definite and the field is lifted into a compatible form.*

Despite all the above efforts, the ensuing isomathematics was still inapplicable to the proof of the EPR argument because it lacked the crucial *invariance over time*, namely, the prediction of the same interior dynamical systems under the same conditions but at different times.

The above occurrence forced the construction of the covering of the Newton-Leibnitz differential calculus into the covering *isodifferential isocalculus* [21] [44] (Section I-3.6) with basic *isodifferential* (Figure 2) [?]]

$$\hat{d}\hat{r} = \hat{T}d[r\hat{I}(r, \dots)] = dr + r\hat{T}d\hat{I}(r, \dots), \quad (7)$$

and corresponding *isoderivative*

$$\frac{\partial \hat{f}(\hat{r})}{\partial \hat{r}} = \hat{I} \frac{\partial \hat{f}(\hat{r})}{\partial r}. \quad (8)$$

In essence, Ref. [21] established the inapplicability of the conventional differential calculus whenever the axioms of numeric fields admit multiplicative units with a dependence on the differentiation variable, with ensuing inapplicability of quantum mechanics, as well as of the objections against the EPR argument, for interior dynamical systems.

The “completion” of the differential calculus into an isotopic form compatible with basic isoproduct (1) finally allowed the achievement of invariance over time (Section I-3.9), thus signaling the achievement of maturity for the apparent proof of the EPR argument reviewed.

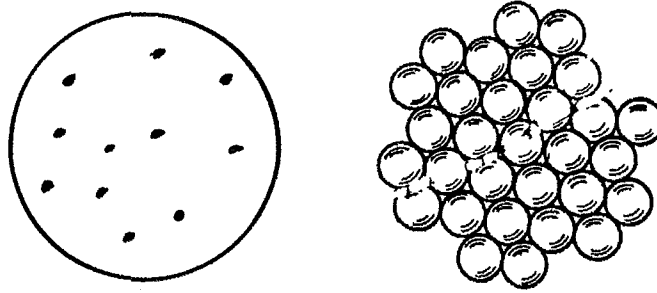


Figure 4: In the l.h.s. of this picture, we present a conceptual rendering of the structure of nuclei as ideal spheres with isolated point-like particles in their interior. This view is an inevitable consequence of the elaboration of quantum mechanics via the conventional differential calculus, resulting in rather serious insufficiencies in nuclear physics outlined in Section I-1.5. In the r.h.s. of this picture, we present a conceptual rendering of the representation of nuclei as occurring in the physical reality, namely, as a collection of extended, therefore deformable charge distributions in condition of partial mutual penetration according to Eq. (4) of isomathematics and related isomechanics, thus permitting the resolution of at least some of the insufficiencies of quantum mechanics in nuclear physics reviewed in Section I-1.5.

In Section 2 of this paper, we complete the methodological needs by outlining and upgrading the time-reversal invariant coverings of conventional spacetime symmetries, known as *isosymmetries*, for systems of extended particles in interior conditions; in Section 3, we review and upgrade the Lie-isotopic $SU(2)$ -spin symmetry and related proofs [10] [11] of the EPR argument.

A few comments on terminologies appear to be recommendable.

The word “completion” is used in these studies to honor the memory of Albert Einstein and should not be intended to indicate “final” theories. In fact, isomathematics and isomechanics admit coverings of Lie-admissible character [12] (Section I-2) that, in turn, admits coverings of hyperstructural character [43], with additional coverings remaining possible in due time.

The terms “non-Hamiltonian interactions” are intended to indicate interactions that are not representable with a Hamiltonian, and are technically identified as interactions violating the integrability conditions for the existence of a Hamiltonian, namely, the *conditions of variational self-adjointness* [25].

When dealing with stable and isolated interior dynamical systems, the

terms “non-conservative forces” are strictly referred to *internal* non-Hamiltonian exchanges verifying conditions (1-55) for the verification of the ten conventional total conservation laws for the total energy, momentum, angular momentum and the uniform motion of the center of mass.

The terms “physical media” refer to media composed by matter in its various states, and are often referred to as *hadronic media*, in the sense that the media are *not* composed by empty space, thus requiring the use of hadronic mathematics and mechanics for their quantitative treatment.

The terms “extended particles” refer to: the wavepacket of elementary particles such as the electron assumed to be of about $1\text{ fm} = 10^{-15}\text{ cm}$; extended charge distributions for protons and neutrons when members of a nuclear structure, also assumed to have a diameter of about 1 fm ; and stable nuclei when considering the structure of stars. Due to its crucial significance for the structure of interior systems, a technical definition of the notion of “extended particles” will be given in Section 3 via the notion of *isoparticle* as isorepresentations of space-time isosymmetries.

2. ISOSYMMETRIES

2.1. Foreword.

In this section, we study the axiom-preserving “completion” (or isotopic lifting) of conventional space-time symmetries, known as *Lie-isotopic symmetries*, or *isosymmetries* for short, which provide the invariance of stable and isolated (thus time reversible) interior dynamical systems of extended particles at mutual distances smaller than their size as occurring, e.g., in nuclear structures (Figure 3).

Lie-isotopic symmetries were first introduced by Santilli in the 1978 Harvard University paper [13] as a particular case of the broader *Lie-admissible symmetries* for irreversible, non-conservative systems [14]. Isosymmetries were then studied in various subsequent works quoted in this section.

The understanding of this section requires a knowledge of the *Lie-Santilli isothory* (Section I-3.7), which was first formulated in monographs [25] [26] over the field of real numbers. Isosymmetries were then formulated in monographs [29] [30] with the full use of isomathematics, including the use isofields [20] [41] and the isodifferential calculus [21] [44] (see Refs. [38] [46] [47] [48] for works on the Lie-Santilli isothory, and Ref. [45] for a general review with applications and experimental verifications).

The assumption at the foundation of isosymmetries is *the preservation of the abstract axioms of 20th century space-time symmetries, and the mere construction of their broadest possible realization permitted by isomathematics.*

Consequently, criticisms of isosymmetries and their novel implications

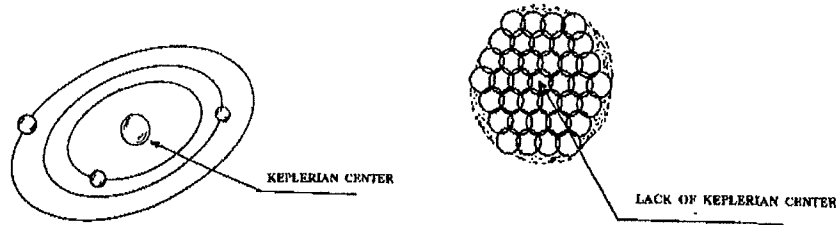


Figure 5: The l.h.s. of this figure illustrates Keplerian systems for which space-time symmetries have been constructed, namely, exterior dynamical systems of point-like masses orbiting in vacuum around a heavier point-like mass known as the Keplerian center. The r.h.s. of this figure illustrates interior systems for which isosymmetries have been built, namely, systems of extended particles in conditions of mutual penetration without any Keplerian center.

are *de facto* criticisms on 20th century space-time symmetries and their implications

2.2. Inapplicability of Lie symmetries for interior systems.

A rather widespread view of 20th century physics is the lack of any difference between exterior and interior dynamical systems on grounds that the latter can be reduced to their elementary constituents, by therefore recovering exterior conditions.

The above view was disproved by R. M. Santilli in his Ph. D. thesis (see the review in Ref. [30]) on numerous grounds, the first being the notorious incompatibility of quantum mechanics with thermodynamics whose resolution motivated the Lie-admissible generalization of Lie algebras and related physical theories [29] [30].

The absence of structural differences between exterior and interior systems was dismissed more directly with the following [26] [28]:

NO REDUCTION THEOREM 2.2.1: *A classical dynamical system with non-conservative interior forces cannot be consistently reduced to a finite number of isolated particles all in conservative conditions and, vice-versa, the latter system cannot reproduce the former under the correspondence or other principles.*

The first direct consequence of the above No Reduction Theorem is the “inapplicability” (rather than the “violation”) for interior dynamical systems of conventional space-time symmetries that have been proved to be so effective for exterior dynamical systems.

Said inapplicability was also proved [*loc. cit.*] from the fact that the

Galileo and the Lorentz-Poincaré symmetries can only provide a non-relativistic and relativistic characterization, respectively, of *Keplerian systems*, namely, systems of point-like masses orbiting in vacuum around a heavier mass called the *Keplerian nucleus* [26].

However, interior dynamical systems do not admit a Keplerian structure because *nuclei have no nuclei* [33] and the same happens for hadrons, stars and gravitational collapse (Figure 5).

It is then possible to prove, e.g., via the imprimitivity theorem, that the lack of existence of a Keplerian structure implies the lack of exact validity of conventional space-time symmetries [26] [30].

On more technical grounds, Lie's theory is known to be solely applicable to exterior systems of point-like particles in vacuum with ensuing sole possible, linear, local and Hamiltonian interactions.

Experimental evidence on interior dynamical systems, e.g., on nuclear volumes compared to the volumes of individual nucleons, establishes that nuclei are composed of extended charge distributions in conditions of partial mutual penetration/entanglement with the ensuing existence of additional, non-linear, non-local and non-Hamiltonian interactions under which Lie's theory is inapplicable.

Hence, the transition of particles from exterior to interior conditions implies the inapplicability of the $SU(2)$ -spin symmetry with consequential inapplicability of Bell's inequality [3] and other objections against the EPR argument [6] in favor of suitable covering vistas [10] [11].

In any case, the $SU(2)$ symmetry, while unquestionable effective for exterior dynamical systems, has been unable to provide a consistent representation of the spin of particles and nuclei, thus warranting the search for a suitable "completion."

2.3. The fundamental theorem on isosymmetries.

The construction of isosymmetries requires the full use of isomathematics with particular reference to the Lie-Santilli isothory formulated on isospaces over isofield and elaborated via the isodifferential calculus (Section I-2.7).

Said construction can be done with the following theorem (for brevity, see the proof in Section 1.2, Vol. I of Refs. [36]):

THEOREM 2.3.1: *Let G be an N -dimensional Lie symmetry of the line element of a k -dimensional metric or pseudo-metric space $S(x, m, I)$ over a numeric field F*

with coordinates x , metric m over a numeric field F with conventional unit I ,

$$\begin{aligned} G : \quad x' &= \Lambda(w)x, \quad y' = \Lambda(w)y, \quad x, y \in S, \\ (x' - y')^\dagger \Lambda^\dagger m \Lambda (x' - y') &\equiv (x - y)^\dagger m (x - y), \\ \Lambda^\dagger(w) m \Lambda(w) &\equiv m. \quad w \in F. \end{aligned} \quad (9)$$

Then, all infinitely possible (non-singular) Lie-Santilli isotopies $\hat{G}$ of G on isospace $\hat{S}(\hat{x}, \hat{M}, \hat{I})$ with isocoordinates

$$\hat{x} = xI, \quad (10)$$

isometric

$$\hat{M} = \hat{m}\hat{I} = (\hat{T}_i^k m_{kj})\hat{I}, \quad (11)$$

and isounit

$$\hat{I} = 1/\hat{T} > 0, \quad (12)$$

over an isofield $\hat{F}$ with isounit $\hat{I}$ leave invariant the isoline element of the isospace $\hat{S}(\hat{x}, \hat{M}, \hat{I})$:

$$\begin{aligned} \hat{G} : \quad \hat{x}' &= \hat{\Lambda}(\hat{w}) \star \hat{x}, \quad \hat{y}' = \hat{\Lambda}(\hat{w}) \star \hat{y}, \quad \hat{x}, \hat{y} \in \hat{S}, \\ (\hat{x}' - \hat{y}')^\dagger \star \hat{\Lambda}^\dagger \star \hat{M} \star \hat{\Lambda} \star (\hat{x}' - \hat{y}') &\equiv (x - y)^\dagger \hat{m} (x - y), \\ \hat{\Lambda}^\dagger(\hat{w}) \star \hat{M} \star \hat{\Lambda}(\hat{w}) &\equiv \hat{M}. \end{aligned} \quad (13)$$

All infinitely possible so constructed isosymmetries $\hat{G}$ are locally isomorphic to the original symmetry G .

The reader should note that, while a given Lie symmetry G is unique as well known, there can be an infinite number of covering isosymmetries $\hat{G}$ with generally different explicit forms of the isotransformations due to the infinite number of possible isotopic elements representing the infinitely different internal interactions of extended particles within physical media.

Note also that all possible isotopic images of a given Lie symmetry can be explicitly and uniquely constructed via the sole knowledge of the original Lie symmetry and of the isotopic element $\hat{T} > 0$, or of the isounit $\hat{I} = 1/\hat{T}$, which property shall be hereon tacitly assumed.

2.4. Isospaces and isogeometries.

As it is well known, the fundamental representation space of relativistic space-time symmetries is the conventional *Minkowski space* $M(x, \eta, I)$ formulated on the field of real numbers $\mathcal{R}$ with coordinates $x = (x^1, x^2, x^3, x^4 = ct)$, metric $\eta = \text{Diag.}(1, 1, 1, -1)$, unit $I = \text{Diag}(1, 1, 1, 1)$ and invariant

$$\begin{aligned} x^2 &= (x^\mu \eta_{\mu\nu} x^\nu) I = \\ &= (x_1^2 + x_2^2 + x_3^2 - c^2 t^2) I, \end{aligned} \quad (14)$$

where the trivial multiplication by the conventional unit $I = \text{Diag.}(1, 1, 1, 1)$ is done for compatibility with isomathematics.

The fundamental isospaces of space-time isosymmetries are given by the infinite family of *iso-Minkowski isospaces*, also called *Minkowski-Santilli isospaces*, $\hat{M}(\hat{x}, \hat{\Omega}, \hat{I})$ formulated on the isofield of isoreal isonumbers $\hat{\mathcal{R}}$. (Section I-3.9), which isospaces were first introduced by R. M. Santilli in Ref. [23] of 1983 and then treated in details in works [29] [30].

Iso-Minkowskian isospaces are characterized by *space-time isocoordinates* $\hat{x} = x\hat{I}$; *isounit* $\hat{I} = 1/\hat{T}$, *isometric*

$$\hat{\Gamma} = (\hat{T}_\mu^\rho \eta_{\rho\nu}) \hat{I}, \quad (15)$$

(where one should note the necessary structure of an isomatrix [29]), positive-definite *isotopic element* (4) representing a system of extended particles in interior dynamical conditions with a restricted functional dependence on local quantities such as coordinates x , momenta p , energy E , frequency ν , density α , temperature τ , pressure pi , wavefunction ψ , etc., under the conditions

$$n_\mu = n_\mu(x, p, E, \nu, \alpha, \tau, \pi, \psi, \partial\psi, \dots) > 0, \quad \mu = 1, 2, 3, 4, \quad (16)$$

$$\Gamma(x, p, E, \nu, \alpha, \tau, \pi, \psi, \partial\psi, \dots) \geq 0, \quad (17)$$

$$\hat{T} = e^{-\Gamma} \ll 1. \quad (18)$$

Iso-Minkowskian isospaces are characterized by the infinite family of isoinvariants (I-28) with isotopic element (4) that, for the case of one single extended particle can be written

$$\begin{aligned} \hat{x}^2 &= \hat{x}^\mu \star \hat{\Omega}_{\mu\nu} \star \hat{x}^\nu = (x^\mu \hat{\eta}_{\mu\nu} x^\nu) \hat{I} = \\ &= \left(\frac{x_1^2}{n_1^2} + \frac{x_2^2}{n_2^2} + \frac{x_3^2}{n_3^2} - t^2 \frac{c^2}{n_4^2} \right) \hat{I}, \end{aligned} \quad (19)$$

where the exponential $\exp-\Gamma$ has been absorbed in the characteristic quantities n_μ , and the final multiplication by the isounit is necessary for the

isoinvariant to be an isoscalar, namely, an element of the isoreal isofield [20] (Section I-3-5).

The following aspects treated in Paper I are important for the understanding of the apparent proof of the EPR argument:

1. The characteristic quantities n_1^2, n_2^2, n_3^2 , admit the first interpretation as representing the deformable semi-axes of elementary or composite particles normalized to the values for the sphere $n_1^2 = n_2^2 = n_3^2 = 1$.

2. The characteristic quantity n_4^2 admit the first interpretation as representing the *density* of the hadronic medium normalized to the value $n_4 = 1$ for the vacuum.

3. The function $\Gamma \geq 0$ provides an invariant representation (Section I-3-9) of all non-linear, non-local and non-Hamiltonian interactions.

4. Property (18) is verified for all applications of isosymmetries to date [10] to [68].

5. The correct elaboration of iso-Minkowskian isospaces requires the use of the *isospherical and isohyperbolic isocoordinates* (see Refs. [29] [30]).

6. Isoinvariant (19) provides a unified representation of both exterior and interior gravitational problems. In fact, K. Schwartzchild wrote in 1916 *two* important papers, the first paper [49] on the *exterior gravitational problem* which became world famous for its initiation of gravitational singularities, and the second paper [50] in the *interior gravitational problem* which has been vastly ignored, except rare studies (such as that in Section 23.2, page 609, Ref. [52]). Such an oblivion is essentially due to the fact that Schwartzchild's second paper is not aligned with the widespread tendency of reducing masses to point-like constituents, in which case all differences between exterior and interior gravitational problems disappear to the detriment of the depth of the gravitational analysis. Readers should keep in mind the full parallelism between exterior and interior dynamical problems for *particles and gravitation*.

7. The *exterior gravitational interpretation* of isoinvariant (19) is given by the following identical representation of Schwartzchild's exterior metric [50]

$$\hat{T}_{kk} = \frac{1}{1 - \frac{2M}{r}}, \quad \hat{T}_{44} = 1 - \frac{2M}{r}. \quad (20)$$

The corresponding *interior gravitational representation* is given by the fol-

lowing isotopy of Schwartzchild exterior metric

$$\hat{T}_{kk} = \frac{1}{(1 - \frac{2M}{r})n_k^2}, \quad \hat{T}_{44} = (1 - \frac{2M}{r})/n_4^2. \quad (21)$$

In view of the arbitrariness of the functional dependence of the characteristic quantities n_μ , it is easy to prove that Schwartzchild's interior metric [50] is a particular case of the much broader class of interior gravitational models (21).

8. The geometry of the iso-Minkowskian isospaces, first presented by Santilli in Ref. [24] under the name of *iso-Minkowskian isogeometry*, contains the machinery of the Riemannian geometry (due to the dependence of the isometric $\hat{\eta}$ on the local coordinates x), although such a machinery is formulated for consistency over isofields [20] and elaborated via the isodifferential isocalculus [21] (Section I-3.5). Hence, *the isominkowskian isogeometry can unify exterior and interior problems for both particles and gravitation.*

9. Recall that iso-Minkowskian isospaces are locally isomorphic to the conventional Minkowski space (Refs. [23] [24] and Theorem 2.3.1). Therefore, *the iso-Minkowskian isogeometry has a null curvature.* This is due to the fact that, under isotopic lifting, the conventional Minkowski metric $\eta = \text{Diag.}(1, 1, 1, -1)$ is lifted into a coordinate-dependent isometric $\hat{T}(x)\eta = \hat{\eta}(x)$ which is *identical* to any given Riemannian metric

$$\eta \rightarrow \hat{\eta}(x) = \hat{T}(x)\eta = g(x). \quad (22)$$

Jointly, the original unit of the Minkowski space $\hat{I} = \text{Diag.}(1, 1, 1, 1)$ is lifted by the *inverse* amount

$$I > 0 \rightarrow \hat{I}(x) = 1/\hat{T}(x) > 0, \quad (23)$$

resulting in no actual curvature. The above features have suggested the introduction of the new notion of *isoflat isospace*, referred to an isospace that has null curvature when formulated on isofields, while recovering conventional curvature when formulated on conventional fields. Readers should be aware that the achievement of the universal symmetry of (non-singular) Riemannian line elements studied in the next sections are due precisely to the isoflatness of the iso-Minkowski isospace since no such symmetry is possible for a conventional Riemannian space, as well known.

Recall that the fundamental representation space of symmetries in 3-space dimensions is the conventional Euclidean space $E(r, \times, I$ with coor-

dinates $r = (x^1, x^2, x^3)$, metric $\delta = \text{Diag.}(1, 1, 1)$ and unit $I = \text{Diag.}(1, 1, 1)$ on the conventional field of real numbers.

Similarly, the fundamental representation space of isosymmetries in 3-dimensions is the *iso-Euclidean isospace* $\hat{E}(\hat{r}, \hat{\delta}, \hat{I})$, also called *Euclid-Santilli isospace* (Refs. [14] [26] [29] and Section I-3.5) which is the space component of the iso-Minkowskian isospace. As such, the iso-Euclidean isospace is hereon tacitly assumed to be known.

2.5. Lorentz-Poincaré-Santilli isosymmetries.

2.5.1. Main references. Following, and only following the construction of the isotopies of Lie's theory, Santilli conducted systematic studies on the isotopies of the various aspects of the Lorentz-Poincaré symmetry for the achievement of the universal invariance of spacetime isoinvariant (19), including:

- 1) The classical isotopies $\hat{SO}(3.1)$ of the Lorentz symmetry $SO(3.1)$ [53];
- 2) The operator isotopies $\hat{SO}(3.1)$ of the Lorentz symmetry $SO(3.1)$ [54];
- 3) The isotopies $\hat{SO}(3)$ of the rotational symmetry $SO(3)$ [55] [56] [57];
- 4) The isotopies $\hat{SU}(2)$ of the $SU(2)$ spin symmetry [10] [58];
- 5) The isotopies $\hat{P}(3.1)$ of the Poincaré symmetry $P(3.1)$ [59] [60], which included the universal symmetry of (non-singular) Riemannian line elements;
- 6) The isotopies $\hat{\mathcal{P}}(3.1)$ of the spinorial covering $\mathcal{P}(3.1)$ of the Poincaré symmetry [61] [62];
- 7) The isotopies $\hat{M}(3.1)$ of the Minkowskian geometry $M(3.1)$ [24].

A general presentation is available in the 1995 monographs [29] [30] with the full use of isomathematics, including isofields and isodifferential calculus, with upgrades in the 2008 monographs [36].

The resulting infinite family of isosymmetries $\hat{SO}(3.1)$ are known as the *Lorentz-Santilli (LS) isosymmetries* while the broader isosymmetries $\hat{P}(3.1)$ and $\hat{\mathcal{P}}(3.1)$ are known as *Lorentz-Poincaré-Santilli isosymmetries* (see Refs. [37] [42] [45] and papers quoted therein).

Experimental verifications of LPS isosymmetries for interior dynamical systems are available in monographs [31] and in Section 3 of the more recent review [63].

In inspecting the subsequent sections, the reader should be aware of the "direct universality" of the LPS isosymmetries for the considered infinite family of interior dynamical systems [64], including the treatment of exterior and interior, particle and gravitational problems (Section 4).

2.5.2. Basic definitions. As it is well known, the conventional Lorentz-Poincaré (LP) symmetry is the symmetry of line element (14) which we rewrite in the form

$$\begin{aligned} (x - y)^2 &= (x^\mu - y^\mu) \eta_{\mu\nu} (x^\nu - y^\nu) I = \\ &= [(x_1 - y_1)^2 + (x_2 - y_2)^2 + (x_3 - y_3)^2 - (t_1 - t_2)^2 c^2] I, \quad (24) \\ \eta &= \text{Diag.}(1, 1, 1, -c^2), \quad I = \text{Diag.}(1, 1, 1, 1), \end{aligned}$$

where the exponential component $\exp -\Gamma$ is again embedded for simplicity in the characteristic quantities n_μ^2 .

The LPS isosymmetry is the universal symmetry of the isoline element (19) in the iso-Minkowski isospace $\hat{M}(\hat{x}, \hat{\Omega}, \hat{I})$ over the isoreal isonumbers $\hat{\mathcal{R}}$ rewritten in the form

$$\begin{aligned} (\hat{x} - \hat{y})^{\hat{2}} &= [(\hat{x}^\mu - \hat{y}^\mu) \star \hat{\Omega}_{\mu\nu} \star (\hat{x}^\nu - \hat{y}^\nu)] = \\ &= [x^\mu - y^\mu] \hat{\eta}_{\mu\nu} (x^\nu - y^\nu) \hat{I} = \\ &= \left[\frac{(x_1 - y_1)^2}{n_1^2} + \frac{(x_2 - y_2)^2}{n_2^2} + \frac{(x_3 - y_3)^2}{n_3^2} - (t_1 - t_2)^2 \frac{c^2}{n_4^2} \right] \hat{I}, \quad (25) \\ \hat{\eta} &= \hat{T} \eta, \quad \hat{T} = \text{Diag.}(\frac{1}{n_1^2}, \frac{1}{n_2^2}, \frac{1}{n_3^2}, \frac{1}{n_4^2}), \end{aligned}$$

$$n_\mu = n_\mu(x, v, a, E, d, \omega, \tau, \psi, \partial\psi, \dots) > 0, \quad \hat{I} = 1/\hat{T} > 0.$$

2.5.3. Isotransformations. By following Theorem 2.3.1, the isotransformations of the LPS isosymmetries can be written

$$\hat{x}' = \hat{\Lambda}(\hat{w}) \star \hat{x}, \quad (26)$$

where $\hat{\Lambda}(\hat{w}) = \Lambda(\hat{w}) \hat{I}$, resulting in generally *non-linear* isotransformations, including isotranslations of the type

$$\hat{x}' = \hat{x} + \hat{A}(\hat{x}, \dots), \quad (27)$$

verifying the following property

$$\hat{\Lambda}^\dagger \star \hat{\eta} \star \hat{\Lambda} = \Lambda \hat{\eta} \Lambda^\dagger. \quad (28)$$

Under the condition of isomodularity

$$\hat{Det}(\hat{\Lambda}) = +\hat{I}, \quad (29)$$

we have the *isoconnected LS isosymmetries* $\hat{S}\hat{O}^0(3.1)$ and the *isoconnected LPS isosymmetries* $\hat{P}^0(3.1)$.

Consider the conventional generators of the Poincaré symmetry

$$(J_k) = (J_{\mu\nu}), P_\mu, k = 1, 2, 3, 4, 5, 6, \mu, \nu = 1, 2, 3, 4. \quad (30)$$

By keeping in mind isoexponentiation (I-16), the isotransformations of $\hat{S}\hat{O}^0(3.1)$ can be written [60]

$$\begin{aligned} \hat{x}' &= (\hat{e}^{iJ_k w_k}) \star \hat{x} \star (\hat{e}^{-iJ_k w_k}) = \\ &= \left[(e^{iJ_k \hat{T} w_k}) x (e^{-i w_k \hat{T} J_k}) \right] \hat{I}, \end{aligned} \quad (31)$$

and the isotranslations $\hat{A}(3.1)$ can be written

$$\begin{aligned} \hat{x}' &= (\hat{e}^{iP_\mu a_\mu}) \star \hat{x} \star (\hat{e}^{-iP_\mu a_\mu}) = \\ &= \left[(e^{iP_\mu \hat{T} a_\mu}) x (e^{-i a_\mu \hat{T} P_\mu}) \right] \hat{I}. \end{aligned} \quad (32)$$

It is evident that the above isotransformations do constitute Lie-Santilli isogroups according to Theorem I-2.7.3.

2.5.4. Isocommutation rules. As recalled earlier, the total quantities of an isolated, stable, interior system must be conserved for consistency.

In order to represent this evidence, the Lie-Santilli isothory was constructed [26] in such a way to preserve conventional generators, because they represent total conservation laws, and isotopically lift their product.

By expanding the preceding finite isotransforms in terms of the isounit, the *LPS isoalgebra* $\hat{s}\hat{o}^0(3.1)$ is characterized by the conventional generators of the LP algebra and the isocommutation rules [30] [60] (here written in their projection on conventional spaces over conventional fields)

$$\begin{aligned} [J_{\mu\nu}, \hat{J}_{\alpha\beta}] &= \\ &= i(\hat{\eta}_{\nu\alpha} J_{\beta\mu} - \hat{\eta}_{\mu\alpha} J_{\beta\nu} - \hat{\eta}_{\nu\beta} J_{\alpha\mu} + \hat{\eta}_{\mu\beta} J_{\alpha\nu}), \end{aligned} \quad (33)$$

$$\begin{aligned} [J_{\mu\nu}, \hat{P}_\alpha] &= i(\hat{\eta}_{\mu\alpha} P_\nu - \hat{\eta}_{\nu\alpha} P_\mu) \\ [P_\mu, \hat{P}_\nu] &= 0, \end{aligned} \quad (34)$$

$$\hat{\eta}_{\mu\nu} = \hat{T} \eta = (\hat{T}_\mu^\rho \eta_{\rho\nu}). \quad (35)$$

where one should note the appearance of the *structure functions* $\hat{\eta}(x, p, E, \nu, \alpha, \tau, \psi, \dots)$, rather than the traditional structure constants (Theorem I-2.7.2).

The presence of structure functions $\hat{\eta}$ in isocommutation rules (33)-(35), Theorem I-3.7.2 and the analysis of Section I-3.8 imply the following important property (Section I-3.8):

LEMMA 2.5.1: LPS isosymmetries cannot be derived via non-unitary transformations of the conventional LP symmetry.

Despite the above non-equivalence, the property $\hat{T} > 0$, the topological structure $(+1, +1, +1, -1)$ of the isometric $\hat{\eta} = \hat{T}\eta$ and Theorem 2.3.1 imply that:

LEMMA 2.5.2. All LPS isosymmetries are locally isomorphic to the conventional LP symmetry.

Recall from Section I-1 that an important limitation of quantum mechanics for the study of the EPR argument is the inability to achieve a consistent and effective treatment of non-linear interactions that are expected in the structure of hadrons, nuclei and stars. In Section I-4.12, we have shown that the isotopic "completion" of quantum mechanics into hadronic mechanics does indeed allow a consistent and effective treatment of non-linear interactions via their embedding in the isotopic element $\hat{T}$.

Due to the unrestricted functional dependence of the isotopic element $\hat{T}$ and, therefore, of the isometric $\hat{\eta} = \hat{T}\eta$, it is easy to see that the LPS isosymmetries are indeed non-linear as a necessary condition to provide the invariance of non-linear dynamical equations.

Note that *isolinear isomomenta* $\hat{P}_\mu$ *isocommute on isospaces over isofields, but they do not commute on conventional spaces over conventional fields*, Eqs. (35), thus confirming that *the LPS isosymmetry is isolinear, that is, linear on isospaces over isofields but generally non-linear in their projection on conventional spaces over conventional fields*.

This important property can be illustrated by recalling the isolinear isomomentum (I-79) on a Hilbert-Myung-Santilli isospace $\hat{\mathcal{H}}$ with isostates $\hat{\psi} >$ over the isocomplex isonumbers $\hat{\mathcal{C}}$

$$\hat{P}_\mu \star |\hat{\psi} > = -i\hat{I}\partial_\mu |\hat{\psi} > . \quad (36)$$

Isocommutators (35) on $\hat{\mathcal{H}}$ over $\hat{\mathcal{C}}$ can then be explicitly written

$$\begin{aligned} [\hat{P}_\mu, \hat{P}_\nu] \star |\hat{\psi}\rangle &= (\hat{P}_\mu \star \hat{P}_\nu - \hat{P}_\nu \star \hat{P}_\mu) \star |\hat{\psi}\rangle = \\ &= (-i\hat{I}\partial_\mu)\hat{T}(-i\hat{I}\partial_\nu) - (-i\hat{I}\partial_\nu)\hat{T}(-i\hat{I}\partial_\mu)\hat{T}|\hat{\psi}\rangle = \\ &= (i\hat{I}\partial_\mu\partial_\nu - i\hat{I}\partial_\nu\partial_\mu)|\hat{\psi}\rangle = 0. \end{aligned} \quad (37)$$

By contrast, the projection of the same isocommutators (35) on a conventional Hilbert space $\mathcal{H}$ over the field of complex numbers $\mathcal{C}$ no longer commutes,

$$\begin{aligned} [\hat{P}_\mu, \hat{P}_\nu]|\hat{\psi}\rangle &= (\hat{P}_\mu\hat{P}_\nu - \hat{P}_\nu\hat{P}_\mu)|\hat{\psi}\rangle = \\ &= (-i\hat{I}\partial_\mu)(-i\hat{I}\partial_\nu) - (-i\hat{I}\partial_\nu)(-i\hat{I}\partial_\mu)|\hat{\psi}\rangle \neq 0. \end{aligned} \quad (38)$$

because, in general, $\partial_\mu\hat{I} \neq \partial_\nu\hat{I}$, and this proves the isolinear character of the isomomentum.

Besides a direct relevance for the structure of hadrons, nuclei and stars, the above isolinearity has important implications, such as a new consistent operator form of gravitation, a new grand unification and other advances [35].

The presence of the structure functions in the isocommutation rules, the capability to provide the invariance under non-linear interactions and other features and applications outlined in Section 4 illustrate the non-triviality of the Lie-Santilli isothory.

2.5.5. Iso-Casimir Isoinvariants. The simple direct use of isocommutation rules (33)-(35) establishes that the *iso-Casimir-isoinvariants* of $\hat{p}^0(3.1)$ are given by [60]

$$\begin{aligned} \hat{C}_1 &= \hat{I}((t, r, p, E, \mu, \tau, \psi, \partial\psi, \dots)) > 0, \\ \hat{C}_2 &= \hat{P}^2 = \hat{P}_\mu \star \hat{P}^\mu = (\hat{\eta}_{\mu\nu} P^\mu P^\nu) \hat{I} = \\ &= (\sum_{k=1,2,3} \frac{1}{n_k^2} P_k^2 - \frac{e^2}{n_4^2} p_4^2) \hat{I}, \\ \hat{C}_3 &= \hat{W}^2 = \hat{W}_\mu \star \hat{W}^\mu, \quad \hat{W} = W \hat{I}, \\ \hat{W}_\mu &= \hat{\epsilon}_{\mu\alpha\beta\rho} \star J^{\alpha\beta} \star P^\rho, \end{aligned} \quad (39)$$

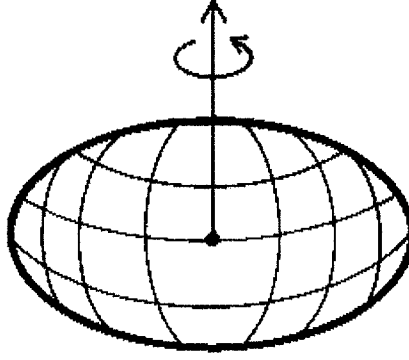


Figure 6: It was generally believed in the 20th century physics that the rotational symmetry is broken for ellipsoids. Santilli isorotational isosymmetry has restored the exact character of the rotational symmetry for all possible (topology preserving) deformations of the sphere [30].

and they are at the foundation of classical and operator *relativistic isomechanics* (Section I-4) with deep implications for structure models of interior dynamical systems [31].

2.5.6. Isorotations. By using isotransforms (32), the explicit form of the isorotations $\hat{SO}(3)$, first derived in Refs. [55] [56], can be written in the isoplane $(\hat{x}^1, \hat{x}^2)$ of iso-Euclidean isospaces $\hat{E}(\hat{x}, \hat{\Delta}, \hat{I})$ over the isoreals $\hat{\mathcal{R}}$, here formulated for simplicity in their projection on the conventional Euclidean space (see Ref. [30] for the general case)

$$\begin{aligned} x^{1'} &= x^1 \cos[\theta(n_1 n_2)^{-1}] - x^2 \frac{n_2^2}{n_1^2} \sin[\theta(n_1 n_2)^{-1}], \\ x^{2'} &= x^1 \frac{n_2^2}{n_1^2} \sin[\theta(n_1 n_2)^{-1}] + x^2 \cos[\theta(n_1 n_2)^{-1}]. \end{aligned} \quad (40)$$

It was generally believed in the 20th century that the $SO(3)$ symmetry is broken for ellipsoid deformations of the sphere. By contrast, as shown by isotransforms (40) the $\hat{SO}(3)$ isosymmetry achieves the invariance of ellipsoids (Figure 6). But $SO(3)$ and $\hat{SO}(3)$ are locally isomorphic (Theorem 2.3.1). We therefore have the following property [55] [56]:

LEMMA 2.5.3: *The Lie-Santilli $\hat{SO}(3)$ isosymmetry restores the exact character of the rotational symmetry for all ellipsoid deformations of the sphere.*

This property is due to the fact that the mutation of the semiaxes of the sphere occur jointly with the *inverse*, mutation of the related units, thus

maintaining the perfect spherical shape in isospaces over isofields

$$\text{Radius } 1_k \rightarrow 1/n_k^2, \text{ Unit } 1_k \rightarrow n_k^2. \quad (41)$$

Note the crucial role of isonumbers for the reconstruction of the exact rotational symmetry because said reconstruction occurs thanks to the isoinvariant by the isounit.

2.5.7. Lorentz-Santilli isotransforms. The infinite family of isoconnected Lorentz-Santilli (LS) isotransforms $\hat{S}\hat{O}^0(3.1)$ on iso-Minkowskian isospaces $\hat{M}(\hat{x}, \hat{\Omega}, \hat{I})$ over the isoreals $\hat{\mathcal{R}}$, first derived by in Ref. [53] of 1983, can be written in the $(\hat{x}^3, \hat{x}^4)$ -isoplane in their projection in the conventional 1 Minkowski space $M(x, \eta, I)$, as follows (see Ref. [30] for the general case):

$$\begin{aligned} x^{1'} &= x^1, \quad x^{2'} = x^2, \\ x^{3'} &= \hat{\gamma}(x^3 - \hat{\beta}_{\frac{n_3}{n_4}} x^4), \\ x^{4'} &= \hat{\gamma}(x^4 - \hat{\beta}_{\frac{n_4}{n_3}} x^3), \end{aligned} \quad (42)$$

where

$$\hat{\beta} = \frac{v_3/n_3}{c/n_4}, \quad \hat{\gamma} = \frac{1}{\sqrt{1 - \hat{\beta}^2}}. \quad (43)$$

A significant aspect of Ref. [53] is the solution of the *historical Lorentz problem*, namely, the invariance of locally varying speeds of light within physical media

$$C = \frac{c}{n_4}. \quad (44)$$

In fact, Lorentz first attempted the invariance of the speed of light $C = c/n_4$, but had to restrict his study to the invariance of the constant speed of light in vacuum c , due to insurmountable technical difficulties. Santilli has shown that Lorentz's difficulties were due to the use of Lie's theory, because, under the use of the covering Lie-Santilli isothory, the invariance of $C = c/n_4$ was achieved in two pages of the 1983 letter [53].

A second significant aspect of Ref. [53] is the achievement of the first invariant formulation of extended, thus deformable and hyperdense particles, as stated beginning with the title of the quoted paper.

It was generally believed in the 20th century that the Lorentz symmetry $SO^0(3.1)$ is broken for locally varying speed of light within physical media represented with the wiggly circle of Figure 7. Ref. [53] proved that the isosymmetry $\hat{S}\hat{O}^0(3.1)$ achieves the invariance of $C = c/n_4$. But $SO^0(3.1)$

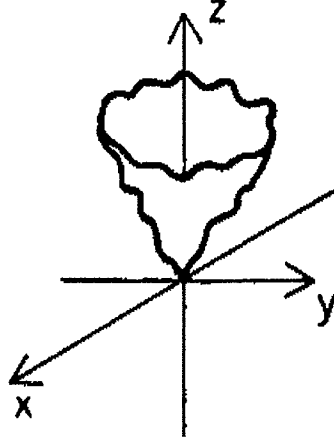


Figure 7: It was generally believed in the 20th century physics that the Lorentz symmetry is broken for locally varying speeds of light within physical media (here represented with a wiggly light cone). The Lorentz-Santilli isosymmetry has restored the exact validity of the Lorentz symmetry for interior dynamical problems [53] [30].

and $\hat{S}\hat{O}^0(3.1)$ are locally isomorphic, thus restoring the exact character of the abstract axioms of the Lorentz for all possible values $C = c/n_4$. We therefore have the following important property [30]:

LEMMA 2.3.5: *The Lie-Santilli $\hat{S}\hat{O}^0(3.1)$ isosymmetry restores the exact validity of Lorentz's axioms for locally varying speeds of light.*

This property is due to the reconstruction of the exact light cone on the iso-Minkowskian isospace over isofields with maximal causal value c , called the *light isocone*,

$$\hat{x}^{\hat{2}} = \hat{x}_3^2 + \hat{x}_4^2 = 0, \quad (45)$$

while its projection on the conventional Minkowski space over conventional fields represents a locally varying speed

$$\hat{x}^2 = \left(\frac{x_3^2}{n_3^2} - t^2 \frac{c^2}{n_4^4} \right) \hat{I} = 0. \quad (46)$$

This property is due to the fact that the mutation of the $\hat{x}^3$ and $\hat{x}^4$ iso-coordinates occurs jointly with the *inverse* mutation of the corresponding isounits, by therefore preserving the original perfect light cone with c as

the maximal causal speed (see the 1966 monograph [30] for details)

$$\begin{aligned} x^3 &\rightarrow \frac{x^3}{n_3}, & I_3 = 1 &\rightarrow \hat{I}_3 = n_3 \\ x^4 = tc &\rightarrow \frac{x^4}{n_4} = t \frac{c}{n_4}, & I_4 = 1 &\rightarrow \hat{I}_4 = n_4. \end{aligned} \quad (47)$$

Another significant aspect of Ref. [53] is the achievement of the first known invariance of non-linear, non-local and non-Hamiltonian interactions thanks to their embedding in the characteristic n -quantities of the isoinvariant (25).

2.5.8. Isotranslations. In view of their non-linearity, isotranslations in four parameters a_μ can be written in their projection in the conventional Minkowski space [30]

$$x'^\mu = x^\mu + A^\mu(a, x, \dots), \quad (48)$$

and can be written via a power series expansion of the general expression

$$A^\mu = a^\mu (n_\mu^{-2} + a^\alpha [n_\mu^{-2}, P_\alpha] / 1! + \dots), \quad (49)$$

The understanding of the isotopic completion of 20th century space-time symmetries requires the knowledge that, when properly written on iso-Minkowskian isospace over isofields, isotranslations recover their conventional form . [30].

2.5.9. Isodilatations. Santilli introduced in Ref. [60] a novel one-dimensional isoinvariance denoted $\hat{D}$ which is given by the dilatation of the isometric caused by its multiplication by as parameter w , while the isounit is jointly subjected to the *inverse* dilatation

$$\begin{aligned} \hat{\Omega} = \hat{\eta} \hat{I} &\rightarrow \hat{w} \star \hat{\Omega} = w \hat{\eta} \hat{I}' \\ \hat{I} &\rightarrow \hat{I}' = \frac{1}{w} \hat{I}, \end{aligned} \quad (50)$$

under which isoinvariant (25) remain manifestly unchanged.

In essence, the new symmetry originates from the fact that, for mathematical consistency, isoinvariants must be elements of isofields, thus having structure (25), namely, isoinvariants must be given by a conventional invariant multiplied by the isounit.

Ref. [60] showed that, by writing conventional invariants with the multiplication, in this case, by the trivial unit 1, the new dilatation symmetry persists for conventional space-time symmetries,

$$\eta \rightarrow \eta' = w\eta, \quad 1 \rightarrow 1' = \frac{1}{w}1. \quad (51)$$

The above properties imply the following:

LEMMA 2.5.5: *The conventional Lorentz-Poincaré symmetry is eleven-dimensional with structure*

$$P^0(3.1) = so^0(3.1) \times A(3.1) \times D, \quad (52)$$

and, consequently, the Lorentz-Poincaré-Santilli isosymmetry is also eleven-dimensional with the structure

$$\hat{P}^0(3.1) = \hat{so}^0(3.1) \star \hat{A}(3.1) \star \hat{D}. \quad (53)$$

The above seemingly trivial property has permitted Santilli the study of a new grand unification of electroweak and gravitational interactions based on the embedding of gravitation in the isotopic degree of freedom of the theory [35].

2.5.10. Isoinversions. The isotopic "completion" of conventional inversions has been studied in details in Refs. [30] and consists of the *isotime isoinversions*

$$\hat{\tau}\hat{t} = (\tau\hat{t})\hat{I} \quad (54)$$

plus the *isospace isoinversions*

$$\hat{\pi}\hat{r} = (\pi\hat{r})\hat{I} \quad (55)$$

where τ and π are conventional time and space inversions, respectively.

Despite their simplicity, Santilli has shown in Ref. [30] that *not only continuous, but also discrete space-time symmetries can be reconstructed as being exact on isospaces over isofields when assumed to be broken on conventional spaces over conventional fields.*

2.5.11. Isospinorial LPS isosymmetry. Recall that the spinorial covering $\mathcal{P}^0(3.1)$ of the connected component of the LP symmetry $P^0(3.1)$ is constructed via the use of the Dirac gamma matrices. In fact, the conventional generators are realized via suitable combination of Dirac gamma matrices.

By following the same historical pattern, Santilli proposed in the 1995 communication [61] of the *Joint Institute for Nuclear Research*, Dubna, Russia (see also the subsequent paper [62]) the following eleven-dimensional isotopic "completion" $\hat{\mathcal{P}}^0(3.1)$ of $\mathcal{P}^0(3.1)$

$$\hat{\mathcal{P}}(3.1) = \hat{S}\hat{L}(2.\hat{C}) \star \hat{A}(3.1) \star \hat{D}, \quad (56)$$

with realization of the generators in terms of the Dirac-Santilli isogamma isomatrices $\hat{\Gamma}_\mu = \hat{\gamma}_\mu \hat{I}$, Eqs. (I-89),

$$\begin{aligned} \hat{S}L(2, \hat{C}) : \hat{R}_k &= \frac{1}{2} \epsilon_{kij} \hat{\Gamma}_i \star \hat{\Gamma}_j, \quad \hat{S}_k = \frac{1}{2} \hat{\Gamma}_k \star \hat{\Gamma}_4, \\ \hat{A}(3.1) : \hat{P}_\mu, \\ k &= 1, 2, 3, 4, 5, 6, \quad \mu = 1, 2, 3, 4. \end{aligned} \tag{57}$$

The verification by the above isogenerators of isocommutation rules (33)-(35) is an instructive exercise for the interested reader. The proof that the Dirac-Santilli isoequations (I-88) transform isocovariantly under $\hat{P}^0(3.1)$ is equally instructive.

2.5.12. Galilean isosymmetries.

As it is well known, the Galileo symmetry $G(3.1)$ characterizes the non-relativistic motion of point particles in vacuum, with consequential absence of resistive or non-potential forces (see the vertical line of Figure 8).

The isotopies of the Galileo symmetry are intended to characterize the non-relativistic motion of extended particles within physical media, by therefore experiencing resistive non-potential forces (see the wiggly line of Figure 8).

The resulting infinite family of isosymmetries $\hat{G}(3.1)$ are here called *Galilean isosymmetries* to stress the preservation of the basic axioms of the Galileo symmetry and the mere construction of the broadest possible realizations permitted by isomathematics.

The Lie-isotopic lifting of the Galileo symmetry were introduced by Santilli in the 1978 paper [12] as a particular case of the covering *Lie-Admissible symmetries*, also called *genosymmetries*, which are intended to characterize the *time rate of variation of physical quantities*.

The first direct study of Galilean isosymmetries was done in Section 5.3, pages 225 on, of the 1981 monograph [26] formulated over conventional fields. These isotopies were then systematically studies and upgraded in the two 1991 volumes [27] [28]. The formulation of Galilean isosymmetries with the full use of isomathematics was done in the 1995 monographs [29] [30] with a final study presented in Ref. [32].

The above studies attracted the attention of Abdus Salam, founder and president of the *International Center for Theoretical Physics (ICTP)*, Trieste, Italy, who invited Santilli in 1991 to deliver at his Center a series of lectures

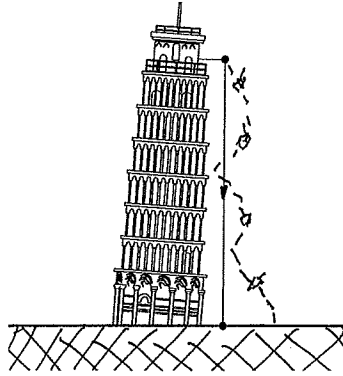


Figure 8: This figure presents a conceptual rendering of the free fall of point-masses in vacuum studied by Galileo (represented with a straight line), and the free fall of extended masses experiencing resistive forces from our atmosphere studied by Santilli (represented with a wiggling line) [26] [30] [37]. It is symptomatic to note that the achievement of the symmetry for extended masses required the construction of a covering of the mathematics used for the point masses with particular reference to the generalization of Newton-Leibnitz differential calculus, from its historical formulation for isolated point, to a covering formulation for volumes [21].

in the isotopies of the Galileo symmetry and relativity, said invitation being apparently the last by Salam prior to his death.

During his visit at the ICTP, Santilli wrote papers [65] through [71]. The notes from Santilli's lectures were collected by A. K. Aringazin, A. Jannussis, F. Lopez, M. Nishioka and B. Vel-janosky and published in volume [37] of 1992.

This work is primarily intended for relativistic isosymmetries. Additionally, all primary applications require relativistic treatments. Therefore, we regret to be unable to review Galilean isosymmetries to prevent a prohibitive length.

Nevertheless, the reader should be aware that an introductory knowledge of the Galilean isosymmetries is suggested, e.g., from the reading of the ICTP papers [65] to [71].

3. APPARENT PROOFS OF THE EPR ARGUMENT

3.1. Foreword.

As it is well known, the conventional Pauli matrices σ_k , $k = 1, 2, 3$, are the fundamental (also called adjoint), irreducible unitary representation of the $SU(2)$ -spin symmetry and play a crucial role for the objections against the

EPR argument [2] - [6] .

In this section, we review the isotopic "completion" of Pauli's matrices into isomatrices

$$\hat{\Sigma}_k = \hat{\sigma}_k \hat{I} \quad (58)$$

which constitute the isofundamental, isoirreducible, isounitary isorepresentation of the Lie-Santilli $\hat{SU}(2)$ isosymmetry and play a crucial role in the apparent proof of the EPR argument for extended particles within physical media studied later on in this section.

By recalling that the $SU(2)$ symmetry characterizes the spin of point-particles in vacuum, the "completed" $\hat{SU}(2)$ isosymmetry is intended to characterize the spin of extended particles within hyperdense media called *hadronic spin*, such as the spin of an electron in the core of a star.

The isotopic "completion" of Pauli's matrices was introduced by Santilli in 1993 while visiting the *Joint Institute for Nuclear Research*, Dubna, Russia [58]. Said "completion" was presented systematically in Refs. [29] [30], used for the apparent proofs of the EPR argument [10] [11], and they are nowadays known as the *Pauli-Santilli isomatrices* [45].

In particular, the preceding studies have shown that, unlike the case for the $SU(2)$ symmetry, the isotopic $\hat{SU}(2)$ isosymmetry admits an explicit and concrete realization of hidden variables λ [3] [4] via realizations of the isotopic element of type $\hat{T} = \text{Diag.}(1, \lambda, \lambda)$ Eq. (3).

In this section, we shall review the construction of $\hat{SU}(2)$ isosymmetry and of Pauli-Santilli isomatrices of regular and irregular type with hidden variables. We shall then use the methods acquired in this and in the preceding paper [9], for the proof that *interior dynamical systems represented via isomathematics and isomechanics appear to admit identical classical counterparts* [10] (Section 3.7), and to *progressively approach the classical EPR determinism [1] in the structure of hadrons, nuclei and stars, while achieving the EPR determinism in the interior of gravitational collapse* [11] (Section 3.8).

A first understanding of this section requires a knowledge of the Lie-Santilli isothory (Section I-2.7) [26] [30] [38] [46] [47] [49]. A technical understanding of this section requires a technical knowledge of hadronic mechanics [29]- [31].

3.2. Pauli matrices.

As it is well known (see, e.g., Ref. [72]), the carrier space of the two-dimensional group of special unitary transformations $SU(2)$ is the two-dimensional complex Euclidean space $E(z, \delta, I)$ with coordinates $z = (z_1, z_2)$, metric $\delta = \text{Diag.}(1, 1)$ and unit $I = \text{Diag.}(1, 1)$.

The two-dimensional, fundamental (also called adjoint), irreducible,

unitary representation of the special unitary Lie algebra $su(2)$ of the $SU(2)$ -spin symmetry is given by the celebrated *Pauli matrices*

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad (59)$$

with commutations rules

$$[\sigma_i, \sigma_j] = \sigma_i \sigma_j - \sigma_j \sigma_i = i2\epsilon_{ijk}\sigma_k, \quad (60)$$

and eigenvalues on a Hilbert space $calH$ over the field of complex numbers $\mathcal{C}$ with basis $|b\rangle$

$$\begin{aligned} \sigma^2 |b\rangle &= (\sigma_1 \sigma_1 + \sigma_2 \sigma_2 + \sigma_3 \sigma_3) |b\rangle = 3 |b\rangle, \\ \sigma_3 |b\rangle &= \pm 1 |b\rangle. \end{aligned} \quad (61)$$

Among the various properties of Pauli's matrices, we should recall their uniqueness in the sense that their expression is invariant under the class of equivalence admitted by quantum mechanics, that under unitary transformation.

We should also recall that Pauli's matrices are also fundamental for the structure of Dirac's equation, Eq. (I-9) since they appear in the very definition of Dirac's gamma matrices, Eqs. (I-89).

3.3. Regular Pauli-Santilli isomatrices.

By following Ref. [58], the carrier isospace of the two-dimensional Lie-Santilli isogroup of isospecial isounitary isotransformations $\hat{SU}(2)$ is the isocomplex iso-Euclidean isospace $\hat{E}(\hat{z}, \hat{\Delta}, \hat{I})$ with isocoordinates

$$\hat{z} = z\hat{I} = (z_1, z_2) = (z_1, z_2)\hat{I}; \quad (62)$$

isounit and isotopic element

$$\hat{I} = \begin{pmatrix} n_1^2 & 0 \\ 0 & n_2^2 \end{pmatrix} = 1/\hat{T} > 0, \quad (63)$$

$$\hat{T} = \begin{pmatrix} n_1^{-2} & 0 \\ 0 & n_2^{-2} \end{pmatrix} \quad (64)$$

isometric

$$\hat{\Delta} = \hat{\delta}\hat{I} = (\hat{T}_i^k \delta_{ki})\hat{I} = \begin{pmatrix} n_1^{-2} & 0 \\ 0 & n_2^{-2} \end{pmatrix} \hat{I}; \quad (65)$$

positive-definite characteristic quantities n_k with unrestricted functional dependence on the variables for interior dynamical problems

$$n_k = n_k(z, \bar{z}, E, \mu, \alpha, \tau, \psi, \partial\psi, \dots) > 0, \quad k = 1, 2; \quad (66)$$

and basic isoinvariant

$$\begin{aligned} \hat{z}_i \star \hat{\Delta}_{ij} \star \hat{z}_j &= (z_i \hat{\delta}_{ij} z_j) \hat{I} = \\ &= \left(\frac{z_1 \bar{z}_1}{n_1^2} + \frac{z_2 \bar{z}_2}{n_2^2} \right) \hat{I}. \end{aligned} \quad (67)$$

By also following Refs. [29] [58], the isogroup of regular, isospecial, isounitary, isotransformations $\hat{S}\hat{U}(2)$ leaving invariant isoline element (67), is characterized by the isotransforms

$$\hat{z}' = \hat{U}(\hat{\theta}) \star z = \hat{U}(\hat{\theta}) \hat{T} \hat{z}, \quad (68)$$

verifying the following conditions [30]:

1. Isounitariness

$$\hat{U}(\hat{\theta}) \star \hat{U}^\dagger(\hat{\theta}) = \hat{U}^\dagger(\hat{\theta}) \star \hat{U}(\hat{\theta}) = \hat{I}; \quad (69)$$

2. Isogroup isoaxioms

$$\begin{aligned} \hat{U}(\hat{\theta}_1) \star \hat{U}(\hat{\theta}_2) &= \hat{U}(\hat{\theta}_1 + \hat{\theta}_2), \\ \hat{U}(\hat{\theta}) \star \hat{U}(-\hat{\theta}) &= \hat{U}(0) = \hat{I}, \quad k = 1, 2, 3; \end{aligned} \quad (70)$$

and

3. Isospecial isounitariness

$$IsoDet \hat{U}(\hat{\theta}) = \hat{I}, \quad Det(U\hat{T}) = 1. \quad (71)$$

The latter condition essentially restricts the isogroup $\hat{S}\hat{U}(2)$ to its iso-connected component $\hat{S}\hat{U}^0(2)$, which is hereon tacitly assumed.

The above conditions imply the local isomorphism

$$\hat{S}\hat{U}(2) \approx SU(2), \quad (72)$$

and the following explicit realization in terms of isoexponential (I-22)

$$\begin{aligned} \hat{U}(\hat{\theta}) &= \Pi_k U_k(\theta_k) \hat{I} = \Pi_k \hat{e}^{i \star J_k \star \hat{\theta}_k} = \Pi_k (e^{i J_k \hat{T} \theta_k}) \hat{I}, \\ U_k(\theta_k) &= e^{i J_k \hat{T} \theta_k}, \quad k = 1, 2, 3, \end{aligned} \quad (73)$$

where $\hat{J}_k$ represents the isogenerators of the Lie-Santilli isoalgebra $\hat{su}(2)$ verifying the conditions

$$IsoTr \hat{J}_k = 0, \quad Tr(\hat{J}_k \hat{T}) = 0, \quad (74)$$

and the isocommutation rules

$$\begin{aligned} [\hat{J}_i, \hat{J}_j] &= \hat{J}_i \star \hat{J}_j - \hat{J}_j \star \hat{J}_i = \\ &= \hat{J}_i \hat{T} \hat{J}_j - \hat{J}_j \hat{T} \hat{J}_i = \epsilon_{ijk} \hat{J}_k. \end{aligned} \quad (75)$$

Note that, in accordance with Theorem I-2.7.2, the isorepresentations here considered are called *regular* because they can be constructed via non-unitary transformations of the conventional $su(2)$ algebra, resulting in the preservation of the conventional structure constants ϵ_{ijk} .

However, as we shall see in the next section, the isotopies of the $su(2)$ algebra imply realizations called *irregular* that cannot be constructed via non-unitary representations of $su(2)$ [58], in which case the structure constants ϵ_{ijk} are replaced by *structure functions* with an arbitrary (non-singular) functional dependence on local variables,

$$\hat{C}_{ijk} = C_{ijk}(z, \bar{z}, E, \nu, \alpha, \tau, \psi, \partial\psi, \dots) \hat{I}. \quad (76)$$

As one can verify, $\hat{su}(2)$ admits the following iso-Casimir isoinvariant

$$\begin{aligned} \hat{J}^2 &= \Sigma_k \hat{J}_k \star \hat{J}_k = \\ &= \hat{J}_1 \hat{T} \hat{J}_1 + \hat{J}_2 \hat{T} \hat{J}_2 + \hat{J}_3 \hat{T} \hat{J}_3. \end{aligned} \quad (77)$$

The maximal set of isocommuting isoooperators is then given by $\hat{J}_3$ and $\hat{J}^2$.

By again following Ref. [58], in order to compute the explicit form of the isorepresentations of $\hat{su}(2)$, we introduce the Hilbert-Myung-Santilli isospace $\hat{\mathcal{H}}$ [19] over the isofield of isocomplex isonumbers $\hat{\mathcal{C}}$ [20] with d -dimensional isobasis $|\hat{b}_k^d\rangle$ verifying isonormalization (I-75),

$$\begin{aligned} \langle \hat{b}_k^d | \star | \hat{b}_k^d \rangle &= \langle \hat{b}_k^d | \hat{T} | \hat{b}_k^d \rangle = \hat{I}, \\ d &= 1, 2, 3, \dots, N, \quad k = 1, 2, 3. \end{aligned} \quad (78)$$

From the local isomorphism $\hat{su}(2) \approx su(2)$ we know that the isoeigenvalue equations have the structure

$$\begin{aligned} \hat{J}_k \star | \hat{b}_k^d \rangle &= b_k^d | \hat{b}_k^d \rangle, \\ \hat{J}^2 \star | \hat{b}_k^d \rangle &= \Sigma_k b_k^d (b_k^d + W) | \hat{b}_k^d \rangle, \end{aligned} \quad (79)$$

$$W = Det \hat{T} = 1/n_1^2 n_2^2,$$

where $W = 1$ for regular isorepresentation, otherwise W is an arbitrary function of local quantities to be identified via subsidiary constraints from the medium in which extended particles are immersed.

The explicit form of the isorepresentations of $\hat{su}(2)$ is then given by the simple isotopy of the conventional case [72]

$$\begin{aligned}\hat{J}_\pm &= \hat{J}_1 \pm \hat{J}_2, \\ (\hat{J}_1)_{ij} &= i\frac{1}{2} \langle \hat{b}_k^d | \star (\hat{J}_- - \hat{J}_+) \star | \hat{b}_k^d \rangle, \\ (\hat{J}_2)_{ij} &= i\frac{1}{2} \langle \hat{b}_k^d | \star (\hat{J}_- + \hat{J}_+) \star | \hat{b}_k^d \rangle, \\ (J_3)_{ij} &= \langle \hat{b}_k^d | \star \hat{J}_3 \star | \hat{b}_k^d \rangle.\end{aligned}\tag{80}$$

By continuing to follow Ref. [58], we now restrict our attention to the two-dimensional isofundamental (isoadjoint) isorepresentation of $\hat{su}(2)$ occurring for $d = 2$, in which case we assume

$$\hat{J}_k = \frac{1}{2} \hat{\sigma}_k, \quad k = 1, 2, 3,\tag{81}$$

and select the basic isounitary isotransform according to Sections I-2.8 and I-2.9

$$UU^\dagger = f(W) > 0, \quad W = \text{Det.} \hat{I} = n_1^2 n_2^2,\tag{82}$$

where $f(W)$ is a smooth function such that $f(1) = 1$.

By using the above procedure, we have the following *regular Pauli-Santilli isomatrices* first introduced by Santilli in Ref. [58], Eqs. (3.2) (where the isometric elements are denoted $g_{kk} = n_k^{-2}$, $k = 1, 2$,

$$\begin{aligned}\hat{\Sigma}_k &= \hat{\sigma}_k \hat{I}, \\ \hat{\sigma}_1 &= (n_1 n_2) \begin{pmatrix} 0 & n_1^{-2} \\ n_2^{-2} & 0 \end{pmatrix}, \quad \hat{\sigma}_2 = (n_1 n_2) \begin{pmatrix} 0 & -i n_1^{-2} \\ i n_2^{-2} & 0 \end{pmatrix}, \\ \hat{\sigma}_3 &= (n_1 n_2) \begin{pmatrix} n_2^{-2} & 0 \\ 0 & -n_1^{-2} \end{pmatrix}.\end{aligned}\tag{83}$$

with isocommutation rules

$$[\hat{\sigma}_i, \hat{\sigma}_j] = i 2 \epsilon_{ijk} \hat{\sigma}_k,\tag{84}$$

in which the 'regular' character of the isomatrices is established by the presence of the conventional (constant) structure constants.

We then have the isoeigenvalues isoequations

$$\begin{aligned}\hat{\sigma}_3 \star |\hat{b}_m^2\rangle &= \hat{\sigma}_3 \hat{T} |\hat{b}_m^2\rangle = \pm \frac{1}{n_1 n_2} |\hat{b}_m^2\rangle \\ \hat{\sigma}^2 &= (\sigma_1 \hat{T} \hat{\sigma}_1 + \sigma_2 \hat{T} \hat{\sigma}_2^\dagger \sigma_3 \hat{T} \hat{\sigma}_3) \hat{T} |\hat{b}_m^2\rangle = \\ &= 3 \frac{1}{n_1^2 n_2^2} |\hat{b}_m^2\rangle,\end{aligned}\tag{85}$$

showing that the regular Pauli-Santilli isomatrices preserve the conventional structure constants ϵ_{ijk} of Pauli matrices, but exhibit structure (84) with generalized isoeigenvalues containing two characteristic quantities n_1^2, n_2^2 .

It is evident that, under isounimodularity condition (71),

$$\text{Det} \hat{T} = 1, \quad n_1 = 1/n_2,\tag{86}$$

isomatrices (83) reduce to

$$\begin{aligned}\hat{\sigma}_1 &= \begin{pmatrix} 0 & n_1^{-2} \\ n_2^{-2} & 0 \end{pmatrix}, \quad \hat{\sigma}_2 = \begin{pmatrix} 0 & -in_1^{-2} \\ in_2^{-2} & 0 \end{pmatrix}, \\ \hat{\sigma}_3 &= \begin{pmatrix} n_2^{-2} & 0 \\ 0 & -n_1^{-2} \end{pmatrix},\end{aligned}\tag{87}$$

by verifying conventional commutation rules (84) and conventional eigenvalues

$$\begin{aligned}\hat{\sigma}_3 \star |\hat{b}_m^2\rangle &= \pm |\hat{b}_m^2\rangle \\ \hat{\sigma}^2 \star |\hat{b}_m^2\rangle &= 3 |\hat{b}_m^2\rangle.\end{aligned}\tag{88}$$

In order to search for additional realizations of regular Pauli-Santilli isomatrices, we now assume the following non-unitary transform

$$U = \begin{pmatrix} n_1 & 0 \\ 0 & n_2 \end{pmatrix} = U^\dagger,\tag{89}$$

under which we have the following second realization of regular Pauli-Santilli isomatrices

$$\begin{aligned}\hat{\sigma}_k &= U \sigma_k U^\dagger, \\ \hat{\sigma}_1 &= \begin{pmatrix} 0 & n_1 n_2 \\ n_1 n_2 & 0 \end{pmatrix}, \quad \hat{\sigma}_2 = \begin{pmatrix} 0 & -in_1 n_2 \\ in_1 n_2 & 0 \end{pmatrix}, \\ \hat{\sigma}_3 &= \begin{pmatrix} n_1^2 & 0 \\ 0 & -n_2^2 \end{pmatrix}.\end{aligned}\tag{90}$$

It is an instructive exercise for the interested reader to verify that the above isomatrices verify the isocommutation rules (84) and conventional isoeigenvalue (99), namely, *the second realization of the Pauli-Santilli isomatrices, Eqs. (83), also admit conventional structure constants and eigenvalues despite the degrees of freedom permitted by the two characteristic quantities n_1^2, n_2^2 .*

We now assume the following non-diagonal realization of the non-unitary transform

$$U = \begin{pmatrix} 0 & n_1 \\ n_2 & 0 \end{pmatrix}, U^\dagger = \begin{pmatrix} 0 & n_2 \\ n_1 & 0 \end{pmatrix}, \quad (91)$$

$$UU^\dagger = \hat{I} = 1/\hat{T} > 0,$$

which characterizes the following third realization of the regular Pauli-Santilli isomatrices

$$\begin{aligned} \hat{\sigma}_1 &= \begin{pmatrix} 0 & n_1 n_2 \\ n_1 n_2 & 0 \end{pmatrix}, \quad \hat{\sigma}_2 = \begin{pmatrix} 0 & -in_1 n_2 \\ in_1 n_2 & 0 \end{pmatrix}, \\ \hat{\sigma}_3 &= \begin{pmatrix} -n_1^2 & 0 \\ 0 & n_2^2 \end{pmatrix}. \end{aligned} \quad (92)$$

It is easy to see that the above third realization of the regular Pauli-Santilli isomatrices also verify conventional commutation rules (84) and eigenvalues (88).

Note that, while Pauli's matrices are invariant under unitary transforms, there exist a number of Pauli-Santilli isomatrices each of which is invariant under isounitary isotransforms (Section I-3.9).

3.4. Irregular Pauli-Santilli isomatrices.

3.4.1. Historical notes. One of the most fundamental, yet unresolved processes in nature is the synthesis of the neutron from the hydrogen in the core of stars, which is a pre-requisite for the production of light and, therefore, for the existence of life itself.

In this section, we would like to outline the main historical aspects on the synthesis of the neutron and identify the open problems because truly fundamental for the construction of the new mathematics needed for their solution.

Recall that stars initiate their life as an aggregate of hydrogen and grow via the accretion of hydrogen existing in interstellar spaces.

In 1910, H. Rutherford [73] conjectured that, when the pressure and temperature at the core of the star reaches certain values, the hydrogen

atom is “compressed” into a neutral particle n which is called the *neutron* according to the reaction (see Section I-4)



Rutherford hypothesis was experimentally confirmed by J. Chadwick in 1932 [74], and the neutron became part of scientific history.

Following the experimental verification that the neutron has the same spin $1/2$ of the electron and of the proton, in an attempt at maintaining the conservation of the angular momentum, E. Fermi [75] suggested that the synthesis of the neutron occurs with the emission of a hypothetical, massless and chargeless particle ν with spin $1/2$ which he called the *neutrino* (meaning “little neutron” in Italian), according to the reaction widely accepted by the scientific community for about one century



After joining Harvard University in September 1977 under DOE support, R. M. Santilli [15]-[17] noted that, despite the salvaging of space-time symmetries and related conservation laws, reaction (93) is not compatible with quantum mechanical laws because the rest energy of the neutron E_n is 0.782 MeV bigger than the sum of the rest energies of the proton E_p and of the electron E_e ,

$$\begin{aligned} E_p &= 938.272 \text{ MeV}, \quad E_e = 0.511 \text{ MeV}, \quad E_n = 939.565 \text{ MeV}, \\ E_n - (E_p + E_e) &= 0.782 \text{ MeV} > 0. \end{aligned} \quad (95)$$

Therefore, Santilli presented a number of arguments according to which the synthesis of the neutron is clear evidence of Einstein’s view on the lack of “completion” of quantum mechanics (see Einstein’s name in the title of the 1981 paper [17] released from the Department of Mathematics of Harvard University). Subsequently, Santilli achieved in Ref. [76] (see the independent review [?]] the non-relativistic representation of *all* characteristics of the neutron in its synthesis from the hydrogen representation, with a relativistic representation subsequently achieved in Ref. [62] (see the independent review in ref. [45]).

The technically most difficult problem of the above representation was the identification of the spin-orbit coupling for the electron when totally immersed inside the proton which was first solved non-relativistically in Ref. [76] and relativistically in Ref. [62] (see the review in Ref. [30], Chapter 6 in particular).

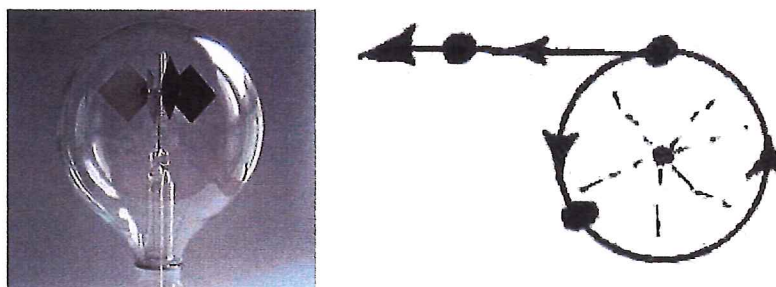


Figure 9: To illustrate some of isosymmetries for interior systems, in the left we show the conversion of linear momentum into angular momentum for the case of photons causing the rotation of a small propeller in a vacuum chamber. In the right, we show the opposite conversion of angular momentum into linear momentum, as it is the case for the sling shot. The left view illustrates that the neutrino hypothesis is not necessary for the synthesis of the neutron when particles are represented as being extended. The right view illustrates that the emission of an antineutrino is not necessary in the neutron decay because the internal angular momentum of the electron can be converted into its external linear momentum without any violation of physical laws.

Recall that quantum mechanics has an excellent consistency for bound states with *negative potentials* causing a *mass defect*. Santilli's first argument is that a representation of experimental data (95) via quantum mechanics is impossible because it would require a *positive potential* capable of producing a *mass excess*, which features imply the loss of physical consistency of Schrödinger equation for *bound states* (and not for free particles with positive kinetic energy) because the indicial equation of Schrödinger equation admits no consistent solutions for positive potential energies, (see Section I-4).

The inability of Dirac's and other quantum mechanical equations for the representation of experimental data (95) then followed.

Various conjectures, aimed at maintaining for the neutron structure the theory so effective for the hydrogen atom, were proved not to be consistent. For instance, the hypothesis that the missing energy of 0.782 MeV is provided by the star via a relative energy between the electron and the proton had to be dismissed because the cross section $e - p$ at 0.782 MeV is essentially null, thus preventing any fusion between the electron and the proton.

Similarly, the hypothesis that the missing energy is provided by the

antineutrino $\bar{\nu}$ via reactions of the type

$$e^- + \bar{\nu} + p^+ \rightarrow n, \quad (96)$$

had to be equally abandoned because the cross section between neutrinos or antineutrinos and individual particles is identically null.

As it is well known, the neutrino hypothesis is necessary for the quantum mechanical treatment of synthesis (94), namely, for the point-like characterization of the proton, the electron and the neutron. Ref. [17] indicated that the neutrino hypothesis cannot any longer be consistently applied for the neutron synthesis whenever particles are represented as being extended.

Independently from that, there exist no known conventional possibility of identifying the *energy* needed for the creation of the neutrino since synthesis (94) already misses 0.782 MeV for the synthesis of the neutron.

Another argument of Ref. [17] is that the conservation of the angular momentum is necessary for the synthesis of bound states with a Keplerian center under the validity of conventional space-time symmetries, such as for the synthesis of the hydrogen atom from an electron and a proton.

However, said conservation is no longer necessary for bound states at short distances without a Keplerian center, since in that case we have the validity of space-time isosymmetries for which the angular momentum can be transformed into linear momentum and vice-versa without any violation of physical laws (see Section 2.2 and Figure 9).

But *the neutron has no Keplerian center*, with the consequential lack of applicability of the Lorentz-Poincaré symmetry, and the ensuing lack of necessary conservation of the angular momentum in favor of alternative hypotheses.

In view of the above (and other) insufficiencies of the neutrino hypothesis, Santilli suggested in Ref. [78] the introduction in the *l.h.s.* (rather than the *r.h.s.*) of the synthesis of the mass-less, charge-less and spin-less particle called the *etherino* and denoted with the symbol a (from the Latin *aether*)

$$p^+ + a + e^- \rightarrow n, \quad (97)$$

whose scope is to represent the delivery of the missing 0.782 MeV to the neutron.

An intriguing aspect is that *the etherino hypothesis can be shown to be compatible with the experimental data of the so-called "neutrino experiments," of course, under the condition of abandoning point-like abstractions of hadrons and representing them as they are in the physical reality, i.e., extended, deformable and hyperdense.*

By recalling that there is no known consistent way of accounting for the missing 0.782 MeV as originating from the star, Ref. [?] submitted the hypothesis that the missing energy may originate from the ether as a universal medium of extremely high energy density for the characterization and propagation of particles and electromagnetic waves.

However, it should be stressed that *the etherino is not intended to be a particle, but to be an "impulse" representing the mechanism of supplying the missing 0.782 MeV for the neutron, the origin of the missing energy from the ether being only one among other possibilities.*

In view of the above insufficiencies of quantum mechanics for the synthesis of the neutron, Santilli initiated in Refs. [15] - [17] the search for a "completion" of quantum mechanics into hadronic mechanics with particular reference to the "completion" of Lie's theory at large, and the $SU(2)$ -spin symmetry in particular, for the characterization of the spin of the electron when "compressed" inside the hyperdense proton.

It should also be recalled that, during the same period, Santilli conducted a post Ph. D. Seminar Course at the Lyman Laboratory of Physics of Harvard University with a technical treatment of the insufficiency of quantum mechanics for the neutron synthesis via the *conditions of variational self-adjointness* for the existence of a Lagrangian or a Hamiltonian. This Seminar Course was eventually published by Springer-Verlag in monographs [25] [26] whose primary aim is the first known presentation of the axiom-preserving Lie-Santilli isothory and the axiom-inducing Lie-Santilli genotheory.

In fact, possible representations of experimental data (95) for the neutron synthesis violate the conditions of variational self-adjointness, thus mandating the search for a covering theory.

The subsequent 1995 papers [55] [56] [58] achieved the regular isotopies $\hat{S}U(2)$ of the spin symmetry (reviewed in the preceding section).

However, *regular isotopies of the $SU(2)$ spin symmetry are insufficient for the neutron synthesis because it requires alterations (called mutations) of conventional eigenvalues that can be solely represented via irregular isorepresentations.*

The irregular isorepresentations of the $SU(2)$ -spin symmetry were identified, apparently for the first time, by Santilli in the 1990 paper [78] and used to achieve the non-relativistic representation of *all* characteristics of the neutron in its synthesis from the hydrogen.

In the 1995 paper [62], Santilli presented a relativistic study of $\hat{S}U(2)$ as an isosubalgebra of the irregular isospinorial covering of the Lorentz-Poincaré symmetry (Section 2.5.11) and used the results to achieve a relativistic representation of the neutron synthesis.

The indicated irregular representations of the $SU(2)$ -spin symmetry were then instrumental for the apparent confirmations of the EPR argument in Refs.[10] [11] reviewed in this section.

3.4.2. Non-relativistic formulation. The first irregular isotopies of Pauli's matrices, today known as *irregular Pauli-Santilli isomatrices* [45], have been introduced in Eqs. (2.32) of Ref. [78] via the use of the isorepresentations of $\hat{SU}(2)$ worked out in the preceding papers [55] [56], and are given by

$$\begin{aligned}\hat{\sigma}_1 &= \begin{pmatrix} 0 & -n_1 \\ n_2 & 0 \end{pmatrix}, \quad \hat{\sigma}_2 = \begin{pmatrix} 0 & -in_1 \\ in_2 & 0 \end{pmatrix}, \\ \hat{\sigma}_3 &= \frac{1}{n_1 n_2} \begin{pmatrix} n_1^2 & 0 \\ 0 & -n_2^2 \end{pmatrix},\end{aligned}\tag{98}$$

with irregular isocommutation rules

$$[\hat{\sigma}_i, \hat{\sigma}_j] = 2i \frac{1}{n_1 n_2} \epsilon_{ijk} \hat{\sigma}_k, \quad i, j, k, = 1, 2, 3\tag{99}$$

and isoeigenvalues

$$\begin{aligned}\hat{\sigma}_3 \star |\hat{u}\rangle &= \pm \frac{1}{n_1 n_2} |\hat{u}\rangle, \\ \hat{\sigma}^2 \star |\hat{u}\rangle &= \frac{1}{n_1 n_2} \left(\frac{1}{n_1 n_2} + 2 \right) |\hat{u}\rangle.\end{aligned}\tag{100}$$

It is easy to see that, when the hyperdense medium surrounding the immersed particle is homogeneous and isotropic, the characteristics quantities can be normalized to the values $n_1 = n_2 = n_3 = 1$, in which case isoeigenvalues (100) are conventional. We therefore have the following

LEMMA 3.1: *Irregular isorepresentations of the Lie-Santilli isosymmetry $\hat{su}(2)$ represent the inhomogeneity and anisotropy of media in which extended particles are immersed.*

Among a number of additional irregular Pauli-Santilli isomatrices with isotopic element $\hat{T}$ in Eq. (64) we quote Eqs. (3.2) of Ref. [10]

$$\begin{aligned}\hat{\sigma}_1 &= n_1 n_2 \begin{pmatrix} 0 & n_1^{-2} \\ n_2^{-2} & 0 \end{pmatrix}, \quad \hat{\sigma}_2 = n_1 n_2 \begin{pmatrix} 0 & -in_1^{-2} \\ in_2^{-2} & 0 \end{pmatrix}, \\ \hat{\sigma}_3 &= n_1 n_2 \begin{pmatrix} n_2^{-2} & 0 \\ 0 & -n_1^{-2} \end{pmatrix},\end{aligned}\tag{101}$$

with irregular isocommutation rules

$$[\hat{\sigma}_i, \hat{\sigma}_j] = 2i \frac{1}{n_1 n_2} \hat{\sigma}_k, \quad (102)$$

and isoeigenvalues

$$\begin{aligned} \hat{\sigma}_3 \star |\hat{u}\rangle &= \pm \frac{1}{n_1 n_2} |\hat{u}\rangle, \\ \hat{\sigma}^{\hat{2}} \star |\hat{u}\rangle &= 3 \frac{1}{n_1^2 n_2^2} |\hat{u}\rangle, \end{aligned} \quad (103)$$

The above isorepresentation appears to be significant when the medium causes a proportional alteration/mutation of both the third component as well as the total value of the spin of a particle having the value 1/2 in vacuum.

Another example of irregular Pauli-Santilli isomatrices is given by Eqs. (3.7) of Ref. [10]

$$\begin{aligned} \hat{\sigma}_1 &= \begin{pmatrix} 0 & n_2 \\ n_1 & 0 \end{pmatrix}, \quad \hat{\sigma}_2 = \begin{pmatrix} 0 & -in_2 \\ in_1 & 0 \end{pmatrix}, \\ \hat{\sigma}_3 &= \begin{pmatrix} n_1^2 & \\ 0 & -n_2^2 \end{pmatrix}, \end{aligned} \quad (104)$$

with irregular isocommutation rules

$$\begin{aligned} [\hat{\sigma}_1, \hat{\sigma}_2] &= 2i \frac{1}{n_1^2 n_2^2} \hat{\sigma}_3, \quad [\hat{\sigma}_2, \hat{\sigma}_3] = 2i \hat{\sigma}_1, \\ [\hat{\sigma}_3, \hat{\sigma}_1] &= 2i \hat{\sigma}_2, \end{aligned} \quad (105)$$

and mutated isoeigenvalues

$$\begin{aligned} \hat{\sigma}_3 \star |\hat{u}\rangle &= \pm |\hat{u}\rangle, \\ \hat{\sigma}^{\hat{2}} \star |\hat{u}\rangle &= \frac{2}{n_1^2 n_2^2} |\hat{u}\rangle. \end{aligned} \quad (106)$$

The above isorepresentation may be useful when the anisotropy and inhomogeneity of the medium maintain the spin value 1/2 along the third axis, yet they are such to deform the remaining components.

Additional example of irregular Pauli-Santilli isomatrices are available from Refs. [10] and [58], and can be readily constructed by interested readers.

3.4.3. Relativistic formulation. Consider the iso-Minkowskian isospace $\hat{M}(\hat{x}, \hat{\Omega}, \hat{I})$ with isometric

$$\begin{aligned}\hat{\Omega} &= \hat{\eta} \hat{I}, \quad \bar{\eta} = \hat{T} \eta, \\ \hat{T} &= \text{Diag.}(\frac{1}{m_1^2}, \frac{1}{m_2^2}, \frac{1}{m_3^2}, \frac{1}{m_4^2}),\end{aligned}\tag{107}$$

$$m_\mu = m_\mu(r, p, E, \nu, \alpha, \tau, \pi, \psi, \dots) > 0. \quad \mu = 1, 2, 3, 4,$$

where the new characteristic quantities m_μ have been introduced to avoid confusion with the previously used symbols n_μ .

The relativistic formulation of the irregular isorepresentation of $\hat{S}U(2)$ were derived, apparently for the first time, in Eqs. (6.4c)-(6.4d) of Ref. [62], and can be written

$$J_k = \frac{1}{2} \epsilon_{kij} \hat{\gamma}_i \star \gamma_j,\tag{108}$$

where $\hat{\gamma}$ are the regular Dirac-Santilli isomatrices (I-89), i.e.,

$$\begin{aligned}\hat{\gamma}_k &= \frac{1}{m_k} \begin{pmatrix} 0 & \hat{\sigma}_k \\ -\hat{\sigma}_k & 0 \end{pmatrix}, \\ \hat{\gamma}_4 &= \frac{i}{m_4} \begin{pmatrix} I_{2 \times 2} & 0 \\ 0 & -I_{2 \times 2} \end{pmatrix},\end{aligned}\tag{109}$$

and $\hat{\sigma}_k$ are the *regular Pauli-Santilli isomatrices*.

The irregular character of isorepresentation (108) is established by the presence of structure functions in the isocommutation rules, Eqs. (6.4c) of Ref. [62],

$$[J_i, J_j] = \epsilon_{ijk} \frac{1}{m_k^2} J_k,\tag{110}$$

and in the irregular isoeigenvalues

$$\begin{aligned}J_3 \star |\hat{\psi} > &= \pm \frac{1}{2} \frac{1}{m_1 m_2} |\hat{\psi} >, \\ J^2 \star |\hat{\psi} > &= \frac{1}{4} \left(\frac{1}{m_1 m_2} + \frac{1}{m_2 m_3} + \frac{1}{m_3 m_1} \right) |\hat{\psi} >,\end{aligned}\tag{111}$$

that, as shown in Ref. [62], permit a relativistic representation of the spin of the neutron in its synthesis from the hydrogen.

Again one should note that, when the medium is homogeneous and isotropic, isoeigenvalues (101) are conventional.

Note that the assumption of mutated spin for an extended particle within a hyperdense medium implies the *inapplicability (rather than the violation) of the Fermi-Dirac statistics, Pauli's exclusion principle and other quantum mechanical laws* with the understanding that said mutations are internal, thus solely testable under *external* strong interactions, as indicated beginning with the title of Harvard's 1978 paper [15].

3.5. Isotopies of hadronic spin and angular momentum.

3.5.1. Historical notes. An electron orbiting in vacuum around the proton in the hydrogen atom experiences no resistive forces, thus verifying known symmetries and conservation laws.

When the same electron has been "compressed" inside the proton according to Rutherford [73], Santilli [78] argued that the sole possible angular momentum is that permitted by *constraints* exercised on the electron by the internal medium.

Since the electron is about 2,000 times lighter than the proton, *the most stable configuration is that for which the electron is "constrained" to orbit with a value of the angular momentum equal to the proton spin*, since any different configuration would imply big resistive forces (Figure 9).

Needless to say, fractional angular momenta are anathema for the quantum mechanical description of point-particles in vacuum.

However, the angular momentum of extended particles immersed within hyperdense hadronic media can acquire values other than integers, depending on the local physical conditions of the medium surrounding the particle, such as pressure, density, anisotropy, inhomogeneity, etc.

The first known quantitative study of *constrained angular momenta* of extended particles within hyperdense hadronic media was done at the non-relativistic level by Santilli in Ref. [78] of 1990 following the preceding isotopies of the rotational symmetry, Refs. [55] [56]. The study was then extended to the relativistic level in Ref. [62] of 1990.

These studies are crucial for quantitative representations of the synthesis of hadrons providing apparent verifications of the EPR argument, and can be summarized as follows.

3.5.2. Non-relativistic representation. Recall the central assumption of isosymmetries according to which conventional generators are preserved (because representing conventional total conservation laws), and only their product is lifted into the isotopic form (1) (to represent the extended character of the particles and their non-Hamiltonian interactions).

Hence, the definition of the *isoangular isomomentum*, also called *hadronic angular momentum*, on an iso-Euclidean isospace is the same as that of

quantum mechanics

$$L_k = \epsilon_{ijk} \hat{r}_i \star \hat{p}_j = \epsilon_{ijk} r_i p_j, \quad (112)$$

although it is defined on a Hilbert-Myung-Santilli isospace $\hat{\mathcal{H}}$ with isostates $|\hat{\psi}\rangle$ on an isocomplex isofield $\hat{\mathcal{C}}$, with *isolinear isomomentum* Eqs. (I-79), and isocommutation rules are then given by Eqs. (I-81).

It is then easy to verify the following isocommutation rules for the hadronic angular isomomentum, Eqs. (2.22b) [78]

$$[L_i, \hat{L}_j] = i\hat{I} \epsilon_{ijk} L_k, \quad (113)$$

where, as one can see, the characteristics of the medium, represented by the isounit $\hat{I}$, enter directly in the isocommutation rules.

The use of the *isospherical isoharmonic isofunctions* (see page 240 of Ref. [30] for details)

$$\begin{aligned} \hat{Y}_{\ell m}(\hat{\theta}, \hat{\phi}) &= UY(\theta, \phi)U^\dagger = \hat{T}^{-1}Y_{\ell m}(\theta, \phi), \\ UU^\dagger &= \hat{I} = 1/\hat{T} \neq I, \end{aligned} \quad (114)$$

where $Y_{\ell m}(\theta, \phi)$ are the conventional spherical harmonic functions, yields the following isoeigenvalues, Eqs. (2.25), Ref. [78],

$$\begin{aligned} L_3 \star \hat{Y}_{\ell m}(\hat{\theta}, \hat{\phi}) &= \hat{I} m \hat{Y}_{\ell m}(\hat{\theta}, \hat{\phi}), \\ L^2 \star \hat{Y}_{\ell m}(\hat{\theta}, \hat{\phi}) &= \hat{I} \ell(\ell + 1) \hat{Y}_{\ell m}(\hat{\theta}, \hat{\phi}), \\ m &= \ell, \ell - 1, \dots, -\ell, \quad m = 1, 2, 3, \dots \end{aligned} \quad (115)$$

where one can see again the mutation of the eigenvalues caused by the surrounding medium.

Applications to particle physics then require specific realizations of the isounit $\hat{I}$, such as the simple assumption of expressions (4) used in Ref. [78]

$$\rho = |\hat{I}| \approx |e^\gamma|, \quad (116)$$

where ρ is a function of all possible or otherwise needed local variables of the medium.

3.5.3. Isotopies of non-relativistic spin-orbit coupling. As one can see, isoeigenvalues (115) do not allow a representation of the constrained hadronic angular momentum of the electron when compressed inside the proton (Figure 9).

In view of this insufficiency, Santilli conducted in Ref. [76] (see Also Ref. [30], Chapter 6) a study of the eigenvalues of the combined spin and angular momentum of the electron in the indicated interior conditions.

We consider then the *total hadronic momentum*

$$J_{tot} = L_\ell \hat{\otimes} J_s, \quad (117)$$

with corresponding basis $|\hat{Y} \hat{\otimes} \hat{u} \rangle$ and isoexpectation values, Eqs. (2.34), Ref. [78],

$$\begin{aligned} J_{3,tot} |\hat{Y} \hat{\otimes} \hat{u} \rangle &= (\rho_m(\ell) \pm \frac{m(s)}{n_1 n_2}) |\hat{Y} \hat{\otimes} \hat{u} \rangle \\ J_{tot}^2 \star |\hat{Y} \hat{\otimes} \hat{u} \rangle &= (\rho \pm \frac{s}{n_1 n_2})(\rho \pm \frac{s}{n_1 n_2} + 1) |\hat{Y} \hat{\otimes} \hat{u} \rangle \\ \ell &= 0, 1, 2, 3, \dots \quad s = 0, \frac{1}{2}, 1, \frac{3}{2}, \dots, \end{aligned} \quad (118)$$

$$m(\ell) = \ell, \ell - 1, \dots, -\ell, \quad m(s) = s, s - 1, \dots, -s.$$

Following a laborious journey initiated in 1977, isoeigenvalues (118) finally permitted Santilli to achieve the desired solution for $\ell = 1$ and $s = \frac{1}{2}$, Eq. (2.36), Ref. [78],

$$\rho = \frac{1}{2} \frac{1}{n_1 n_2}, \quad (119)$$

for which the *total hadronic angular momentum of the electron in the synthesis of the neutron is identically null*, $J_{tot} = 0$, and the *spin of the neutron coincides with that of the proton*.

More detailed studies pertaining to electric and magnetic dipoles excluded the alternative $J = 1$ of eigenvalues (118), as well as total hadronic angular momenta of the electron other than zero.

The preceding studies permitted a quantitative non-relativistic representation of the spin of the neutron in its synthesis from the hydrogen atom. A representation of the remaining characteristics of the neutron (mass, radius, charge, dipole moments, etc.) is reviewed in Section 4.5.

3.5.4. Isotopies of relativistic spin-orbit couplings. The hadronic spin $\hat{S} = S\hat{I}$ is a realization of the $\hat{S}U(2)$ isosubalgebra of $\hat{\mathcal{P}}(3.1)$ with generators (57), while the hadronic angular momentum $\hat{L} = L\hat{I}$ is a realization of the isorotational $\hat{S}O(3)$ isosubalgebra. Their relativistic formulation on iso-Minkowskian isospace (107) has been first derived in Eqs. (6.4a) (6.4b), Ref. [62] and are given by

$$S_k = 2\epsilon_{kij} \hat{\gamma}_i \star \hat{\gamma}_j, \quad (120)$$

$$L_k = \epsilon_{kij} r_i \star p_j,$$

where $\hat{\gamma}_k$ are the Dirac-Santilli isomatrices.

We then have the irregularisocommutation rules

$$[S_i, \hat{S}_j] = \epsilon_{kij} m_k^2 \hat{S}_k, \quad (121)$$

$$[L_i, \hat{L}_j] = \epsilon_{ijk} m_k^2 L_k,$$

and isoeigenvalues, Eqs, (6.4d) Ref. [62]

$$\begin{aligned} \hat{S}_3 \star |\hat{\psi}\rangle &= \pm \frac{1}{m_1 m_2} |\hat{\psi}\rangle, \\ \hat{S}^2 \star |\hat{\psi}\rangle &= (m_1^{-2} m_2^{-2} + m_2^{-2} m_3^{-2} + m_3^{-2} m_1^{-2}) |\hat{\psi}\rangle, \\ \hat{L}_3 \star |\hat{\psi}\rangle &= \pm m_1 m_2 |\hat{\psi}\rangle, \\ \hat{L}^2 \star |\hat{\psi}\rangle &= (m_1^2 m_2^2 + m_2^2 m_3^2 + m_3^2 m_1^2) |\hat{\psi}\rangle. \end{aligned} \quad (122)$$

The most salient difference between relativistic isoeigenvalues (122) and their non-relativistic counterparts (155) is that *the former admit fractional hadronic angular momenta while the latter do not.*

In fact, for the following values admitted by a homogeneous and isotropic medium [62]

$$m_1 = m_2 = m_3 = \frac{1}{\sqrt{2}}, \quad (123)$$

isoeigenvalues (122) become

$$\begin{aligned} \hat{S}_3 \star |\hat{\psi}\rangle &= \pm \frac{1}{2} |\hat{\psi}\rangle, \\ \hat{S}^2 \star |\hat{\psi}\rangle &= \frac{3}{4} |\hat{\psi}\rangle, \\ \hat{L}_3 \star |\hat{\psi}\rangle &= \pm \frac{1}{2} |\hat{\psi}\rangle, \\ \hat{L}^2 \star |\hat{\psi}\rangle &= \frac{3}{4} |\hat{\psi}\rangle. \end{aligned} \quad (124)$$

Consequently, *isoeigenvalues (122) permit a quantitative representation of the hadronic angular momentum of the electron as being constrained to be equal to the proton spin [61] [62] (Figure zzzz).*

In this case too, the total hadronic angular momentum of the electron is null because the only stable hadronic spin-orbit coupling is in singlet, and the spin of the electron can be assumed in first good approximation not to be mutated since the electron is about 2,000-times lighter than the proton.

Hadronic spins, hadronic angular momenta and hadronic spin-orbit couplings were studied in detail Chapter 6 of Ref. [30] resulting in Lemma 6.12.1 here reproduced without proof:

LEMMA 3.2: *When immersed within hadrons or nuclei with spin 1/2, an elementary particle having spin 1/2 in vacuum can only have a null total hadronic angular momentum.*

As we shall see in Section 4.6, the above configuration of the synthesis of the neutron from the hydrogen is an apparent verification of the EPR argument.

3.6. Realization of hidden variables.

As recalled in Section 1.1, the conventional quantum mechanical realization of the Lie symmetry $SU(2)$ does not allow a consistent representation of hidden variables λ [3] [4].

It is easy to see that, despite the local isomorphism $\hat{SU}(2) \approx SU(2)$, the Lie-Santilli isosymmetry $\hat{SU}(2)$ does indeed allow explicit and concrete realizations of hidden variables thanks to the degree of freedom permitted by the isotopic element (1) in the structure of the Lie-Santilli isoproduct (2) with realizations of the isotopic element of type (3).

In this section, we review the explicit and concrete realization of *regular hidden variables*, namely, realizations that can be derived via non-unitary transforms of the Lie algebra $su(2)$, and then review *irregular hidden variables*, namely, realizations that do not admit such a simple derivation.

Regular and irregular realizations of hidden variables have been first identified by Santilli in Ref. [58] of 1993, and then used for the proof of the EPR argument [10] reviewed in Section 3.7.

Realizations of regular hidden variables are easily provided by Pauli-Santilli isomatrices (83) with the identifications

$$n_1^2 = \lambda_1, \quad n_2^2 = \lambda_2, \quad (125)$$

yielding the desired realization, Eqs. (3.9), Ref. [58],

$$\begin{aligned} \hat{\sigma}_1 &= (\lambda_1 \lambda_2) \begin{pmatrix} 0 & \lambda_1^{-1} \\ \lambda_2^{-1} & 0 \end{pmatrix}, \quad \hat{\sigma}_2 = (\lambda_1 \lambda_2) \begin{pmatrix} 0 & -i\lambda_1^{-1} \\ i\lambda_2^{-1} & 0 \end{pmatrix}, \\ \hat{\sigma}_3 &= (\lambda_1 \lambda_2) \begin{pmatrix} \lambda_2^{-1} & 0 \\ 0 & -\lambda_1^{-1} \end{pmatrix} \end{aligned} \quad (126)$$

verifying isocommutation rules

$$[\hat{\sigma}_i, \hat{\sigma}_j] = i\epsilon_{ijk} \hat{\sigma}_k, \quad (127)$$

and isoeigenvalue isoequations

$$\begin{aligned}\hat{\sigma}_3 \star |\hat{b}\rangle &= \pm(\lambda_1\lambda_2)|\hat{b}\rangle \\ \hat{\sigma}^{\hat{2}} &= 3(\lambda_1\lambda_2)^2|\hat{b}\rangle.\end{aligned}\tag{128}$$

We consider now the particular case of Eq. (3), i.e.,

$$Det.\hat{T} = 1, \quad n_1^2 = 1/n_2^2 = \lambda,\tag{129}$$

derivable via the basic non-unitary transformation

$$\hat{T} = (UU^\dagger)^{-1} = \begin{pmatrix} \lambda^{-1} & 0 \\ 0 & \lambda \end{pmatrix}.\tag{130}$$

In this case, isomatrices (83) become (Eqs. (3.9) of [58])

$$\begin{aligned}\hat{\sigma}_1(\lambda) &= \begin{pmatrix} 0 & \lambda^{-1} \\ \lambda & 0 \end{pmatrix}, \quad \hat{\sigma}_2(\lambda) = \begin{pmatrix} 0 & -i\lambda^{-1} \\ i\lambda & 0 \end{pmatrix}, \\ \hat{\sigma}_3(\lambda) &= \begin{pmatrix} \lambda & 0 \\ 0 & -\lambda^{-1} \end{pmatrix}.\end{aligned}\tag{131}$$

It is an instructive exercise for the interested reader to verify that the above realization of the regular Pauli-Santilli isomatrices verifies isocommutation rules with the same stricture constants of the $SU(2)$ algebra

$$[\hat{\sigma}_i(\lambda), \hat{\sigma}_j(\lambda)] = i2\epsilon_{ijk}\hat{\sigma}_k(\lambda),\tag{132}$$

and admit conventional eigenvalues

$$\begin{aligned}\hat{\sigma}_3(\lambda) \star |\hat{b}\rangle &= \pm|\hat{b}\rangle \\ \hat{\sigma}(\lambda)^{\hat{2}} &= 3|\hat{b}\rangle.\end{aligned}\tag{133}$$

Consequently, we have the following property [58]:

LEMMA 3.3. *Regular Pauli-Santilli isomatrices provide an explicit and concrete realization of regular hidden variables directly in the spin 1/2 algebra.*

Note that, besides being positive-definite, hidden variables have an unrestricted functional dependence on all needed local variables, Eqs. (66).

An example of irregular hidden variables is provided by the $\hat{S}U(2)$ -component of the spinorial covering of the Lorentz-Poincaré-Santilli isosymmetry.

To illustrate this realization, introduce *three* additional hidden variables for the characterization of isospace (107)

$$m_\mu = \lambda_\mu, \quad \mu = 1, 2, 3, 4. \quad (134)$$

Realization (108) then implies the following irregular Dirac-Santilli isomatrices

$$\begin{aligned} \hat{\gamma}_k(\lambda) &= \frac{1}{\gamma_k} \begin{pmatrix} 0 & \hat{\sigma}_k(\lambda) \\ -\hat{\sigma}_k & 0 \end{pmatrix}, \\ \hat{\gamma}_4 &= \frac{i}{m_4} \begin{pmatrix} I_{2 \times 2} & 0 \\ 0 & -I_{2 \times 2} \end{pmatrix}, \end{aligned} \quad (135)$$

where $\hat{\sigma}_k$ are the *regular or irregular Pauli-Santilli isomatrices*, with isocommutation rules

$$[S_i(\lambda), \hat{S}_j(\lambda)] = \epsilon_{ijk} \frac{1}{\lambda_k} S_k, \quad (136)$$

and isoeigenvalues

$$\begin{aligned} S_3 \star |\hat{\psi}\rangle &= \pm \frac{1}{2} \frac{1}{\sqrt{\lambda_1 \lambda_2}} |\hat{\psi}\rangle, \\ S^{\hat{2}} \star |\hat{\psi}\rangle &= \frac{1}{4} \left(\frac{1}{\sqrt{\lambda_1 \lambda_2}} + \frac{1}{\sqrt{\lambda_2 \lambda_3}} + \frac{1}{\sqrt{\lambda_3 \lambda_1}} \right) |\hat{\psi}\rangle. \end{aligned} \quad (137)$$

Consequently, we have the following property [62]

LEMMA 3.4: *The axioms of Dirac's equation admit up to five generally different, regular or irregular hidden variables.*

Additional realizations of irregular hidden variables can be found in Eqs. (3.11) of Ref. [58] or can be easily derived from the preceding realization of the Pauli-Santilli isomatrices.

3.7. Apparent admission of classical counterparts.

As it is well known, Bell's inequality [3] [4], von Neumann's theorem [5], and the theory of local realism at large (see review [6] with a comprehensive literature) are generally assumed to be evidence of the impossibility of "completing" quantum mechanics into a broader theory, with ensuing rejection of the EPR argument [1].

Following decades of preparatory works reviewed in Paper I [9] and in the preceding sections of this paper, Santilli proved in Ref. [10] of 1998 (see also the detailed study in Ref. [30], particularly Chapter 4 and Appendix 4C, page 166) that:

1) Bell's inequality, von Neumann's theorem and related studies are indeed valid, but under the *tacit* assumption of representing particles as being point-like, with ensuing sole admission of linear, local and potential interactions (exterior dynamical problems).

2) Bell's inequality, von Neumann's theorem and related studies are *inapplicable* (rather than being violated) for extended particles within physical media, due to the presence of additional non-linear, non-local and non-potential interactions (interior dynamical systems).

3) The latter systems represented with the axiom-preserving "completion" of 20th century applied mathematics into isomathematics and the ensuing "completion" of quantum mechanics into hadronic mechanics [29]-[31] verify Statement 2 and admit well defined classical counterparts.

To review the preceding advances, consider two quantum mechanical particles with spin 1/2 denoted 1 and 2 which verify the $SU(2)$ spin symmetry.

Assume that, as a result of some interaction, the two particles have antiparallel spins represented in the Hilbert space $\mathcal{H}$ over the field of complex numbers $\mathcal{C}$. The total state in $calH$ is then given by

$$|S_{1-2} \rangle = \frac{1}{\sqrt{2}}(|S_{1\uparrow} \rangle \times |S_{2\downarrow} \rangle - |S_{1\downarrow} \rangle \times |S_{2\uparrow} \rangle), \quad (138)$$

with conventional normalization

$$\langle S_{1-2} | S_{1-2} \rangle = 1, \quad (139)$$

where $\times$ is the conventional associative product.

Let a_1, b_1 and a_2, b_2 be unit vectors along the z -axis of a conventional Euclidean space $E(r, \delta, I)$ for particle 1 and 2, respectively. Introduce the quantum mechanical probability

$$P(a_1, b_1) = \langle S_{1-2} | (\sigma_1 \otimes a_1) \times (\sigma_2 \otimes b_1) | S_{1-2} \rangle = -a_1 \otimes b_1, \quad (140)$$

where $\otimes$ is the conventional scalar product.

Then, Bell's inequality can be written [4] (see Ref. [6] for numerous equivalent formulations)

$$D_{Bell}^{QM} = \text{Max} |P(a_1, b_1) - P(a_1, b_2) + P(a_2, b_1) + P(a_2, b_2)| \leq 2, \quad (141)$$

and implies the following property:

LEMMA 3.5: *Particles in vacuum verifying the Lie symmetry $SU(2)$ admit no classical counterparts.*

PROOF: The classical counterpart of Bell's inequality is given by

$$D_{Max}^{Classical} = Max|a_1 \otimes b_1 - a_1 \otimes b_2| + |a_2 \otimes b_1 + a_2 \otimes b_2| = 2\sqrt{2}. \quad (142)$$

But the quantum mechanical value of D_{Bell}^{QM} is *always* smaller than its classical counterpart $D_{Max}^{Classical}$,

$$D_{Bell}^{QM} < D_{Max}^{Classical}, \quad (143)$$

by therefore establishing the impossibility for an $SU(2)$ -invariant system to admit identical classical images. Q. E. D.

Santilli [10] has shown that inequality (141) is inapplicable for the same particles when they are in interior dynamical conditions, e.g., when they are in the core of a star, or at the limit, when they are in the interior of a gravitational collapse.

Considers two extended particles also denoted 1 and 2. Suppose that said particles verify the regular $\hat{S}U(2)$ isosymmetry with spin 1/2 (Section 3.3), thus implying the elaboration via isomathematics (Section I-3) and the verification of the isotopic branch of hadronic mechanics (Section I-4).

Suppose that the two extended particles with spin 1/2 are characterized by the following isotopic elements:

$$\begin{aligned} \text{Particle 1 : } \hat{T}_1 &= \text{Diag}(\lambda_1, 1/\lambda_1), \\ \text{Particle 2 : } \hat{T}_2 &= \text{Diag}(\lambda_2, 1/\lambda_2), \end{aligned} \quad (144)$$

with realization (83) of the Pauli-Santilli isomatrices.

Suppose that, due to preceding interactions, the two extended particles are in single overlapping/entanglement thus having opposite spins.

Let $\hat{I}_1$ and $\hat{I}_2$ be the isounits for particles 1 and 2, respectively. The systems of the assumed two isoparticles is then characterized by the total isounit

$$\hat{I}_{tot} = \hat{I}_1 \times \hat{I}_2 = \frac{1}{\hat{T}_{tot}} = \frac{1}{\hat{T}_1 \times \hat{T}_2}. \quad (145)$$

In this case, the total isostate on the Hilbert-Myung-Santilli isospace $\hat{\mathcal{H}}$ [19] over the isofield of isocomplex isonumbers $calC$ [20] is given by

$$|\hat{S}_{1-2} \rangle = \frac{1}{\sqrt{2}}(|\hat{S}_{1\uparrow} \rangle \hat{\times} |\hat{S}_{2\downarrow} \rangle - |\hat{S}_{1\downarrow} \rangle \hat{\times} |\hat{S}_{2\uparrow} \rangle). \quad (146)$$

The lack of validity of inequality (141) for irregular isorepresentations of $\hat{S}U(2)$ is evident (e.g., because of the anomalous spin isoeigenvalues) and, as such, it is ignored.

A significant aspect of Ref. [10] is the proof of the inapplicability of inequality (141), not only for regular isorepresentation of $\hat{SU}(2)$, but also when such isorepresentations are isounimodular, Eqs. (144).

Let a_1, b_1, a_2, b_2 be unit vectors along the z -axis of an iso-Euclidean isospace. Introduce the isoprobability (Eq. (32.39), page 99, Ref. [30])

$$\begin{aligned}\hat{P}(a, b) &= \langle \hat{S}_{1-2} | \star (\hat{\Sigma}_1 \hat{\otimes}_1 a) \times (\hat{\Sigma}_2 \hat{\otimes}_2 b) | \hat{S}_{1-2} \rangle \hat{I}_{tot} = \\ &= \langle \hat{S}_{1-2} | \star (\hat{\sigma}_1 \otimes a) \times (\hat{\sigma}_2 \otimes b) | \hat{S}_{1-2} \rangle \hat{I}_{tot},\end{aligned}\quad (147)$$

with isonormalization (here referred to individual diagonal elements of isotopic elements and isounits)

$$\langle \hat{S}_{1-2} | \star | \hat{S}_{1-2} \rangle = \langle \hat{S}_{1-2} | \hat{T}_{tot} | \hat{S}_{1-2} \rangle = \hat{I}_{tot} \quad (148)$$

where: $\star$ is the total isoproduct; $\hat{\otimes}_k$, $k = 1, 2$, is the isoscalar isoproduct; and we have used simplifications of the type

$$\hat{\Sigma}_1 \hat{\otimes}_1 a = (\hat{\sigma}_1 \hat{I}_1)(\hat{T}_1 \otimes) a = \hat{\sigma}_1 \otimes a. \quad (149)$$

An isotopy of the conventional case yields the following isobasis, Eq. (6.5) of Ref. [10],

$$|S_{1-2}\rangle = \frac{1}{2} \left\{ \begin{pmatrix} \lambda_1^{-1/2} \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ \lambda_2^{1/2} \end{pmatrix} - \begin{pmatrix} 0 \\ \lambda_2^{1/2} \end{pmatrix} \begin{pmatrix} \lambda_1^{-1/2} \\ 0 \end{pmatrix} \right\}. \quad (150)$$

The appropriate use of products and isoproducts then yield expression (5.6) Ref. [10], i.e.,

$$\begin{aligned}\langle \hat{S}_{1-2} | \hat{T}_{tot} (\hat{\sigma}_1 \otimes_1 a) \times (\hat{\sigma}_2 \otimes_2 b) \hat{T}_{tot} | \hat{S}_{1-2} \rangle &= \\ &= -a_x b_x - a_y b_y - \frac{1}{2} (\lambda_1 \lambda_2^{-1} + \lambda_1^{-1} \lambda_2) a_z b_z.\end{aligned}\quad (151)$$

The continuation of the isotopy of the conventional case, yields the main result, Eq. (5.8) of Ref. [10], which provides the following *isotopic "completion" of Bell's inequality*,

$$\begin{aligned}\hat{D}_{Max}^{HM} &= D_{Max}^{HM} \hat{I}_{tot} = \\ Max | \hat{P}(a_1, b_1) - \hat{P}(a_1, b_2) + \hat{P}(a_2, b_1) + \hat{P}(a_2, b_2) | &= \\ &= [\frac{1}{2} (\lambda_1 \lambda_2^{-1} + \lambda_1^{-1} \lambda_2) D_{Bell}^{QM} \hat{I}_{tot},\end{aligned}\quad (152)$$

with consequential:

LEMMA 3.6. *Extended particles within physical media that are invariant under the Lie-Santilli isosymmetry $\hat{S}U(2)$ admit identical classical counterparts.*

PROOF: Isoinequality (141) establishes the lack of universal validity of Bell's inequality (128) because the factor $\frac{1}{2}(\lambda_1\lambda_2^{-1} + \lambda_1^{-1}\lambda_2)$ can have values *bigger* than one, thus implying

$$D_{Max}^{HM} \geq D_{Bl}^{QM}. \quad (153)$$

Consider then a classical iso-Euclidean isospace $\hat{E}(\hat{r}, \hat{\delta}, \hat{I})$ representing motion of classical extended particles 1 and 2 within physical media [30] with isometric elements

$$\hat{\delta}_{11} = 1, \quad \hat{\delta}_{22} = 1, \quad \hat{\delta}_{33} = \frac{1}{2}(\lambda_1\lambda_2^{-1} + \lambda_1^{-1}\lambda_2) = 2, \quad (154)$$

in which case

$$D_{Max}^{HM} \equiv \hbar t D_{Max}^{Classical}, \quad (155)$$

by therefore establishing that systems of extended particles within physical media verifying the $\hat{S}U(2)$ isosymmetry admits an identical classical counterpart along the EPR argument. Q.E.D.

It is an instructive exercise for the interested reader to prove that the above lemma also holds for different isorenormalizations, e.g., Eqs. (171) of next section, with the understanding that different isorenormalizations imply different isobasis and different hidden variable terms in Eqs. (151).

Note the crucial role of hidden variables for the proof of Lemma 3.6. It is an instructive exercise for interested readers to prove that Lemma 3.6 holds for any other regular, isounimodular isorepresentation of the isotopic $\hat{S}U(2)$ symmetry in terms of hidden variables presented in Section 3.3.

The proof of the lack of applicability of von Neumann's theorem [5] for extended particles in interior conditions is elementary. Recall that von Neumann's theorem is based on the uniqueness of the eigenvalues E of a Hermitean operator H , $H|\psi\rangle = E|\psi\rangle$ under unitary transformation on $\mathcal{H}$,

$$UH|\psi\rangle U^\dagger = UE|\psi\rangle U^\dagger = EU|\psi\rangle U^\dagger, \quad UU^\dagger = U^\dagger U = I, \quad (156)$$

under the tacit assumption of point particles in vacuum.

By contrast, when the same particles is in interior conditions, it is subjected to an infinite number of different physical different interactions with

the medium represented by the isotopic element $\hat{T}$ with ensuing isoeigenvalue equation (Section I-4), [9],

$$abel1H \star |\hat{\psi}_{\hat{T}} r \rangle = HT |\hat{\psi}_{\hat{T}} \rangle = E_{\hat{T}} |\hat{\psi}_{\hat{T}} \rangle, \quad (157)$$

thus establishing that a given quantum mechanical operator H representing the energy of an extended particle in interior conditions has an infinite number of *generally different* isoeigenvalues $E_{\hat{T}}$ depending on the infinite number of different interior conditions.

Note that, for each given $\hat{T}$ the isoeigenvalue $E_{\hat{T}}$ is invariant under isounitary isotransformations (Section I-3-9).

3.8. Apparent admission of classical determinism.

Consider a point-like particle in empty space represented in the 3-dimensional Euclidean space $E(r, \delta, I)$, where r represents coordinates, $\delta = \text{Diag.}(1, 1, 1)$ represents the Euclidean metric and $I = \text{Diag.}(1, 1, 1)$ represents the space unit.

Let the operator representation of said point-like particle be done in a Hilbert space $\mathcal{H}$ over the field of complex numbers $\mathcal{C}$ with states $\psi(r)$ and familiar normalization

$$\langle \psi(r) | \psi(r) \rangle = \int_{-\infty}^{+\infty} \psi(r)^{\dagger} \psi(r) dr = 1. \quad (158)$$

As it is well known, the primary objections against the EPR argument [2]- [6] were based on Heisenberg uncertainty principle according to which *the position r and the momentum p of said particle cannot both be measured exactly at the same time.*

By introducing the *standard deviations* Δr and Δp , the uncertainty principle is generally written in the form

$$\Delta r \Delta p \geq \frac{1}{2} \hbar, \quad (159)$$

which is easily derivable via the vacuum expectation value of the canonical commutation rule

$$\Delta r \Delta p \geq \left| \frac{1}{2i} \langle \psi | [r, p] | \psi \rangle \right| = \frac{1}{2} \hbar. \quad (160)$$

Standard deviations have the known form (see, e.g., Ref. [79]) with $\hbar = 1$

$$\begin{aligned} \Delta r &= \sqrt{\langle \psi(r) | [r - (\langle \psi(r) | r | \psi(r) \rangle)]^2 | \psi(r) \rangle}, \\ \Delta p &= \sqrt{\langle \psi(p) | [p - (\langle \psi(p) | p | \psi(p) \rangle)]^2 | \psi(p) \rangle}, \end{aligned} \quad (161)$$

where $\psi(r)$ and $\psi(p)$ are the wavefunctions in coordinate and momentum spaces, respectively.

We consider now an extended particle, this time, in interior conditions, e.g., in the core of a star, classically represented by the *iso-Euclidean isospace* $\hat{E}(\hat{r}, \hat{\delta}, \hat{I})$ with isounit $\hat{I} = 1/\hat{T} > 0$, isocoordinates $\hat{r} = r\hat{I}$, isometric

$$\hat{\delta} = \hat{T}\delta, \quad (162)$$

and isotopic element (4)) under conditions (5).

For simplicity, we assume that the extended particle has no Hamiltonian interactions due to the dominance of the latter interactions over the former.

Consequently, we can represent the extended particle in the isospace $\hat{\mathcal{H}}$ over the isofield $\hat{\mathcal{C}}$ and introduce the time independent *isoplanewave* [18]

$$\begin{aligned} \hat{\psi}(\hat{r}) &= \tilde{\psi}(\hat{r})\hat{I} = \\ &= \hat{N} \star (\hat{e}^{i\hat{k}\hat{r}})\hat{I} = N(e^{ik\hat{T}\hat{r}})\hat{I}, \end{aligned} \quad (163)$$

where $\hat{N} = N\hat{I}$ is an *isonormalization isoscalar*, $\hat{k} = k\hat{I}$ is the *isowavenumber*, and the isoexponentiation is given by Eq. (I-22) [26].

The corresponding representation in isomomentum isospace is given by

$$\tilde{\psi}(\hat{p}) = \hat{M} \star \hat{e}^{i\hat{n}\hat{p}}, \quad (164)$$

where $\hat{M} = M\hat{I}$ is an *isonormalization isoscalar* and $\hat{n} = n\hat{I}$ is the *isowave-number* in isomomentum isospace.

The *isoprobability isofunction* is then given by (Ref. [30] page 99)

$$\hat{\mathcal{P}} = \hat{<} \star | \hat{>} = \langle \hat{\psi}(\hat{r}) | T | \hat{\psi}(\hat{r}) \rangle \hat{I}, \quad (165)$$

that, written in terms of isointegrals (Ref. [29] page 354), becomes

$$\begin{aligned} \int_{-\infty}^{+\infty} \hat{\psi}(\hat{r})^\dagger \star \hat{\psi}(\hat{r}) \star \hat{d}\hat{r} &= \\ &= \int_{-\infty}^{+\infty} \tilde{\psi}(\hat{r})^\dagger \tilde{\psi}(\hat{r}) (dr + r\hat{T}d\hat{I}), \end{aligned} \quad (166)$$

where one should keep in mind that the isodifferential $\hat{d}\hat{r}$ given by Eqs. (I-29).

The *isoexpectation isovalues* of a Hermitean operator $\hat{Q}$ are then given by [30]

$$\begin{aligned}\hat{<}| \star \hat{Q} \star | \hat{>} &= \langle \hat{\psi}(\hat{r}) | \star \hat{Q} \star | \hat{\psi}(\hat{r}) \rangle = \\ &= \int_{-\infty}^{+\infty} \hat{\psi}(\hat{r})^\dagger \star \hat{Q} \star \hat{\psi}(\hat{r}) \hat{d}\hat{r} = \\ &= \int_{-\infty}^{+\infty} \tilde{\psi}(\hat{r})^\dagger \hat{Q} \tilde{\psi}(\hat{r}) \hat{d}\hat{r},\end{aligned}\tag{167}$$

with corresponding expressions for the isoexpectation isovalues in isomomentum isospace.

Santilli then introduced apparently for the first time in Ref. [11] the *isotopic operator*

$$\hat{\mathcal{T}} = \hat{T} \hat{I} = I, \tag{168}$$

that, despite its seemingly irrelevant value, is indeed the correct operator formulation of the isotopic element for the "completion" of the isoproduct from its scalar form (1) to the isoscalar form

$$\hat{n}^{\hat{2}} = \hat{n} \star \hat{n} = \hat{n} \star \hat{\mathcal{T}} \star \hat{n} = n^2 I. \tag{169}$$

In Sections 3.6, 3.7, we have shown that the Lie-Santilli isosymmetry $\hat{S}U(2)$ admits an explicit and concrete realization of hidden variables that allowed the construction of identical classical counterparts for interior dynamical systems.

Ref. [11] introduced the isoexpectation isovalue of the isotopic operator

$$\begin{aligned}\hat{<}| \star \hat{\mathcal{T}} \star | \hat{>} &= \langle \hat{\psi}(\hat{r}) | \star \hat{\mathcal{T}} \star | \hat{\psi}(\hat{r}) \rangle = \hat{I} = \\ &= \int_{-\infty}^{+\infty} \tilde{\psi}(\hat{r})^\dagger \hat{\mathcal{T}} \tilde{\psi}(\hat{r}) \hat{d}\hat{r},\end{aligned}\tag{170}$$

and assumed the isonormalization (again, intended for diagonal matrix elements)

$$\begin{aligned}\hat{<}| \star \hat{\mathcal{T}} \star | \hat{>} &= \\ &= \int_{-\infty}^{+\infty} \hat{\psi}(\hat{r})^\dagger \hat{\mathcal{T}} \hat{\psi}(\hat{r}) \hat{d}\hat{r} = \hat{T}.\end{aligned}\tag{171}$$

Consider then the *isostandard isodeviation* for isocoordinates $\Delta\hat{r} = \Delta r \hat{I}$ and isomomenta $\Delta\hat{p} = \Delta p \hat{I}$, where Δr and Δp are the standard deviations in our space.

By using isocanonical isocommutation rules (I-81), we obtain the expression

$$\begin{aligned}\Delta\hat{r} \star \Delta\hat{p} &= \Delta r \Delta p \hat{I} \approx \frac{1}{2} | \langle \hat{\psi}(\hat{r}) | \star [\hat{r}, \hat{p}] \star | \hat{\psi}(\hat{r}) \rangle | \hat{I} = \\ &= \frac{1}{2} | \langle \hat{\psi}(\hat{r}) | \hat{T} [\hat{r}, \hat{p}] \hat{T} | \hat{\psi}(\hat{r}) \rangle | \hat{I},\end{aligned}\tag{172}$$

Studies on the EPR argument, I: Basic methods

One should note the replacement of the symbol $\geq$ in Eq. (160) with the symbol $\approx$ in Eq. (172). This is due to the fact that the historical arguments applying for a point-like particle in vacuum no longer apply for an interior system because the pressure exercised by the medium on the particle (Figure 4) reduce the lower limit of Eq. (160) to the approximate value of Eq. (172).

Under the above assumptions, by eliminating the common isounit $\hat{I}$, Ref. [11] achieved the desired result here called *isodeterministic isoprinciple*

$$\begin{aligned}\Delta r \Delta p &\approx \frac{1}{2} | \langle \hat{\psi}(\hat{r}) | \star [\hat{r}, \hat{p}] \star | \hat{\psi}(\hat{r}) \rangle = \\ &= \frac{1}{2} | \langle \hat{\psi}(\hat{r}) | \hat{T} [\hat{r}, \hat{p}] \hat{T} | \hat{\psi}(\hat{r}) \rangle = \\ &\frac{1}{2} \int_{-\infty}^{+\infty} \hat{\psi}(\hat{r})^\dagger \hat{T} \hat{\psi}(\hat{r}) d\hat{r} = \frac{1}{2} T \ll 1\end{aligned}\quad (173)$$

where the property $\Delta r \Delta p \ll 1$ follows from the fact that the isotopic element $\hat{T}$ has values smaller than 1 in the fitting of all experimental data dealing with hadronic media such as hadrons, nuclei and stars, and null value for gravitational collapse [31].

In the event Eq. (35), page 14 of Ref. [11] should be compatible with Eq. (173) above, it is sufficient to turn into a comma the sign $=$ in the right of the central expression of Eq. (35), or absorb the factor $1/2$ of Eq. (173) into the isorenormalization.

In this way, thanks to a laborious scientific journey initiated at Harvard University in late 1977, and thanks to contributions by numerous mathematicians, theoreticians and experimentalists, Santilli reached the following verification of the EPR argument [11]:

LEMMA 3.7 (ISODETERMINISTIC PRINCIPLE): *The isostandard isodeviations for isocoordinates $\Delta \hat{r}$ and isomomenta $\Delta \hat{p}$, as well as their product, progressively approach classical determinism for extended particles in the interior of hadrons, nuclei, and stars, and achieve classical determinism at the extreme densities in the interior of gravitational collapse.*

PROOF: Define the isostandard isodeviations via the following isotopy of quantum mechanical expressions (161) (where we ignore the common multiplication by the isounit)

$$\begin{aligned}\Delta r &= \sqrt{\langle \hat{\psi}(\hat{r}) | [\hat{r} - \langle \hat{\psi}(\hat{r}) | \star \hat{r} \star | \hat{\psi}(\hat{r}) \rangle]^2 | \hat{\psi}(\hat{r}) \rangle}, \\ \Delta p &= \sqrt{\langle \hat{\psi}(\hat{p}) | [\hat{p} - \langle \hat{\psi}(\hat{p}) | \star \hat{p} \star | \hat{\psi}(\hat{p}) \rangle]^2 | \hat{\psi}(\hat{p}) \rangle},\end{aligned}\quad (174)$$

where the differentiation between the isotopic elements for isocoordinates and isomomenta is ignored for simplicity. But the isotopic element represents the interactions of the particle with the physical medium and tends toward null values for gravitational collapse, Eqs. (I-91) (I-92). Therefore, isosquare in expression (171) implies the expressions

$$\begin{aligned}\Delta r &= \sqrt{\hat{T} < \hat{\psi}(\hat{r}) | [\hat{r} - < \hat{\psi}(\hat{r}) | \star \hat{r} \star | \hat{\psi}(\hat{r}) >]^2 | \hat{\psi}(\hat{r}) >}, \\ \Delta p &= \sqrt{\hat{T} < \hat{\psi}(\hat{p}) | [\hat{p} - < \hat{\psi}(\hat{p}) | \star \hat{p} \star | \hat{\psi}(\hat{p}) >]^2 | \hat{\psi}(\hat{p}) >},\end{aligned}\tag{175}$$

that approach indeed null value under the indicated limit conditions of gravitational collapse

$$\begin{aligned}\lim_{\hat{T}=0} \Delta r &= 0, \\ \lim_{\hat{T}=0} \Delta p &= 0,\end{aligned}\tag{176}$$

Q.E.D.

3.9. Apparent removal of quantum divergencies.

Recall from Section I-4.13 that, under condition (I-96), corresponding to condition (173), there is a rapid convergence of isoserries (I-97), as well as the removal of the singularity of Dirac's delta distribution, Eq. (I-98) (Figure I-14).

The above properties can be now formalized according to the following:

COROLLARY 3.7.1. Einstein's determinism according to Lemma 3.7 implies the removal of quantum mechanical divergencies.

PROOF. Lemma 3.7 is based on values of the isotopic element $\hat{T}$ being smaller than 1, which values imply in turn the rapid convergence of perturbative series without divergencies (Section I-4-13).

Q.E.D.

The removal of quantum divergencies, that have been cause of controversies for about one century, illustrates the far reaching implications of Einstein's determinism for interior dynamical systems.

3. CONCLUDING REMARKS.

Following the study of basic methods in Paper I, in this paper we have provided an apparent confirmation of proofs [10] [11] of the EPR argument [1] for extended, thus deformable particles within hyperdense media with

ensuing linear and non-linear, local and non-local and potential as well as non-potential/non-Hamiltonian interactions.

This study has been conducted via the use of isomathematics and isomechanics admitting a conventional Hamiltonian H or Lagrangian L for the invariant representation of linear, local and potential interactions, plus the isotopic element $\hat{T}$ of isoproducts $A \star B = A\hat{T}B$, Eq. (1), for the invariant representation of non-linear, non-local and non-Hamiltonian interactions.

Following the outline and upgrade of isosymmetries for time-reversal invariant interior systems (Section 3), we have apparently confirmed the proof of Ref. [10] according to which extended particles in interior dynamical conditions admit identical classical counterpart.

We have then apparently confirmed the proof of Ref. [11] according to which extended particles progressively approach classical determinism when in the interior of hadrons, nuclei and stars, and achieve full determinism at the limit of gravitational collapse, essentially as predicted by A. Einstein, B. Podolsky and N. Rosen [1].

To illustrate the far reaching implications of what appears to be Einstein's most important legacy, we have shown for the first time that the recovering of Einstein's determinism for interior conditions appears to imply the removal of quantum divergencies due to the rapid convergence of the isoserries of hadronic mechanics, the removal of the singularity in Dirac's delta distribution and other features.

A number of illustrations and novel applications in mathematics, physics and chemistry are presented in the forthcoming Paper III.

Acknowledgments. The writing of these papers has been possible thanks to pioneering contributions by numerous scientists we regret not having been able to quote in this paper (see Vol. I, Refs. [?] for comprehensive literature), including:

The mathematicians: H. C. Myung, for the initiation of iso-Hilbert spaces; S. Okubo, for pioneering work in non-associative algebras; N. Kamiya, for basic advances on isofields; M. Tomber, for an important bibliography on non-associative algebras; R. Ohemke, for advances in Lie-admissible algebras; J. V. Kadeisvili, for the initiation of isofunctional isoanalysis; Chun-Xuan Jiang, for advances in isofield theory; Gr. Tsagas, for advances in the Lie-Santilli theory; A. U. Klimyk, for the initiation of Lie-admissible deformations; Raul M. Falcon Ganfornina and Juan Nunez Valdes, for advances in isomanifolds and isotopology; Svetlin Georgiev, for advances in the isodifferential isocalculus; A. S. Muktibodh, for advances in the isorepresentations of Lie-Santilli isoalgebras; Thomas Vou-

giouklis, for advances on Lie-admissible hypermathematics; and numerous other mathematicians.

The physicists and chemist: A. Jannussis, for the initiation of isocreation and isoannihilation operators; A. O. E. Animalu, for advances in the Lie-admissible formulation of hadronic mechanics; J. Fronteau, for advances in the connection between Lie-admissible mechanics and statistical mechanics; A. K. Aringazin for basic physical and chemical advances; R. Mignani for the initiation of the isoscattering theory; P. Caldirola, for advances in isotime; T. Gill, for advances in the isorelativity; J. Dunning Davies, for the initiation of the connection between irreversible Lie-admissible mechanics and thermodynamics; A. Kalnay, for the connection between Lie-admissible mechanics and Nambu mechanics; Yu. Arestov, for the experimental verification of the iso-Minkowskian structure of hadrons; A. Ahmar, for experimental verification of the iso-Minkowskian character of Earth's atmosphere; L. Ying, for the experimental verification of nuclear fusions of light elements without harmful radiation or waste; Y. Yang, for the experimental confirmation of magnecules; D. D. Shillady, for the isorepresentation of molecular binding energies; R. L. Norman, for advances in the laboratory synthesis of the neutron from the hydrogen; I. Gandzha for important advances on isorelativities; A. A. Bhalekar, for the exact representation of nuclear spins; Simone Beghella Bartoli important contributions in experimental verifications of hadronic mechanics; and numerous other physicists.

I would like to express my deepest appreciation for all the above courageous contributions written at a point in time of the history of science dominated by organized oppositions against the pursuit of new scientific knowledge due to the trillions of dollars of public funds released by governmental agencies for research on pre-existing theories.

Special thanks are due to A. A. Bhalekar, J. Dunning-Davies, S. Georgiev, R. Norman, T. Vougiouklis and other colleague for deep critical comments.

Additional special thanks are due to Simone Beghella Bartoli for a detailed critical inspection of this Paper I.

Special thanks and appreciation are due to Arun Muktibodh for a very detailed inspection and control of all three papers, although I am solely responsible for their content due to numerous revisions implemented in their final version.

Thanks are also due to Mrs. Sherri Stone for an accurate linguistic control of the manuscript.

Finally, I would like to express my deepest appreciation and gratitude to my wife Carla Gandiglio Santilli for decades of continued support of my research amidst a documented international chorus of voices opposing the

study of the most important vision by Albert Einstein.

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Studies on A. Einstein, B. Podolsky and N. Rosen argument that “quantum mechanics is not a complete theory,” III: Illustrative examples and applications

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Abstract

In the preceding Papers I and II of this series, we have presented a review and upgrade of novel mathematical, physical and chemical methods, and shown their use for a confirmation of the apparent proof of the EPR argument that extended particles within physical media admit classical counterparts, while Einstein’s determinism appears to be progressively verified with the increase of the density of the medium. In this third paper, we have additionally shown, apparently for the first time, the validity of the EPR final statement to the effect that *the wavefunction [of quantum mechanics] does not provide a complete description of the physical reality.* In fact, we have studied the axiom-preserving “completion” of the quantum mechanical wavefunction due to deep wave-overlapping when represented via isomathematics, and shown that it permits an otherwise impossible representation of the attractive force between identical electrons pairs in valence coupling, as well as the representation of *all* characteristics of various physical and chemical systems existing in nature.

Keywords: EPR argument, isomathematics, isomechanics.

2010 AMS subject classifications: 05C15, 05C60. ¹

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¹Received 26th of April, 2020, accepted 19th of June, 2020, published 30th of June, 2020,
doi 10.23755/rm.v38i0.517, ISSN 1592-7415; e-ISSN 2282-8214. ©Ruggero Maria Santilli

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3. CONCLUDING REMARKS.

Acknowledgments.

References.

1. INTRODUCTION.

1.1. The EPR argument.

As it is well known, Albert Einstein did not accept quantum mechanical uncertainties as being final, for which reason he made his famous quote "God does not play dice with the universe."

Einstein communicated his views to B. Podolsky and N. Rosen and they jointly published in 1935 the historical paper [1] that became known as the *EPR argument*.

Objections against the EPR argument have been voiced by numerous scholars, including by N. Bohr [2], J. S. Bell [3] [4], J. von Neumann [5] and others (see Ref. [6] for a review and comprehensive literature).

The field became known as *local realism* and included the dismissal of the EPR argument based on claims that quantum axioms do not admit *hidden variables* λ [7] [8].

1.2. Outline of preceding works.

Following various preparatory works, in Ref. [9] of 1998, R. M. Santilli:

1) Assumed the validity of quantum mechanics, with consequential validity of the objections against the EPR argument [2] - [6], for point-like particles in empty space under linear, local and potential interactions (*exterior dynamical problems*);

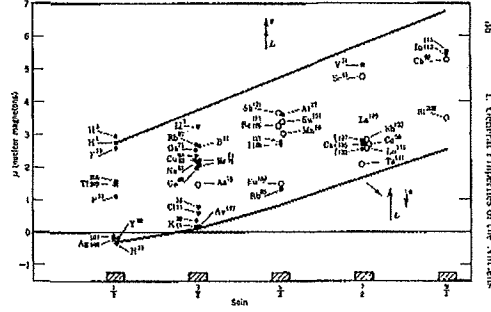
2) Proved the inapplicability (and not their violation) of said objections for the broader class of extended, deformable and hyperdense particles within physical media under the most general known linear and non-linear, local and non-local and potential as well as non-potential interactions (*interior dynamical problems*); and

3) Provided the apparent proof that *interior dynamical systems admit classical counterparts in full accordance with the EPR argument* via the representation of interior systems with of *isomathematics* also called *isotopoic branch of hadronic mathematics*, and *isomechanics*, also called *isotopic branch of hadronic mechanics*.

In the 2019 paper [10], Santilli provided the apparent proof that *Einstein's determinism is progressively approached in the interior of hadrons, nuclei and stars and it is fully achieved in the interior of gravitational collapse*.

In Ref. [11], herein referred to as Paper I, we provided a review of isomathematics and isomechanics.

In the subsequent Ref. [12], hereinafter referred as Paper II, we provided a review of isosymmetries, with particular reference to the isotopies of the spin and rotational symmetries, and provided an apparent confirmation of the proofs [9] and [10] of the EPR argument.



$\hat{T} > 0$, but possesses otherwise an unrestricted functional dependence on all needed local variables, including wavefunctions and their derivatives.

20th century applied mathematics and quantum mechanics are reformulated in an axiom-preserving, thus isotopic form, via isoproduct (1) (Paper I and Section II-2).

3) The axiom-preserving isotopy of the various branches of Lie's theory, first achieved by Santilli in Refs. [18] [23] and are today known as the *Lie-Santilli isothory* [24] (Section I-3.7) with Lie-Santilli algebras of the type [9]

$$[X_i, X_j] = X_i \star X_j - X_j \star X_i = C_{ij}^k X_k. \quad i, j = 1, 2, \dots, N. \quad (2)$$

and ensuing systematic isotopy of space-time and internal symmetries (Section II-2).

3) Immediate explicit and concrete realizations of "hidden variables" [7] [8] of the type

$$\hat{T} = \text{Diag.}(1/\lambda, \lambda), \quad \text{Det}\hat{T} = 1. \quad (3)$$

Ref. [9], therefore establishing that, contrary to objections [2] to [6], *the abstract axioms of quantum mechanics do indeed admit explicit and concrete realizations of hidden variables.*

4) Representation of extended particles in conditions of mutual penetration via realizations of the isotopic element $\hat{T}$ of isoproduct (1) of the type [25]

$$\hat{T} = \Pi_{k=1, \dots, N} \text{Diag.}(\frac{1}{n_{1k}^2}, \frac{1}{n_{2k}^2}, \frac{1}{n_{3k}^2}, \frac{1}{n_{4k}^2}) e^{-\Gamma}, \quad (4)$$

$$k = 1, 2, \dots, N, \quad \mu = 1, 2, 3, 4,$$

where n_1^2, n_2^2, n_3^2 , (called *characteristic quantities*) represent the deformable semi-axes of the particle normalized to the values $n_k^2 = 1, 'k = 1, 2, 3$ for the sphere; n_4^2 represents the *density* of the particle considered normalized to the value $n_4 = 1$ for the vacuum; and Γ represents non-linear, non-local and non-Hamiltonian interactions caused by mutual penetrations/entanglement of particles.

In particular, the isotopic element $\hat{T}$ has resulted to have numeric values *smaller* than 1 in all known applications [23]-

$$|\hat{T}| \leq 1, \quad (5)$$

which property permitted Ref. [10] to show that *the standard deviations Δr and Δp progressively tend to zero with the increase of the density of the medium of interior problems.*

5) Isosymmetries do not preserve over time the basic unit 1 of conventional numeric fields $F(n, \times, 1)$ with consequential lack of experimental

verifications. This occurrence mandated Santilli to formulate isomathematics on *isofields* $\hat{F}(\hat{n}, *, \hat{I})$ [26] [27] (Section I-3.3) with *isonumbers* $\hat{n} = n\hat{I}$ equipped with isoproduct (1) and basic *isounit*

$$\hat{I} = 1/\hat{T} > 0, \quad (6)$$

which remains numerically invariant under isosymmetries as necessary for consistency (Sections I-3.8, I-3.9 and II-2).

5) Recall that the Newton-Leibnitz differential calculus provides the ultimate characterization of the point-like character of particles admitted by quantum mechanics due to the known feature that said calculus can be solely defined as a finite number of isolated points.

This occurrence mandated Santilli to construct the covering of the Newton-Leibnitz differential calculus into the *isodifferential isocalculus* (Section I-3.6) with basic *isodifferential* [28] [29]

$$\hat{d}\hat{r} = \hat{T}d[r\hat{I}(r, \dots)] = dr + r\hat{T}d\hat{I}(r, \dots), \quad (7)$$

and corresponding *isoderivative*

$$\frac{\partial \hat{f}(\hat{r})}{\partial \hat{r}} = \hat{I} \frac{\partial \hat{f}(\hat{r})}{\partial \hat{r}}, \quad (8)$$

which are defined on *volumes* represented by the isotopic element $\hat{T}$, rather than points.

Recall that Bell's inequality [3] D_{Bell}^{qm} does not admit a classical counterpart D^{class} because always smaller than the classical counterpart,

$$D_{Bell}^{qm} < D^{class}. \quad (9)$$

In Ref. [9], Eqs. (5.8), page 189, proved that, thanks to the existence in hadronic mechanics of hidden variables of type (3), the corresponding inequality in hadronic mechanics (hm), D^{hm} is bigger than Bell's inequality, and always admits a classical counterpart

$$D^{hm} \equiv D^{class}, \quad (10)$$

resulting in the following:

LEMMA II-3.6 (EXISTENCE OF CLASSICAL COUNTERPART). *Extended particles within physical media that are invariant under the Lie-Santilli isosymmetry $\hat{SU}(2)$ admit identical classical counterparts.*

The main result of Ref. [10], Eqs. (35), page 14,

$$\begin{aligned}\Delta r \Delta p &\approx \frac{1}{2} | \langle \hat{\psi}(\hat{r}) | \star [\hat{r}, \hat{p}] \star | \hat{\psi}(\hat{r}) \rangle = \\ &= \frac{1}{2} | \langle \hat{\psi}(\hat{r}) | \hat{T} [\hat{r}, \hat{p}] \hat{T} | \hat{\psi}(\hat{r}) \rangle = \\ &\int_{-\infty}^{+\infty} \hat{\psi}(\hat{r})^\dagger \hat{T} \hat{\psi}(\hat{r}) d\hat{r} = T \ll 1,\end{aligned}\tag{11}$$

verifying Einstein's determinism, was expressed with the following:

LEMMA II-3.7 (EINSTEIN DETERMINISM): *The isostandard isodeviations for isocoordinates $\Delta \hat{r}$ and isomomenta $\Delta \hat{p}$, as well as their product, progressively approach classical determinism for extended particles in the interior of hadrons, nuclei and stars, and achieve classical determinism at the extreme densities in the interior of gravitational collapse.*

By recalling that isoperturbative series of hadronic mechanics has based on isoproduct (1) (Section I-4),

$$A(0) = \hat{I} + (A\hat{T}H - H\hat{T}A)/1! + \dots\tag{12}$$

and that $\hat{T} \ll 1$ from Lemma II-3.7, the following consequential property was introduced, apparently for the first time in Paper II:

COROLLARY 3.7.1 (LACK OF DIVERGENCIES) *Einstein's determinism according to Lemma II-3.7 implies the lack of quantum mechanical divergencies in hadronic mechanics.*

It should be indicated that the technical understanding of the verifications and applications presented in this paper require a knowledge of isomathematics and isomechanics.

2. VERIFICATIONS AND APPLICATIONS.

2.1. Foreword.

Being dimensionless, *Newtonian massive points cannot experience resistive or contact force of any type.* By recalling that Newton's equation have been the foundations of physics for the past four centuries, 20th century mainstream particle physics has been developed without the notion of resistive force, with ensuing lack of treatment of the *pressure* exercised by a medium on a particle in its interior, contrary to clear evidence that a proton in the core of a star is exposed to extremely big pressures (Figure 1).

In this section, we shall illustrate the fact that, once admitted, the pressure exercised on extended particles characterizes in their interior characterizes standard deviations $\Delta\hat{r}$ and $\Delta\hat{p}$ that, being *constrained* by said pressure, verify the isodeterministic principle of Lemma II-3.7 as well as the rapid convergence of isoperturbative series according to Corollary II-3.7.1, by progressively approaching classical determinism with the increase of the pressure, up to the apparent achievement of classical determinism for the interior of gravitational collapse as predicted by Einstein, Podolsky and Rosen [1].

A physically important notion emerging from the examples provided below is that *the EPR argument appears to be verified by strong interactions* because, as indicated by Santilli in the 1978 paper [19], contact non-Hamiltonian interactions responsible for the synthesis of the neutron and other hadrons are short range, strongly attractive, charge independent and non-Hamiltonian (technically identified as *variationally non-selfadjoint interactions* [22]), thus providing a conceivable, first known, explicit and concrete representation of strong interactions.

The models outlined in this section were first proposed by Santilli in Ref. [19] in their time irreversible form, as requested for decaying bound states, thus being elaborated with Lie-admissible genomathematics, in which case, the need for a "completion" of quantum mechanics is beyond scientific doubt (Section I-1.3).

However, the objections against the EPR argument [2] - [6] have been formulated for conventional quantum axioms, thus implying the sole consideration of time-reversal invariant states. In this section, we illustrate the need for a "completion" of quantum mechanics also for time-reversal invariant systems of extended particles in interior conditions.

Therefore, unstable strongly interacting particles (hadrons) are hereon studied for such a small period of time to allow their time-reversal invariant approximation.

2.2. Particles under pressure.

One of the simplest illustrations of Lemma II-3.7 is given by a particle in the center of a star, thus being under extreme pressures π from the surrounding hadronic medium in all radial directions (Figure 1).

By ignoring particle reactions in first approximation, the conditions here considered can be rudimentarily represented for very short periods of time by assuming that the function $\Gamma > 0$ in the exponent of the isotopic element (4) is linearly dependent on the pressure π , resulting in a

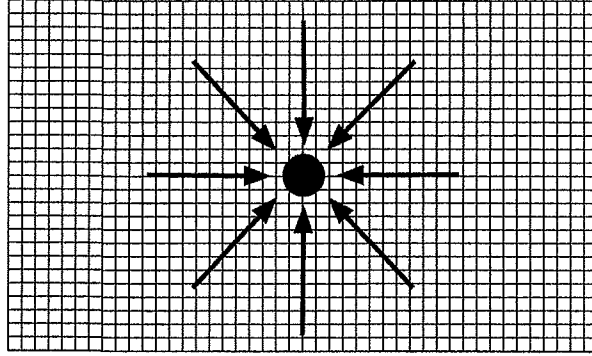


Figure 2: In this figure, we present a conceptual rendering of the central notion used for the verification of the EPR argument, namely, the pressure experienced by extended particles immersed in hyperdense media, such as a proton in the core of a star, which pressure evidently restricts uncertainties in favor of Einstein's determinism (Lemma II-3.7). Note that the notion of pressure does not exist in 20th century physics due to the approximation of particles as being point-like.

realization of the isotopic element of the simple type

$$\hat{T} = e^{-w\pi} \ll 1, \quad \hat{I} = e^{+w\pi} \gg 1, \quad (13)$$

where w is a positive constant.

Isodeterministic principle (11) for the considered particle is then given by

$$\Delta r \Delta p \approx \frac{1}{2} e^{-w\pi} \ll 1, \quad (14)$$

and tends to null values for diverging pressures.

The above example illustrates the consistency of isorenormalization (II-171) because a constant isotopic element verifies the isonormalization

$$\begin{aligned} \hat{\psi}(\hat{r}) | \hat{T} | \hat{\psi}(\hat{r}) &> \hat{I} = \\ &= T < \hat{\psi}(\hat{r}) | \hat{\psi}(\hat{r}) > \hat{I} = \\ &= < \hat{\psi}(\hat{r}) | \hat{\psi}(\hat{r}) >, \end{aligned} \quad (15)$$

but not necessarily other isorenormalizations.

Note that we have considered an individual extended particle immersed in a hadronic medium, rather than the bound state of extended particles in condition of mutual penetration that are studied in the next sections.

Consequently, isotopic element (14) represents a *subsidiary constraint on standard deviations* caused, as indicated, by the pressure of the surrounding hadronic medium on the particle considered.

It is easy to see that, since $\Gamma(\pi) > 0$, more complex functional dependence on the pressure π continue to verify Lemma II-3.7 as well as Corollary II-3.7.1.

2.3. Non-relativistic hadronic bound states.

As recalled in Paper I, the dynamical equations of quantum mechanics, such as the Schrödinger and the Dirac equation, are characterized by the conventional differential calculus which can solely be defined at a finite set of isolated points on a given representation space. Consequently, quantum mechanical bound states are solely possible for *point-like constituents* under linear, local and potential interactions (technically identified as *variationally self-adjoint interactions* [22]). This it is the case for the familiar Schrödinger equation for the bound state of two point-like particles of mass m with Coulomb potential $V(r)$ in a Euclidean space $E(r, \delta, I)$ formulated on a Hilbert space $\mathcal{H}$ over the field of complex numbers $\mathcal{C}$

$$\begin{aligned} i \frac{\partial}{\partial t} \psi(t, r) &= H \psi(t, r) = \left[\frac{\hbar^2}{m} \sum_k p_k p_k - V(r) \right] \psi(t, r) = \\ &= \left[\frac{1}{m} \sum_k (-i \hbar \partial_k) (-i \hbar \partial_k) - V(r) \right] \psi(t, r) = \\ &= \left[-\frac{\hbar^2}{m} \Delta_r - V(r) \right] \psi(t, r) = E \psi(t, r). \end{aligned} \quad (16)$$

By contrast, *hadronic bound states* are bound states of *extended particles* at mutual distances smaller or equal to the *hadronic horizon* (Figure I-13)

$$R = \frac{1}{b} \approx 10^{-13} \text{ cm}. \quad (17)$$

In such a region, bound states verify hadronic mechanics, can be represented with isomathematics (Section I-3) and isomechanics (Section I-4) for the case of time-reversal invariant bound states (such as the deuteron), or genomathematics and genomechanics for time irreversible bound states (such as all hadrons produced in particle physics laboratories).

By using the methods outlined in Paper I and in the preceding sections, when assumed to be stable in first approximation (thus being time reversal invariant), hadronic bound states are characterized by the following main features:

1) The bound states occur between *isoparticles*, namely, isoirreducible, isounitary isorepresentations of the isospinorial covering of the Galileo-Santilli isosymmetry $\hat{G}$ for non-relativistic treatments (Sections II-2.5.1 and II-3.9) or of the Lorentz-Poincaré-Santilli isosymmetry $\hat{P}$ for relativistic treatments (Sections II-2.5.11 and II-3.9) represented by hadronic mechanics including the verification of Einstein's determinism (Lemma II-3.7) and the absence of divergencies (Corollary II-3.7.1).

2) The representation of the extended character of the isoparticles is done with isoproduct (1) and isotopic element (4), resulting in iso-Schrödinger equations of type (I-80), while the deep mutual penetration of the wavepackets and/or charge distributions of isoparticles generates novel non-linear, non-local and non-potential interactions represented by the exponent of the isotopic element (4) and other means. Note that the latter interactions are short range, strongly attractive, charge independent, and non-Hamiltonian according to all studies conducted to date in the field, thus allowing an initial yet explicit and concrete realization of strong interactions [19].

3) By recalling that isosymmetries $\hat{G}$ and $\hat{P}$ are all *irregular* realizations of the Lie-Santilli isothory (Sections I-2.7 and II-2.5.4), a necessary condition for the invariance of hadronic dynamical equations under isosymmetries is that contact interactions *cannot* be derived via non-unitary transforms of quantum mechanical potentials, thus being *basically new interactions*. The physically equivalent property is that, as it is well known, *strong interaction cannot be derived via non-unitary or other known transformations of electromagnetic interactions*, thus confirming the necessary use of the irregular Lie-Santilli isothory and hadronic dynamical equations.

The notion of hadronic bound states was proposed, apparently for the first time, by Santilli in the 1978 Ref. [19] and was extensively studied hereafter in various works by various authors (see the 2001 monograph [30], the 2011 review [31], and papers quoted therein).

It is important to review the derivation of the basic non-relativistic and relativistic isoequations for hadronic bound states to show their apparent verification of the isodeterministic principle of Lemma 3.7.

The fundamental *non-relativistic, irregular isoequation of a time-reversal invariant hadronic bound state* of two isoparticles of mass m at mutual distances of the order of the hadronic horizon $R = 10^{-13} \text{ cm}$ in an iso-Euclidean isospace $\hat{E}(\hat{r}, \hat{\delta}, \hat{I})$ formulated on a Hilbert-Myung-Santilli isospace $\hat{H}$ over

the isofield of isocomplex isonumbers $\hat{\mathcal{C}}$ can be written

$$\begin{aligned} i \frac{\partial}{\partial \hat{t}} \hat{\psi}(\hat{t}, \hat{r}) &= \left[(\hat{1}/\hat{m}) \star \Sigma_k \hat{p}_k \star \hat{p}_k \pm \hat{V}(\hat{r}) - \hat{S}(\hat{\psi}) \right] \star \hat{\psi}(\hat{t}, \hat{r}) = \\ &= \left[(1/m) \Sigma_k \hat{p}_k \hat{T} \hat{p}_k \hat{T} \pm V(\hat{r}) - S(\hat{\psi}) \right] \hat{\psi}(\hat{t}, \hat{r}) = \\ \hat{E} \star \hat{\psi}(\hat{t}, \hat{r}) &= E \hat{\psi}(\hat{t}, \hat{r}), \end{aligned} \quad (18)$$

where $\hat{V}(\hat{r}) = V(\hat{r})\hat{I}$; $\hat{S}(\hat{\psi}) = S(\hat{\psi})\hat{I}$ represents the novel short range, strongly attractive force; the value $-\hat{V}(\hat{r})$ occurs for bound states with opposite charge (as it is the case for the synthesis of hadrons reviewed below); the value $+\hat{V}(\hat{r})$ occurs for isoparticles with the same charge (as occurring for valence electron bonds reviewed below); and one should note that isoeigenvalues can always be reduced to conventional eigenvalues, thus allowing experimental verifications.

Due to the large representational capabilities of isoequations (182), we use the following simplifying assumptions:

- 1) The isotime is equal to the conventional time, $\hat{t} = t\hat{I}_t = t$, $\hat{I}_t = 1$;
- 2) Being extremely small, the orbits of the isoparticles are assumed to be nearly constant circles, thus implying that the n_k characteristic quantities of then isotopic element (4) can be normalized to the sphere, $n_k = 1$, $k = 1, 2, 3$;
- 3) The isotopic element is assumed to be given by the exponential term of Eq. (4) with realizations of the non-linear, non-local and non-potential interactions of the type (Eq. (4.7), page 170 Ref. [30])

$$\hat{T} = e^{-\Gamma} = e^{-N\psi/\hat{\psi}} \approx 1 - N\psi/\hat{\psi}, \quad (19)$$

where ψ behaves like the solution of quantum equation (180),

$$\psi(r) \approx W_1 e^{-br}, \quad (20)$$

and $\hat{\psi}$ behaves like the solution of the hadronic equation expected to be of the type

$$\hat{\psi} \approx W_2 (1 - e^{\frac{(1-br)}{r}}), \quad (21)$$

where W_1 and W_2 are positive normalization constants.

We therefore have the following explicit form of the isotopic element

$$\hat{T} = e^{-W_2 \frac{e^{-br}}{(1-e^{-br})/r}} \approx 1 - W \frac{e^{-br}}{(1-e^{-br})/r}, \quad (22)$$

exhibiting the *Hulten potential*

$$V_{Hult} = W_2 \frac{e^{-br}}{1 - e^{-br}}, \quad (23)$$

directly in the exponent of the isotopic element, where W is normalization constants.

It should be recalled that Santilli suggested the use of the Hulten potential in the 1978 paper [19], Eq. (5.1.6), page 833, as an *initial yet explicit and concrete representation of strong interactions*.

Under the above assumptions, isotopic element (186) verifies the central condition for the validity of the isodeterministic principle as well as the rapid convergence of isoperturbative series inside the hadronic horizon (Lemma II-3.7 and Corollary II-3.7.1), in a way fully compatible with the validity of conventional uncertainties as well as divergence of perturbative series outside said horizon

$$|\hat{T}| \ll 1, \quad (24)$$

$$\lim_{r \gg R} \hat{T} = 1.$$

The use of the isolinear isomomentum (I-79), the projection of isodynamical equation (II-182) into our Euclidean space can be written in the form first derived in Eq. (5.1.9), page 833, Ref [19]

$$\begin{aligned} i \frac{\partial}{\partial t} \hat{\psi}(t, r) &= \left[\frac{1}{m} \Sigma_k \hat{p}_k \star \hat{p}_k \star \pm V \frac{e^2}{r} - W \frac{e^{-br}}{1 - e^{-br}} \right] \hat{\psi}(t, r) = \\ &= \left[\frac{1}{m} \Sigma_k (-i \hat{I} \partial_k) (-i \hat{I} \partial_k) \pm W_1 \frac{e^2}{r} - W_2 \frac{e^{-br}}{1 - e^{-br}} \right] \hat{\psi}(t, r) = \\ &= \left[-\frac{1}{\bar{m}} \Delta_r \pm V \frac{e^2}{r} - W \frac{e^{-br}}{1 - e^{-br}} \right] \hat{\psi}(t, r) = E \hat{\psi}(t, r), \end{aligned} \quad (25)$$

where: W_1 and W_2 are renormalization constants; E is the binding energy of the hadronic bound state; the total energy E_{tot} is given by

$$E_{tot} = E_1 + E_2 + -E, \quad (26)$$

where $E_k, k = 1, 2$ are the total energies of particles 1 and 2, respectively, and

$$\bar{m} = \frac{m}{\hat{I}^2}. \quad (27)$$

The nonrelativistic expression of the mean-life of the hadronic bound state can be derived via an isotopy of the quantum mechanical form, yielding the expression (Ref. [19], Eq. (5.1.13), page 835)

$$\tau^{-1} = 2\pi\lambda^2 |\hat{\psi}(0)|^2 \frac{\alpha^2 E_k}{\hbar}, \quad k = 1, 2. \quad (28)$$

The angular component of Eqs. (85) has been studied in detail in Ref. [33] via the isospherical isoharmonics.

The radial component of the non-relativistic, irregular, hadronic isoequation for the characterization of the total energy E_{tot} , mean life τ and charge radius R of a time-reversal invariant hadronic bound state can be written (Ref. [19], Eqs. (5.1.40, page 835)

$$\left[\frac{1}{r^2} \left(\frac{d}{dr} r^2 \frac{d}{dr} \right) + \bar{m}(E \pm W_1 e^{\frac{2}{r}} + W_2 \frac{e^{-br}}{1-e^{-br}}) \right] = 0,$$

$$E_{tot} = E_1 + E_2 - E, \quad \bar{m} = \frac{m}{f^2} \quad (29)$$

$$\tau^{-1} = 2\pi\lambda^2 |\hat{\psi}(0)|^2 \frac{\alpha^2 E_k}{\hbar}, \quad k = 1, 2,$$

$$R = b^{-1},$$

where the last two equations are subsidiary constraints on the first two.

The analytic solution of the above equations has been studied in detail in Section 5, Ref. [19], including boundary conditions requested by the subsidiary constraints, and we cannot review it here for brevity. We merely limit ourselves to the indication that said analytic solution was reduced to the solution of the following two algebraic equations on the parameters k_1 and k_2 (Ref. [19], Eq. (5.1.32), page 840)

$$k_1[k_1 - (k_2 - 1)^3] = \frac{1}{2c} E_{tot} R, \quad (30)$$

$$\frac{(k_2 - 1)^3}{k_1} = \frac{9 \times 10^6}{3\pi c} \frac{R}{\tau}.$$

It is now important to evaluate the numeric value of the binding energy E in Eq. (198). For this purpose, we recall that the Hulthen potential behaves like the Coulomb potential at short distances

$$V_{hp} \approx K \frac{1}{r}, \quad (31)$$

where K is a positive constant. Consequently, *the Hulthen potential can absorb the Coulomb potential resulting in a short range strongly attractive force irrespective of whether the Coulomb force is attractive or repulsive.*

The radial equation can then be reduced in first approximation to the expression (Eq. (5.1.14a), page 836, Ref. [19])

$$\left[\frac{1}{r^2} \left(\frac{d}{dr} r^2 \frac{d}{dr} \right) + \bar{m} \left(E + W_2 \frac{e^{-br}}{1 - e^{-br}} \right) \right] = 0, \quad (32)$$

and its energy spectrum results to be the typical *finite* spectrum of the Hul-ten potential

$$|E_{hp}| = \frac{1}{4R^2\bar{m}} \left(k_2 \frac{1}{n} - n \right)^2, \quad n = 1, 2, 3, \dots \quad (33)$$

An important feature of hadronic bound states is that, since we can ignore Coulomb interactions and solely assume *contact* interactions represented with the exponent of the isotopic element (4), *the value of the binding energy E is expected to be small or null*,

$$|E| = \frac{1}{4R^2\bar{m}} \left(k_2 \frac{1}{n} - n \right)^2 = 0. \quad (34)$$

This is due to the fact that *contact interactions do not carry potential energy* (this is classically the case for a balloon moved by winds in our atmosphere).

Therefore, *the Hul-ten potential for consistent hadronic bound states is expected to admit one single energy level, the ground state* since all possible excited states imply radial distances bigger than the hadronic horizon R , with consequential recovering of quantum mechanics.

In particular, property (34) is solely possible for the following values of the k -[arameters

$$k_1 > 0, \quad k_2 \geq 1. \quad (35)$$

The absence of a spectrum of energies was called in Section 5 of Ref. [19] the *hadronic suppression of quantum mechanical energy spectra* in order to differentiate the *quantum mechanical classification of hadrons into families* (which is characterized by energy spectra) from the *structure of individual hadrons of a given classification family* (which is expected to require different constituents for different particles due to the general difference of spontaneous decays with the lowest mode).

2.4. Relativistic hadronic bound states.

The relativistic counterpart of Eqs. (25) was identified, apparently for the first time, in Refs. [35] [36] and was formulated in the isoproduct of a real-valued iso-Minkowski isospace for orbital motions and a complex valued iso-Euclidean isospace for the hadronic spin

$$\hat{S}_{tot} = \hat{M}(\hat{x}, \hat{\eta}, \hat{I}_{orb}) \star \hat{R}(\hat{z}\hat{\delta}, \hat{I}_{spin}), \quad (36)$$

resulting in the following *irregular extension of the Dirac-Santilli isoequation (I-88)*,

$$\begin{aligned} & [\hat{\Omega}^{\mu\nu} \star \hat{\Gamma}_\mu \star \hat{\partial}_\nu + \hat{M} \star \hat{C} - \hat{V}_{hp}] \hat{\psi}(\hat{x}) = \\ & = (-i\hat{I}\hat{\eta}^{\mu\nu}\hat{\gamma}_\mu\partial_\nu + mC - V_{hp})\hat{\psi}(\hat{x}) = 0, \end{aligned} \quad (37)$$

where $\hat{S} = S\hat{I}_{orb}$ represents strong interactions, and the *Dirac-Santilli isogamma isomatrices* $\hat{\Gamma} = \hat{\gamma}\hat{I}$ are given by

$$\begin{aligned} \hat{\gamma}_k &= \frac{1}{n_k} \begin{pmatrix} 0 & \hat{\sigma}_k \\ -\hat{\sigma}_k & 0 \end{pmatrix}, \\ \hat{\gamma}_4 &= \frac{i}{n_4} \begin{pmatrix} I_{2 \times 2} & 0 \\ 0 & -I_{2 \times 2} \end{pmatrix}; \end{aligned} \quad (38)$$

where $\hat{\sigma}_k$ are the *irregular Pauli-Santilli isomatrices* studied in Section II-3.4 with the following *anti-isocommutation rules*

$$\begin{aligned} \{\hat{\gamma}_\mu, \hat{\gamma}_\nu\} &= \hat{\gamma}_\mu \hat{I} \hat{\gamma}_\nu + \hat{\gamma}_\nu \hat{I} \hat{\gamma}_\mu = \\ &= 2\hat{\eta}_{\mu\nu}. \end{aligned} \quad (39)$$

where $\hat{\eta}$ is the isometric of the orbital iso-Minkowskian isospace.

2.5. Einstein's determinism in the structure of mesons.

2.5.1. Insufficiencies of quark conjectures. While the classification of hadrons into families has received a rather large consensus since its initiation by M. Gell-Mann in the 1960's [75], the conjecture that the hypothetical quarks are the actual physical constituents of hadrons has been controversial since its inception. These problematic aspects were reviewed in detail in the 1979 paper [20] (written at the Department of Mathematics of Harvard University under DOE support), and can be summarized as follows:

1) The quantum mechanical classification of point-like particles permitted by the $SU(3)$ model and, more recently, by the standard model, has indeed achieved a satisfactory *classification* of hadrons into families.

2) Quarks are purely mathematical representations of a purely mathematical unitary symmetry defined on a purely mathematical complex-valued internal space and, as such, quarks cannot be the actual physical constituents of hadrons for numerous insufficiencies or sheer inconsistencies, such as:

2A) By recalling that quarks have to be point-like as a necessary condition to maintain the validity of quantum mechanics in the interior of hadrons, the ensuing conception of the hyperdense hadrons as ideal spheres with point-particles in their interior is not realistic;

2B) Quarks cannot be physical particles in our spacetime, and the same holds for their masses, because they cannot be defined as unitary irreducible representations of the Lorentz-Poincaré symmetry (Section 3.9);

2C) Quarks cannot be rigorously confined inside hadrons (i.e., confined with a rigorously proved, identically null probability of tunnel effect) due to the uncertainty principle;

2D) Quarks have not been directly detected under collisions at the extremely high energies achieved at CERN and at other particle physics laboratories;

2E) The wavepackets of all particles are of the same order of magnitude of the size of all hadrons. Hence, the hyperdense character of hadrons is due to the total mutual penetration of the wavepackets of their constituents, resulting in non-linear, non-local and non-Hamiltonian internal interactions under which the SU(3) and other symmetries cannot be consistently defined (Sections 3, 4).

3) History has thought that the study of atoms (as well as of other natural systems) required *two* different yet compatible models, one for the *classification* of atoms into family, and a different model for the *structure* of each atom of a given classification family. Particularly significant is the fact that the classification of atoms could be achieved via the use of *pre-existing mathematics*, while the structure of atoms required *new mathematics*, such as the Hilbert spaces, that are unnecessary for the classification of atoms.

In order to resolve the insufficiencies of the conjecture that quarks are physical particles, Santilli [20] suggested to follow the teaching of the history of science, and study hadrons via *two* different models, the standard model for the classification of hadrons and a different, yet compatible, model for the structure of individual hadrons of a given classification multiplet.

In particular, the classification of hadrons can be effectively done via quantum mechanics because individual hadrons can be well approximated as being point-like particles in vacuum. By contrast, the structure of hadrons requires a necessary "completion" of quantum mechanics into a covering theory suggested beginning with the title of Ref. [20] in view of the unavoidable, internal, non-linear, non-local, and non-potential interactions.

2.5.2. Hadronic structure model of mesons. A primary aim of papers [18] [19] [20] and monographs [22] [23] of 1978-1979 was the "completion" of quantum mechanics (qm) into the covering hadronic mechanics (hm) for the specific intent of achieving a representation of *all* characteristics of mesons as hadronic bound states of actual, massive, physical particles

Table 1. The Technical Origin of Some of the Controversies in Hadron Physics*

$\{\pi^{\pm} : 1^{-}, 0^{-}, +, 133, 0.8 \times 10^{-23}, \sim 1 F\}, \{\pi^0 : 1^{-}, 0^{-}, +, 139, 2.6 \times 10^{-23}, \sim 1 F\}$ $\{K^{\pm} : \frac{1}{2}, 0^{-}, +, 439, 1.2 \times 10^{-23}, \sim 1 F\}, \{K_L^0 : \frac{1}{2}, 0^{-}, +, 498, 0.9 \times 10^{-23}, \sim 1 F\}$ $\{K_S^0 : \frac{1}{2}, 0^{-}, +, 498, 5.1 \times 10^{-23}, \sim 1 F\}, \{\eta : 0^{-}, 0^{-}, +, 549, \Gamma = 0.8 \text{ keV}, \sim 1 F\}$	
$\pi^{\pm} \rightarrow \gamma\gamma, 98.3\%$	$K^{\pm} \rightarrow \pi^0\pi^0\gamma, 3.7 \times 10^{-4}$
$\pi^{\pm} \rightarrow \gamma e^{\pm} e^{\mp}, 1.15\%$	$K^{\pm} \rightarrow \pi e^{\pm} e^{\mp}, 2.6 \times 10^{-7}$
$\pi^0 \rightarrow \gamma\gamma, < 5 \times 10^{-8}$	$K^0 \rightarrow \pi^0 e^+ e^-, < 1.5 \times 10^{-4}$
$\pi^0 \rightarrow e^+ e^- e^+ e^-, 3.3 \times 10^{-8}$	$K^0 \rightarrow \pi^0 \mu^+ \mu^-, < 2.4 \times 10^{-4}$
$\pi^0 \rightarrow \gamma\gamma\gamma, < 6 \times 10^{-4}$	$K^0 \rightarrow \pi\gamma, < 3.5 \times 10^{-4}$
$\pi^0 \rightarrow e^+ e^-, < 2 \times 10^{-8}$	$K^0 \rightarrow \pi\gamma\gamma, < 3.0 \times 10^{-4}$
$\pi^{\pm} \rightarrow \mu\nu, 100\%$	$K^0 \rightarrow \pi\nu, < 0.6 \times 10^{-4}$
$\pi^{\pm} \rightarrow e\nu, 1.2 \times 10^{-4}$	$K^0 \rightarrow \pi\gamma, < 4 \times 10^{-4}$
$\pi^{\pm} \rightarrow \mu\gamma, 1.2 \times 10^{-4}$	$K^0 \rightarrow e^+ e^- \mu^{\pm}, < 2.8 \times 10^{-6}$
$\pi^{\pm} \rightarrow \pi^0 e^{\pm}, 1.02 \times 10^{-4}$	$K^0 \rightarrow e^+ e^- \mu^{\pm}, < 1.4 \times 10^{-6}$
$\pi^{\pm} \rightarrow e\nu, 3 \times 10^{-4}$	$K^0 \rightarrow \mu\nu, < 6 \times 10^{-4}$
$\pi^{\pm} \rightarrow e^+ e^- e^{\mp}, < 3.4 \times 10^{-4}$	$K_L^0 \rightarrow \pi^+ \pi^-, 68.7\%$
$K^{\pm} \rightarrow \mu\nu, 63.6\%$	$K_L^0 \rightarrow \pi^0 \pi^0, 31.3\%$
$K^{\pm} \rightarrow \pi^0 \pi^{\pm}, 21.05\%$	$K_L^0 \rightarrow \mu^+ \mu^-, < 3.2 \times 10^{-7}$
$K^{\pm} \rightarrow \pi^{\pm} \pi^0, 5.6\%$	$K_L^0 \rightarrow e^+ e^-, < 3.4 \times 10^{-4}$
$K^{\pm} \rightarrow \pi^{\pm} \pi^{\pm}, 1.7\%$	$K_L^0 \rightarrow \pi^0 \pi^0 \gamma, 2.0 \times 10^{-4}$
$K^{\pm} \rightarrow \mu^{\pm} \pi^0, 3.2\%$	$K_L^0 \rightarrow \gamma\gamma, < 0.4 \times 10^{-4}$
$K^{\pm} \rightarrow e^{\pm} \pi^0, 4.8\%$	$K_L^0 \rightarrow \pi^0 \pi^0 \pi^0, 21.4\%$
$K^{\pm} \rightarrow \mu\nu, 5.8 \times 10^{-4}$	$K_L^0 \rightarrow \pi^0 \pi^0 \pi^{\pm}, 12.2\%$
$K^0 \rightarrow \pi^0 \pi^0 \pi^0, 1.8 \times 10^{-4}$	$K_L^0 \rightarrow \pi\mu\nu, 27.1\%$
$K^0 \rightarrow \pi^+ \pi^- \pi^0, 3.7 \times 10^{-4}$	$K_L^0 \rightarrow \pi e\nu, 39.0\%$
$K^0 \rightarrow \pi^0 \pi^0 \pi^{\pm}, < 5 \times 10^{-4}$	$K_L^0 \rightarrow \pi e\nu\gamma, 1.3\%$
$K^0 \rightarrow \pi^0 \pi^{\pm} \pi^{\mp}, 0.9 \times 10^{-4}$	$K_L^0 \rightarrow \pi^+ \pi^-, 0.2\%$
$K^0 \rightarrow \pi^{\pm} \pi^0 \pi^{\mp}, < 3.0 \times 10^{-4}$	$K_L^0 \rightarrow \pi^0 \pi^0, 0.09\%$
$K^0 \rightarrow e\nu, 1.5 \times 10^{-4}$	$K_L^0 \rightarrow \pi^0 \pi^0 \gamma, 6.0 \times 10^{-4}$
$K^0 \rightarrow e\nu\gamma, 1.6 \times 10^{-4}$	$\eta \rightarrow \gamma\gamma, 38.0\%$
$K^0 \rightarrow \pi^0 \pi^0 \gamma, 2.7 \times 10^{-4}$	$\eta \rightarrow \pi^0 \gamma\gamma, 3.1\%$
$K^0 \rightarrow \pi^0 \pi^0 \pi^0 \gamma, 1 \times 10^{-4}$	$\eta \rightarrow 3\pi^0, 29.9\%$
$K^0 \rightarrow \mu^{\pm} \pi^{\mp} \gamma, < 6 \times 10^{-4}$	$\eta \rightarrow \pi^+ \pi^- \pi^0, 23.6\%$
$K_L^0 \rightarrow \pi^0 \gamma\gamma, < 2.4 \times 10^{-4}$	$\eta \rightarrow \pi^+ \pi^- \gamma, 4.9\%$
$K_L^0 \rightarrow \gamma\gamma, 4.9 \times 10^{-4}$	$\eta \rightarrow e^+ e^- \gamma, 0.5\%$
$K_L^0 \rightarrow e\nu, < 2.0 \times 10^{-4}$	$\eta \rightarrow \mu^+ \mu^- e^{\mp}, < 0.04\%$
$K_L^0 \rightarrow \mu^+ \mu^-, 1.0 \times 10^{-4}$	$\eta \rightarrow \pi^+ \pi^-, < 0.15\%$
$K_L^0 \rightarrow \pi^+ \mu^- \gamma, < 7.8 \times 10^{-4}$	$\eta \rightarrow \pi^+ \pi^- e^+ e^-, 0.1\%$
$K_L^0 \rightarrow \mu^+ \mu^- e^{\mp}, < 5.7 \times 10^{-4}$	$\eta \rightarrow \pi^+ \pi^- \pi^0 \gamma, < 6 \times 10^{-4}$
$K_L^0 \rightarrow e^+ e^-, < 2.0 \times 10^{-4}$	$\eta \rightarrow \pi^+ \pi^- \gamma\gamma, < 0.2\%$
$K_L^0 \rightarrow e^+ e^- \gamma, < 2.8 \times 10^{-4}$	$\eta \rightarrow \mu^+ \mu^-, 2.2 \times 10^{-3}$
$K_L^0 \rightarrow \pi^0 \pi^0 e^+ e^-, < 2.2 \times 10^{-4}$	

* In this table we have listed most of the experimental data on the octet of light mesons from the particle data^[19] with only one addition, the inclusion of the (approximate) charge radius of the particles. The data are divided into two groups: those for total

Figure 3: A reproduction of Table 1, page 429, Ref. [20] used to identify the physical constituents of mesons in the massive particles produced free in the spontaneous decays, generally those with the lowest mode. The "completion" of quantum mechanics into hadronic mechanics then becomes recommendable for any quantitative treatment of the indicated structure model.

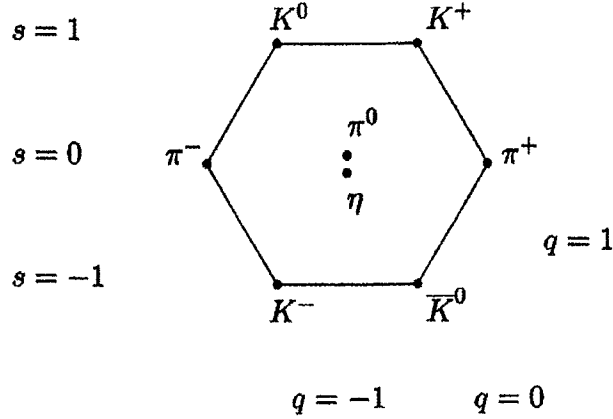


Figure 4: An illustration of the compatibility between the hadronic structure model of mesons, Eqs. (40)-(45), with physical constituents and the conventional $SU(3)$ classification. This compatibility is achieved via the identification of the isounits or isotopic elements per each meson, Eq. (46), and their use as the hyperunit of the $\tilde{SU}(3)$ hypersymmetry in view of its local isomorphism to the conventional symmetry.

produced free in the spontaneous decays, generally those with the lowest mode illustrated in Table 1, page 429, Ref. [20] (reproduced in Figure 2).

The above view suggested the following new structure models for the octet of mesons:

$$\pi^0 = (\tilde{e}^-, \tilde{e}^+)_{hm}, \quad (40)$$

$$\pi^\pm = (\tilde{\pi}^0, \tilde{e}^\pm)_{hm}, \quad (41)$$

$$K^0 = (\tilde{\pi}^0, \tilde{\pi}^0)_{hm}, \quad (42)$$

$$K^\pm = (\tilde{\pi}^0, \tilde{\pi}^\pm)_{hm}, \quad (43)$$

$$K_S = (\tilde{\pi}^-, \tilde{\pi}^+)_{hm}, \quad (44)$$

$$K_L = (\tilde{K}^-, \tilde{\pi}^+)_{hm}. \quad (45)$$

In the above models, the role of positrons as physical constituents of mesons provides the first known numerical representation of their very short meanlives, while all other characteristics are numerically represented via the hadronic bound states of Sections 2.3 and 2.4.

Structure models (40) - (45) are incompatible with quantum mechanics because the rest energy of all particles are *bigger* than the sum of the rest energies of the constituents (thus requiring *positive binding energies*,

with ensuing *mass excesses* that are anathema for quantum mechanics as discussed in Section I.3), and for other reasons.

Necessary conditions to prevent misrepresentations is that models (40) - (45) are treated with hadronic mechanics and their constituents are *isoparticles and anti-isoparticles* (Section II-3.9) hereon denoted with an upper tilde.

Therefore, the elementary constituents of the π^0 in Eq.(40) are *one isoelectron $\tilde{e}^-$ and one isopositron $\tilde{e}^+$* (called *eletons* in Ref. [19]); the constituents of the $\pi^\pm$, Eq. (51) are *one iso-meson $\tilde{\pi}^0$ and one isoelectron or isopositron*; etc.

Following their emission in the spontaneous via isotunnel effects (i.e., tunnel effects in the iso-Hilbert isospace), the isoconstituents assume conventional quantum mechanical characteristics plus possible secondary effects with the emissions of massless particles.

Note that *all models (40)-(45) either are or can be reduced to two-body hadronic bound states*, thus admitting analytic solutions in their representation via Eqs. (29). Note also that models (40)-(45) have a kind of bootstrap structures, since a given meson appears in the structure of heavier mesons.

Note additionally that *models (40)-(45) imply the increase of the number of "elementary" constituents with the increase of the rest energy*. In fact, the π^0 has only two elementary constituents while K_L has eight elementary constituents.

Note finally that the role of isopositrons as actual physical constituents of mesons provides the *only* known mechanism via particle-antiparticle annihilation for the *quantitative representation* of the very small mean-lives of mesons, by keeping in mind that the rest energy of electron-positron bound states is positive in our world and negative in the antimatter world (Section 2.5.3).

Since different structure models are characterized by numerically different isounits, the compatibility of the above hadronic structure model of mesons with their classification is readily achieved at the higher level of the *hyperstructural branch of hadronic mechanics* [33] [37], with the following total multi-valued *hyperunit*

$$\hat{I}_{tot} = \{\hat{I}_{\pi^0}, \hat{I}_{\pi^\pm}, \hat{I}_{K^0}, \hat{I}_{K^\pm}, \hat{I}_{K_S}, \hat{I}_{K_L}\}. \quad (46)$$

2.5.3. Positive energy of particle-antiparticle bound states. The *positive* total energy of particle-antiparticle hadronic bound states has been studied in detail in monograph [38] (see, e.g., Section 2.3.14, page 131), and in various additional works, including all historical references.

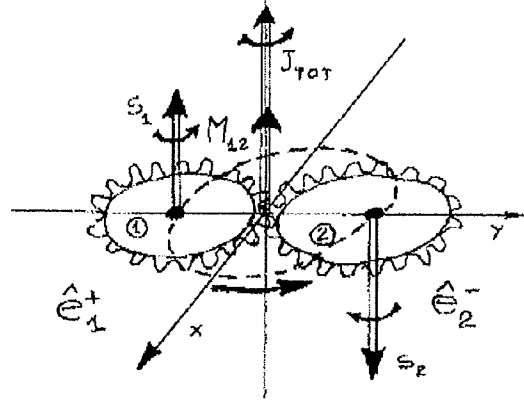


Figure 5: A reproduction of "gear model" used in Section 5, page 852, Ref. [19] to illustrate the strongly attractive character of contact non-Hamiltonian forces when the constituents are in singlet coupling and their strongly repulsive character when in triplet couplings.

We cannot possibly review here the underlying *isodual theory of antimatter*, but an indication of the following main notions appears recommendable to avoid basic misconceptions.

Recall that *negative energies violate causality*, for which reason P. A. M. Dirac was forced to work out his "hole theory."

Santilli resolved the problem of causality by referring all physical quantities, thus including energies, to *isodual units defined on isodual fields with a 'negative' unit*.

Hence, the terms "negative energy of antiparticles" have no meaning in the field here considered, because the correct statement is "negative energies of antiparticles measured to negative units of energy."

Within the above setting, Ref. [38], Eqs. (2.3.77), page 131, shows that *the total energy of a particle-antiparticle bound state is positive when measured in our world (with positive units) and negative when measured in the antimatter word (with negative units)*.

The above property is truly fundamental for a consistent quantitative representation of all characteristics of unstable particles.

In fact, the presence of antiparticles in the structure of mesons and (unstable) baryons, first proposed by Santilli in 1978 [19], appears to be the most plausible origin of the extreme instability of mesons and (unstable) baryons with mean lives such as $\tau = 10^{-16}$ s or less, with no equally effec-

tive or plausible alternative known to this day.

2.5.4. Hadronic structure model of the π^0 meson. The characteristics of the π^0 meson are:

1. Rest energy $E \approx 134.96 MeV$,
- 2) Mean-life $\tau = 0.828 \times 10^{-16} s$,
- 3) Charge radius $R = 10^{-13} cm$,
- 4) Null charge and spin.
- 5) Null electric and magnetic moments,
- 6) Negative parity; and
- 7) Primary decay

$$\pi^0 \rightarrow \gamma + \gamma, \quad 98.85 \%. \quad (47)$$

The above characteristics are *all* numerically represented by hadronic bound state (29)) as a "compressed" form of the positronium (Pos), resulting in hadronic bound state (40) of one isoelectron and one isopositron (Ref. [19], Section 5, page 828 on)

$$Pos = (e^-, e^+)_{qm} \rightarrow \pi^0 = (e^-, e^+)_{hm}, \quad (48)$$

with hadronic structure equations

$$\left[\frac{1}{r^2} \left(\frac{d}{dr} r^2 \frac{d}{dr} \right) + \bar{m} (E \pm W_1 e^{\frac{2}{r}} + W_2 \frac{e^{-br}}{1 - e^{-br}}) \right] = 0,$$

$$E_{tot} = E_{\bar{e}^-} + E_{\bar{e}^+} - E = 135 MeV \quad \bar{m} = \frac{m}{f^2} \quad (49)$$

$$\tau^{-1} = 2\pi\lambda^2 |\hat{\psi}(0)|^2 \frac{\alpha^2 E_{\bar{e}}}{\hbar} = 10^{-16} s$$

$$R = b^{-1} = 10^{-13} cm = 1 fm,$$

and numeric solution

$$k_1 = 0.34, \quad k_2 = 1 + 4.27 \times 10^{-2}, \quad (50)$$

verifying conditions (35) as expected.

In appraising representation (48), the reader should keep in mind the assumption of Section 2.3 of '*constrained, nearly constant and circular orbits of the constituents*' under which the isounit can be considered independent from local coordinates.

The above results confirm all expectations indicated in preceding sections, namely,

- 1) Hadronic spectrum (33) admits one and only one energy level, the π^0 , since all excited states are those of the positronium;

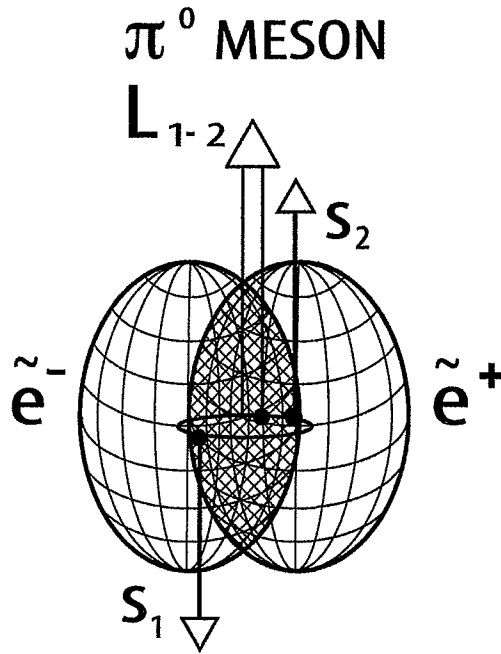


Figure 6: A conceptual rendering of the structure model of the π^0 meson as a hadronic bound state of one isoelectron $\tilde{e}^-$ and one isopositron $\tilde{e}^+$ in singlet coupling with unmutated spins $s_1 = -s_2 = 1/2$ and orbital hadronic momentum in the ground state $L_{1-2} = 0$. The dashed area represents the contact, non-linear, non-local and non-potential interactions responsible for the bound state that, being not representable with a Hamiltonian H , are representable via the isotopic element $\hat{T}$ in the iso-Schrödinger equation $H \star \hat{\psi} = H\hat{T}\hat{\psi} = E\hat{\psi}$.

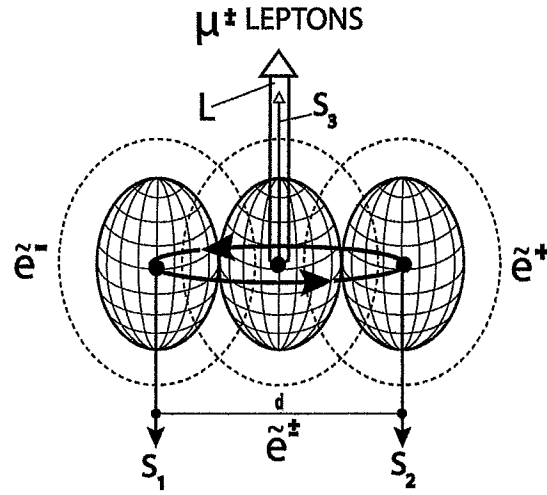


Figure 7: To achieve compatibility with the standard model, it is generally believed that muons are elementary particles in contrast with the experimental evidence that they are naturally unstable with various decay modes (51) that strongly suggest a composite structure. In this figure, we present a conceptual rendering of the structure of the $\mu^\pm$ leptons predicted by hadronic mechanics as a three-body of elementary isoconstituents identified in the muon decay with the lowest mode. The weak interaction character of the particles is derived from a weak hadronic bound state essentially given by the overlapping of the classical radius of the constituents indicated by dashed ellipses. By recalling that all three constituents have a point-like charge, the dimension d of the charge radius of the two peripheral constituents is purely nominal.

2) The presence of the antiparticle $\tilde{e}^+$ in the π^0 structure (40) explains its very short mean-life as well as its main decay (47);

3) Isodeterministic conditions (11) are verified by model (40), as a result of which standard deviations Δr and Δp have individual values smaller than one (Lemma II-3.7) and the isoperturbation series have no divergences (Corollary II-3.7.1).

2.5.5. Hadronic structure model of the remaining mesons. Recall the main characteristics of the muons $\mu^\pm$ are:

- 1) Rest energy 105,658 MeV;
- 2) Charge radius $R = 10^{-13}$ cm;
- 3) Mean-life $\tau = 2.19703 \times 10^{-6}$ s;
- 4) Spin 1/2 and elementary charge;
- 5) Spontaneous decay

$$\begin{aligned}\mu^\pm &\rightarrow e^- + \nu + \bar{\nu}, \\ \mu^\pm &\rightarrow e^- + \gamma, \\ \mu^\pm &\rightarrow e^- + 2\gamma, \\ \mu^\pm &\rightarrow e^- + e^\pm + e^\pm.\end{aligned}\tag{51}$$

For the intent of achieving compatibility with the standard model, the $\mu^\pm$ leptons are considered to be "elementary particles." Santilli [19] cannot accept such a view because in contrast with the experimental evidence that *the muons are naturally unstable particles with decays (51)*.

Therefore, Ref. [19], Section 5, proposed the structure model of the muons with the physical constituents identified by the decay mode (51) with the smallest mode $< 10 \times 10^{-12}$ (Figure 5)

$$\mu^\pm = (e^-, e^\pm, e^\pm)_{hm}.\tag{52}$$

The birth of weak interactions was suggested as being due to a weak form of wave-overlapping essentially that of the classical size of the constituents

$$R_{e^\pm} = 2.28 \times 10^{-13} \text{ cm},\tag{53}$$

which said weak contact interactions can be quantitatively represented by hadronic mechanics via an isotopic element $\hat{T}$ with values close to 1.

A main aspect is that there is the birth of strong interactions in model (52) because we do not have a wave-overlapping inside the hadronic horizon, as it is the case for the π^0 .

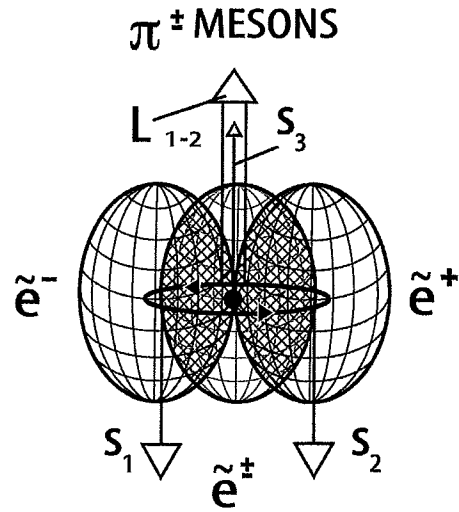


Figure 8: A conceptual rendering of the structure of the $\pi^\pm$ mesons as a three-body hadronic bound states (57) of elementary isoparticles which can be interpreted as a “compressed $\mu^\pm$ lepton” (Figure 6). Note that individual pairs of isoconstituents are coupled in singlet as necessary for consistency. Note also that the isoconstituents have a point-like charge structure. Therefore, the charge diameter of the structure is given by the distance between the centers of the two peripheral isoconstituents.

It is easy to see that model (52) can represent the muon spin 1/2 and decay (51) with the lowest mode as a tunnel effect of the constituents.

The remaining characteristics can be represented via a direct analytic solution of model (52) as a restricted three-body problem.

Ref. [19] suggested the approximation of the model into a two-body structure with weakly bounded constituents and the structure equations

$$\left[\frac{1}{r^2} \left(\frac{d}{dr} r^2 \frac{d}{dr} \right) + \tilde{m}(E \pm W_1 e^{\frac{2}{r}} + W_2 \frac{e^{-br}}{1-e^{-br}}) \right] = 0,$$

$$E_{tot} = E_{100 \text{ MeV}} + E_{e^\pm} - E, = 105 \text{ MeV} \quad \tilde{m} = \frac{m}{I^2} \quad (54)$$

$$\tau^{-1} = 2\pi\lambda^2 |\hat{\psi}(0)|^2 \frac{\alpha^2 E_e}{\hbar} = 10^{-6} \text{ s}$$

$$R = b^{-1} = 10^{-13} \text{ cm},$$

admitting the following values for the the k -parameters

$$k_1 = 0.93, \quad k_2 = 1 + 8.47 \times 10^{-2}, \quad (55)$$

that also verify conditions (35).

The main characteristics of the $\pi^\pm$ mesons are:

- 1) Rest energy 139.570 MeV;
- 2) Charge radius $R = 10^{-15} \text{ cm}$;
- 3) Mean-life $\tau = 2.603 \times 10^{-8} \text{ s}$;
- 4) Spin $J = 0$ and elementary charge;
- 5) Decay with lowest mode $< 3.2 \times 10^{-9}$

$$\pi^\pm \rightarrow e^- + e^\pm + e^+ + \nu\pi^\pm \quad (56)$$

The above characteristics are all represented by the hadronic structure model of the $\pi^\pm$ mesons first proposed in Section 5, Ref. [19]

$$\pi^\pm = (\tilde{e}^-, \tilde{e}^\pm, \tilde{e}^+)_{hm} \approx (\tilde{\pi}^0, \tilde{e}^\pm)_{hm}, \quad (57)$$

for which, unlike the case of the muons, the constituents are elementary isoparticles in condition of deep mutual penetration inside the hadronic horizon (Figure 7).

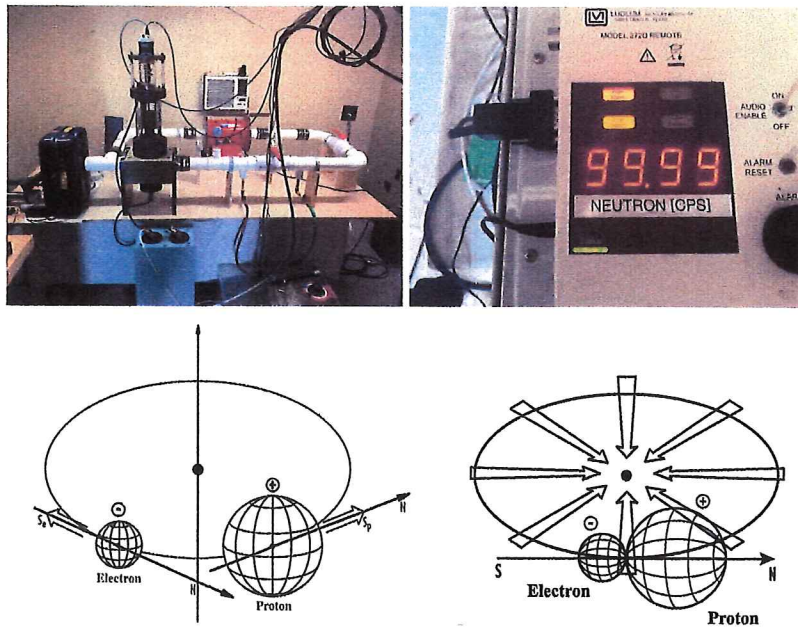


Figure 9: In this figure we show: 1) At the top left a picture of the Directional Neutron Source (DNS) developed by Santilli at Thunder Energies Corporation, now Hadronic Technologies Corporation [39] which synthesizes on demand neutral and negatively charged hadrons from a hydrogen gas with preferred directionality, energy and flux; 2) At the top right a typical case of neutron CPS; 3) At the bottom left a conceptual rendering of the ionization of the hydrogen gas by the electric arc during its activation and the proper axial alignment of the proton and electron with opposite charge and magnetic polarities; and 4) At the bottom right a conceptual rendering of the “compression” of the electron inside the proton by the electric arc during its disconnection.

Studies on the EPR argument, I: Basic methods

The rest energy, mean-life and charge radius of the $\pi^\pm$ are readily represented by model (57) with the hadronic structure equations

$$\left[\frac{1}{r^2} \left(\frac{d}{dr} r^2 \frac{d}{dr} \right) + \bar{m} (E \pm W_1 e^{\frac{2}{r}} + W_2 \frac{e^{-br}}{1-e^{-br}}) \right] = 0,$$

$$E_{tot} = E_{\pi^\pm} + E_{\bar{e}^\pm} - E = 139 \text{ MeV}, \quad \bar{m} = \frac{m}{I^2}$$

$$\tau^{-1} = 2\pi\lambda^2 |\hat{\psi}(0)|^2 \frac{\alpha^2 E_{\pi^\pm}}{\hbar} = 2.603 \times 10^{-8} \text{ s},$$

$$R = b^{-1} = 10^{-13} \text{ cm},$$

admitting solutions for the k -parameters

$$k_1 = 0.34, \quad k_2 = 1 + 3.67 \times 10^{-3}, \quad (59)$$

that again verify conditions (35).

The main recent advance since the above 1978 proposal [19] is the achievement of a consistent representation of the total angular momentum $J = 0$ of the $\pi^\pm$ meson thanks to the irregular $SU(2)$ isosymmetry (Section 3.5). In essence, when the three isoparticles are all compressed inside the hadronic horizon, the orbital motion of the two peripheral isoconstituents L_{1-2} is constrained, for stability, to be equal to the spin $S_{\bar{e}^\pm}$ of the central isoparticle, thus having value $L_{1-2} = S_{tildede^\pm} = 1/2$ with total angular momentum (Sections II-3.4 and II-3.5, Section II-3.5.3 in particular).

$$J_{tot} = s_1 + s_2 + s_3 + L_{1-2} = -1/2 + 1/2 - 1/2 - 1/2 = 0. \quad (60)$$

Note that model (57) can be interpreted as a form of "compressed muons" (52) (see Figures 6 and 7). The orbital motion of the peripheral electrons is unrestricted in model (52), yielding a total angular momentum $1/2$. By contrast, the peripheral isoelectrons and isopositron in model (57) are *constrained to orbit inside the central isoelectron or isopositron*, thus being forced to have the orbital value equal to the spin of the isoparticle. In fact, orbital values different than $1/2$ would imply extreme resistive forces with ensuing excessive instabilities.

In Santilli's view, an important aspect of model (57) is its apparent verification of the isodeterminism of Lemma II-3.7 as well as of the lack of divergencies in the isoperturbation series (Corollary II-3.7.1) due to the small value of the isotopic element assured by values (59).

Interested readers can verify that values (35) remain verified under the use of more accurate experimental values for the characteristics of muons and mesons.

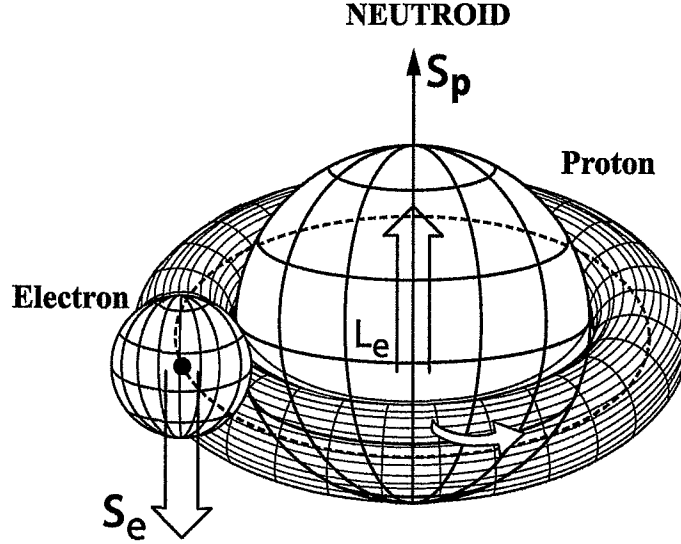


Figure 10: A conceptual rendering of the structure of a particle called the neutroid $\tilde{n} = (e^-, p^+)_{hm}$, Eq. (61), with spin $J = 1$, charge radius $R = 10^{-13}$ cm, mass essentially that of the proton and mean-life of the order of 5 s which has been undetected in all experiments to date [I-90]-[I-95] by neutron detectors, yet has caused nuclear transmutations typical of neutron irradiation. The bound state is created by the extremely strong, $r - p$ Coulomb attraction which is of the order of 10^{24} N plus weak contact interactions implying ignorable mutations of the electron and the proton. The $e - p$ singlet coupling and the orbital motion in the ground state imply the spin $J = 1$ explaining the lack of detection of neutroids by neutron detectors. Note that the neutroid is impossible for quantum mechanics.

It is an instructive exercise for the interested reader to work out the hadronic structure models of the remaining mesons, Eqs. (42)-(45) showing the increase of the k_1 value and the decrease of the k_2 value with the increase of the rest energy (see Section 6.2 of Ref. [31] for an independent review).

2.6. Einstein's determinism in the structure of baryons.

2.6.1. Structure of the neutroid. Stars initiate their lives as being composed of hydrogen; they grow in time via the accretion of interstellar-intergalactic hydrogen; and eventually reach such pressures and tempera-

tures in their core to synthesize the neutron as a "compressed" hydrogen atom according to H. Rutherford [40].

A main aspect of the studies herein considered is that, in Santilli's view [20], the synthesis of the neutron from the hydrogen in the core of stars is one of the best illustrations of the validity of the EPR argument [1] because said synthesis cannot be represented with quantum mechanics for various technical reasons reviewed in Section I-1, in denial of the experimental evidence that, *having opposite charges, the proton and the electron at mutual distances of 10^{-13} cm are attracted to each other with a Coulomb force of about 230 N (See Eq. (107) for its calculation). Such an attractive force is so big for particle standards to dismiss as inapplicable any theory unable of producing a bound state under such an enormous attractive force.*

Consequently, Santilli dedicated decades of mathematical, theoretical, experimental and industrial research on the neutron synthesis due to its truly fundamental character for all quantitative sciences (see Refs. [I-85] to [I-95] and independent reviews [31] [41]).

Besides a number of preceding attempts without neutron detection, the first indirect experimental detection of the synthesis of the neutron from the hydrogen was initiated by in the 1960's by Don Carlo Borghi [76], confirmed in subsequent tests [I-90] - [I-95], and routinely verifiable with the Directional Neutron Source developed by Hadronic Technologies Corporation [39] (Figure 8), indicate the existence of an unstable, neutral, intermediate state called *neutroid* (denoted with the symbol $\tilde{n}$) which is unidentified by neutron detectors, while causing nuclear transmutations typically triggered by neutron irradiation.

Hadronic mechanics suggests the following representation of the structure of the neutroid (Figure 9)

$$\tilde{n} = (\tilde{e}_\downarrow^-, p_\uparrow^+)_{hm}, \quad (61)$$

consisting of an isoelectron $\tilde{e}^-$ with spin $s_1 = 1/2$ in singlet contact coupling with a standard proton p^+ with spin $s_2 = 1/2$ and orbital motion $L_{1-2} = 0$ in the ground state.

These assumptions suggest the following predicted features of the neutroid:

- 1) Rest energy estimated to be about 940 MeV,
- 2) Mean-life estimated to be of at least $\tau = 5$ s,
- 3) Charge radius estimated to be of about $R \approx 10^{-13}$ cm,
- 4) Spin predicted to be $J = 0$, and
- 5) Spontaneous decay

$$\tilde{n} \rightarrow e^- + p \quad (62)$$

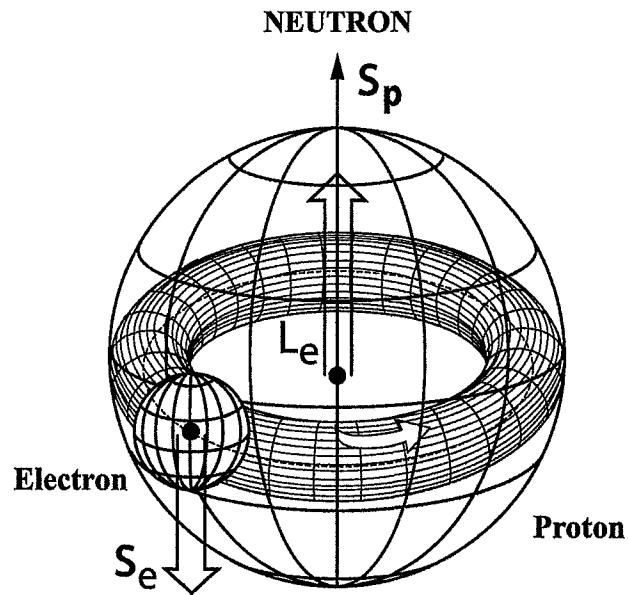


Figure 11: A conceptual rendering of the structure of the neutron as “compressed” hydrogen atoms in the core of stars according to H. Rutherford [40]. When the proton is represented as an extended particle, there is the emergence of a constrained orbital motion of the electron inside the proton verifying the isodeterminism of Lemma 3.7, which allows the first known exact representation of all the characteristics of the neutron at the nonrelativistic [42] and relativistic [36] levels without need for the neutrino hypothesis.

The above features can be represented with hadronic bound state (29) specialized to the indicated values

$$\left[\frac{1}{r^2} \left(\frac{d}{dr} r^2 \frac{d}{dr} \right) + \tilde{m} (E \pm W_1 \frac{e^2}{r} + W_2 \frac{e^{-br}}{1-e^{-br}}) \right] = 0,$$

$$E_{tot} = E_{\bar{e}} + E_p - E = 940 \text{ MeV}$$

$$\tau^{-1} = 2\pi\lambda^2 |\hat{\psi}(0)|^2 \frac{\alpha^2 E_1}{\hbar} = 5 \text{ s}$$

$$R = b^{-1} = 10^{-13} \text{ cm},$$
(63)

with values of the k -parameters

$$k_1 \approx 1, \quad k_2 \approx 1 + 10^{-6},$$
(64)

verifying the crucial condition (35).

Once absorbed by a stable nucleus $N(A, Z, J)$, the neutroid is transformed by strong interactions into a neutron plus secondary emissions, resulting in a generally untabulated unstable nucleus $\tilde{N}(A+1, Z, J+1/2)$ that decays into stable nuclei plus radiations.

The spin 0 of the neutroid explains the impossibility of its detection via conventional neutron detectors.

The decay of the nucleus $\tilde{N}(A+1, Z, J+1/2)$ explains the triggering by undetectable particles of conventional nuclear transmutations normally triggered by neutron irradiations.

Hadronic bound state (61) is clearly impossible for quantum mechanics, but readily possible for hadronic mechanics via structure model (29) due to the combination of strongly attractive Coulomb and contact forces at mutual distances of the order of 10^{-13} cm with isotopic elements of the simple type (19) verifying isodeterministic Lemma II-3.7. and the rapid convergence of isoperturbation series of Corollary II-3.7.1.

2.6.2. Nonrelativistic representation of the neutron synthesis. We consider now the synthesis of the neutron on demand from a hydrogen gas in the needed directionality, (low) energy and CPS.

Such a synthesis has been first achieved by the Directional Neutron Source (DNS) developed by Santilli at Thunder Energies Corporation, now Hadronic Technologies Corporation [39].

Recall that, when seen in an oscilloscope set for milliseconds, electric arcs between carbon electrodes, are continuously connected and disconnected to seek the shortest distance for the discharge.

With reference to Figure 9, the electric arc ionizes the hydrogen gas during its connection and creates an axial alignment along a magnetic line of electrons and protons with opposite charge and magnetic polarities.

By contrast, during their disconnection electric arcs “compress” the electron inside the proton by synthesizing in this way the neutron in a way crucially dependent on the shape of the arc, its power and other factors.

With reference to Figure 11, when compressed inside the hyperdense proton, the much lighter electron is “constrained” to have an orbital motion equal to the proton spin so as to avoid extreme resistive forces with ensuing high instabilities.

Note that the neutroid (figure 10) appears to be an unavoidable intermediate step prior to the full synthesis of the neutron (Figure 11).

The well known main characteristics of the neutron are the following:

- 1) Rest energy 939.565 MeV ;
- 2) Charge radius $R = 1.73 \times 10^{-13} \text{ cm}$;
- 3) Mean-life $\tau = 881 \text{ s (about } 15 \text{ m)}$;
- 4) Spin $1/2$;
- 5) Charge $;0$
- 6) Anomalous magnetic moment $\mu_n = -1.9 \frac{e}{2m_p c}$ and null electric dipole moment;
- 7) Decay

$$n \rightarrow p^+ + e^- + \bar{\nu}. \quad (65)$$

The nonrelativistic representation of *all*— characteristics of the neutron in its synthesis from the hydrogen was first achieved by Santilli in the 1990 paper [43] via hadronic bound state

$$n = (\tilde{e}_\downarrow^-, \tilde{p}_\uparrow^+)_{hm}; \quad (66)$$

where one should note that, unlike the case of the neutroid (61), both the electron and the proton are mutated into isoparticles.

Note also that model (66) represents a *compressed neutroid* in a way much similar to the structure of the $\pi^\pm$ mesons as compressed muons, $\mu^\pm$ (Section 2.5.4).

The representation of the rest energy, charge radius, mean-life, charge, parity and tunnel effect decay of the neutron have been first achieved in

Ref. [43] Eqs. (2.19), page 521, via hadronic two-body bound state

$$\left[\frac{1}{r^2} \left(\frac{d}{dr} r^2 \frac{d}{dr} \right) + \bar{m} (E \pm W_1 \frac{e^2}{r} + W_2 \frac{e^{-br}}{1-e^{-br}}) \right] = 0,$$

$$E_{tot} = E_{\bar{e}} + E_{\bar{p}} - E = 939 \text{ MeV}$$

$$\tau^{-1} = 2\pi\lambda^2 |\hat{\psi}(0)|^2 \frac{\alpha^2 E_{\bar{e}}}{\hbar} = 881 \text{ s (about } 15m)$$

$$R = b^{-1} = 10^{-13} \text{ cm},$$

(67)

with values of the k -parameters

$$k_1 = 2.6, \quad k_2 = 1 + 0.81 \times 10^{-8},$$

(68)

verifying conditions (35) for the validity of isodeterminism inside the neutron and the validity of conventional uncertainties in its outside.

The representation of the spin 1/2 of the neutron was also achieved for the first time in Ref. [43], Eqs. (2.22)-(2.37), thanks to the appearance of a constrained *orbital* motion of the isoelectron when totally “compressed” inside the proton (which orbital motion is completely non-existent for quantum mechanics).

The representation of the spin 1/2 is additionally permitted by the *isotopies of spin-orbit couplings* (see Chapter 6, page 209 on, Ref. [33], for a detailed treatment), which the hadronic angular momentum of the isoelectron is constrained to be equal to the spin of the isoproton, as a necessary condition to avoid big instabilities according to Eqs. (II-118) under constraint (II-119). Hence, $J_{tot}^n = s_p + s_e + L_e = 1/2$, namely, *the spin of the neutron coincides with that of the proton*.

Recall the following experimental values of magnetic moments in nuclear units $\mu_N = \frac{e}{2m_p c}$

$$\mu_n^{exp} = -1.91 \mu_N$$

$$\mu_p^{exp} = 2.7 \mu_N$$

(69)

$$\mu_e^{exp} = 1 \mu_B = 5 \times 10^{-4} \mu_N,$$

where one can see that the magnetic moment of the neutron is “anomalous” because outside the prediction of quantum mechanics both in its direction and numeric value.

The first known exact representation of the anomalous magnetic moment of the neutron was achieved in Ref. [43], Eqs. (2.39)-(2.41), page 526, via the novel contribution of the internal orbital motion of the isoelectron.

By noting from values (233) that the magnetic moment of the electron is very small for nuclear standard, thus being ignorable in first approximation, the main assumption of Ref. [43] is that *the magnetic moment of the neutron of model (66) is given by the magnetic moment of the proton plus the orbital magnetic moment of the isoelectron inside the proton.*

Note that said contribution is completely absent in quantum mechanics due to its point-like approximation of particles as being point-like when at mutual distances *smaller* than their charge diameter.

Note that the electron is negatively charged and, consequently, the contribution to the magnetic moment of the neutron from its rotation inside the proton in the direction of the proton spin is *negative*, thus providing the first known representation of the *anomalous direction* of the neutron magnetic moment.

Calculations done in Ref. [43] for the magnetic moment of the isoelectron orbiting inside the proton yield the value $\mu_e^{orb} \approx -4.6 \mu_N$, by therefore reaching the first known representation of the *anomalous value* of the neutron magnetic moment (Eqs. (2.40), page 526, Ref. [43], see also Ref. [31], Section 6.3.D and paper [41], Section 4, page 41)

$$\begin{aligned} \mu_n &= \mu_p + \mu_e^{orb} + \mu_e^{intr} \approx \\ &\approx \mu_p + \mu_e^{orb} = \\ &= 2.7 \mu_N - 4.6 \mu_N = -1.91 \mu_N = \mu_n^{exp}. \end{aligned} \tag{70}$$

Note that *the negative sign of the neutron magnetic moment can be considered as direct evidence of the presence of an electron in singlet internal coupling in the proton*, since no other known particle besides the electron can provide the internal, rather large contribution $\mu_e^{orb} = -4.6 \mu_N$.

In Santilli's view, *the inability by quantum mechanics to represent both the anomalous direction and value of the neutron magnetic moment constitutes an additional evidence beyond scientific doubt supporting Einstein's vision on the lack of "completion" of quantum mechanics.*

Furthermore, *the impossibility for quantum mechanics to characterize the motion of the electron compressed inside the proton, constitutes evidence that the most important "completion" of quantum mechanics needed to represent experimental values is the characterization of particles with their actual shape and density.*

The parity and null value of the electric dipole moment were represented via a isotopy of conventional lines.

For historical comments on the birth of the neutrino hypothesis in quantum mechanics [13] and its possible replacement with the etherino hypoth-

esis [42] in hadronic mechanics, one may consult Section II-3.4.1 on Irregular Pauli-Santilli isomatrices.

The reader should keep in mind that the neutron is naturally unstable (when isolated). Consequently, the synthesis of the neutron is a time-irreversible process. It then follows that the Lie-isotopic isomechanical treatment presented in this section is an approximation of the broader treatment via the covering Lie-admissible genomechanics outlined in Section 1-2 (see Refs. [32] [33] for extensive treatments).

With reference to the geno-Schrödinger and geno-Heisenberg equations (I-10) and (I-11), respectively, the genomechanical treatment can be studied via the extension of isotopic element (19) into a time-dependent form characterizing the genotopic elements for motions forward and backward in time.

2.6.3. Relativistic representation of the neutron synthesis. The relativistic representation of *all* characteristics of the neutron in synthesis (66) was first achieved by Santilli in Refs. [35] [36] (see independent review [31], Section 6.3, page 342 on) via the isosymmetry of the irregular Dirac-Santilli isoequation (37), namely, the isotopy $\hat{P}(3.1)$ of the spinorial covering of the Poincaré symmetry (Section 2.5.11) and can be outlined as follows.

Recall that the non-potential hadronic representation of strong interactions via the isotopic element (1) (Section 2.3) implies the "absorption" of the Coulomb potential by the Hulten potential, as essentially implied by the charge independence of strong interactions.

This feature permits to ignore Coulomb binding energies in first approximation, resulting in a *weakly bounded* relativistic hadronic structure of the neutron, namely, a state with a small binding energy typical of all non-potential interactions.

Consider first synthesis (61) of the neutroid, and assume its representation in the iso-Minkowski isospace $\hat{M}(\hat{x}, \hat{\eta}, \hat{t})$ with isospacetime (II-19). Assume in first approximation that the proton is perfectly spherical for which $n_1 = n_2 = n_3 = 1$ and assume that the density of the region of neutroid-proton overlapping is close to that of the vacuum, thus implying the value $n_4 = 1$. These assumptions imply the simplified isometric

$$\hat{\eta} = \text{Diag.}(1, 1, 1, -1)e^{-K}, \quad K > 0. \quad (71)$$

The relativistic version of the synthesis (61) with the representation of all characteristics of the neutroid then follows via a simply isotopy of the relativistic treatment of the hydrogen atom.

In the transition to the relativistic treatment of the structure of the neutron, Eq. (66), the isoproton cannot any longer be assumed to be perfectly

spherical and the density of the overlapping region becomes dominant, resulting in values of the characteristic quantities $n_\mu \neq 1$, $\mu = 1, 2, 3, 4$.

In Refs. [35] [36], Santilli assumes that the value of the characteristic quantity n_4 representing the density of the neutron is equal to the density of the proton-antiproton fireball of the Bose-Einstein correlation [46] [47], resulting in the values

$$n_4 = 0.62, \quad b_4 = \frac{1}{n_4} = 1.62, \quad (72)$$

where $b_4 = 1/n_4$ is the notation used in Ref. [36].

The Lorentz-Santilli isotransforms (II-42) then imply the following *isore-normalization of the rest energy of the electron*, namely a renormalization caused by non-potential interactions (Ref. [36], Eqs. (7.1), page 191)

$$E_e = m_e c^2 = 0.511 \text{ MeV} \rightarrow E_{\bar{e}} = m_e \frac{c^2}{n_4^2} = 1.341 \text{ MeV}. \quad (73)$$

As one can see, the above isorenormalization removes the problem of the missing 0.782 MeV energy in the neutron synthesis when represented on the iso-Minkowskian isospace over an isofield, thus rendering consistent the needed isorelativistic isoequations.

It should be stressed that the above isorenormalization continues to be based on the etherino mechanism for the delivery of the missing energy to the neutron [42].

The relativistic representation of the spin of the neutron in synthesis (66) was first achieved in Refs. [35] [36]. Recall from Figure 10 that the spin S_1 of isoelectron is opposite to the spin S_2 of the isoproton. The relativistic spin-orbit coupling implies the *constraint* that the orbital angular momentum L_1 of the isoelectron inside the isoproton be equal to the isoproton spin,

$$L_1 = S_2. \quad (74)$$

The above identity is manifestly impossible for the spinorial covering of the Poincaré symmetry $\mathcal{P}(3.1)$ and relativistic quantum mechanics, but it is indeed possible for the covering isosymmetry $\hat{\mathcal{P}}(3.1)$. In fact, with reference to Section II-3.5.3, Eqs. (II-122), identity (II-241) implies the con-

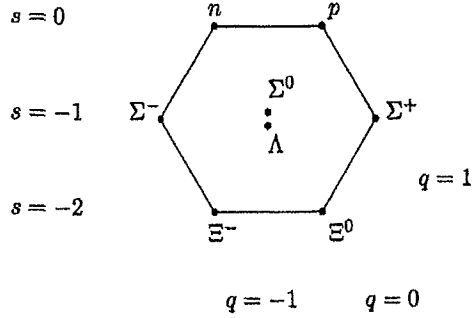


Figure 12: An illustration of the compatibility of the quantum mechanical $SU(3)$ and subsequent classifications of baryons unto families, and the structure of individual baryons as a hadronic bound state with physical constituents generally emitted free in the spontaneous decays with the lowest mode, Eqs. (241). Said compatibility is achieved via the multi-valued hyperunit characterized by the isounits or isotopic elements of the individual baryons, Eqs. (242), and its use to build the hyperstructural image of the applicable Lie symmetry that turns out to be locally isomorphic to the original symmetry due to the positive-definite character of the isounit.

ditions (Ref. [36], Eqs. (7.2), page 192)

$$\begin{aligned}
 \hat{L}_3 \star |\hat{\psi}\rangle &= \pm n_1 n_2 |\hat{\psi}\rangle = \\
 &= \hat{S}_3 \star |\hat{\psi}\rangle = \frac{1}{2} \frac{1}{n_1 n_2} |\hat{\psi}\rangle, \\
 \hat{L}^2 \star |\hat{\psi}\rangle &= (n_1^2 n_2^2 + n_2^2 n_3^2 + n_3^2 n_1^2) |\hat{\psi}\rangle = \\
 &= \hat{S}^2 \star |\hat{\psi}\rangle = \frac{1}{4} (n_1^{-2} n_2^{-2} + n_2^{-2} n_3^{-2} + n_3^{-2} n_1^{-2}) |\hat{\psi}\rangle,
 \end{aligned} \tag{75}$$

admitting the simple solution

$$\begin{aligned}
 n_k^2 &= \frac{1}{\sqrt{2}} = 0.706, \\
 b_k^2 &= \frac{1}{n_k^2} = \sqrt{2} = 1.415, \quad k = 1, 2, 3,
 \end{aligned} \tag{76}$$

where $b_k^2 = 1/n_k^2$ in the notation of Ref. [36].

The relativistic representation of the anomalous magnetic moment of the neutron was also achieved for the first time in Ref. [36], Eqs. (7.4), page 192. The representation is again permitted by the contribution from

the orbital motion of the isoelectron inside the isoproton, and it is given by non-relativistic expression (234) with a more accurate representation of the orbital contribution.

It should be indicated that the non-relativistic and relativistic structure models of the neutron outlined in this section are mere approximation of a much more complex reality in which all characteristics of the constituents are mutated, thus including the charge.

2.6.4. Remaining baryons. A central requirement for the consistency of model (66) is that *the excited states of the neutron are the conventional states of the hydrogen atom*. This illustrates again the hadronic suppression of quantum mechanical energy spectra, since the latter are typical for the classification, rather than the structure of hadrons.

By recalling the condition that the number of elementary constituents of hadrons increases with the increase of the rest energy, the hadronic structure of the remaining baryons is reducible to two isoparticle structures derived from the spontaneous decays generally those with the lowest mode, much along the structure of mesons [21] (see [31], Section 6.3.J, page 366 for an independent review)

$$\begin{aligned}
 p^+(938 \text{ MeV}) &= \text{stable}, \\
 n(940 \text{ MeV}) &= (\tilde{p}^+, \tilde{e}^-)_{hm}, \\
 \Lambda(1115 \text{ MeV}) &= (\tilde{p}^+, \tilde{\pi}^-)_{hm}, \\
 \Sigma^+(1189 \text{ MeV}) &= (\tilde{p}^+, \tilde{\pi}^0)_{hm}, \\
 \Sigma^0(1192 \text{ MeV}) &= (\tilde{n}, \tilde{\pi}^0)_{hm}, \\
 \Sigma^-(1197 \text{ MeV}) &= (\tilde{n}, \tilde{\pi}^-)_{hm}, \\
 \Sigma^0(1314 \text{ MeV}) &= (\tilde{\Lambda}, \tilde{\pi}^0), \\
 \Xi^-(1321 \text{ MeV}) &= (\tilde{\Lambda}, \tilde{\pi}^-)_{hm},
 \end{aligned} \tag{77}$$

by keeping in mind that numerous alternative internal exchanges of isoparticles do indeed occur while keeping constant the total characteristics.

It is an instructive exercise for the interested reader to see that all the above models verify condition (II-199) on the lack of hadronic excited states, as well as conditions (II-188) for the validity of isodeterministic Lemma II-3.7 and related rapid convergence of isoserries.

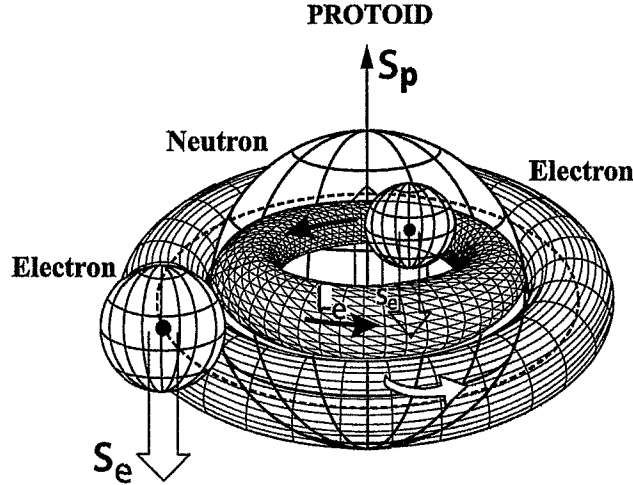


Figure 13: A conceptual rendering of the protoid as weakly bounded electron and neutron in singlet contact coupling, resulting in a negatively charged hadron with spin $S = 0$ with intriguing applications.

The compatibility of hadronic structure models (77) with the $SU(3)$ -color or more recent classifications is achieved at the hyperstructural level [33] [37] with the following ordered hyperunit [21] (Figure 11)

$$\hat{I}_{tot} = \{\hat{I}_p, \hat{I}_n, \hat{I}_\Lambda, \hat{I}_{\Sigma^+}, \hat{I}_{\Sigma^0}, \hat{I}_{\Sigma^-}, \hat{I}_{\Xi^0}, \hat{I}_{\Xi^-}\} \quad (78)$$

Said compatibility is ultimately due to the positive-definiteness of the individual isounits that, in turn, implies the local isomorphism between the hypersymmetry $\tilde{S}U(3)$ and the conventional $SU(3)$ symmetry.

2.6.5. Industrial applications. There is no doubt that the “completion” of quantum mechanics is, by far, Einstein’s most important legacy because of its *basic* implications for mathematics, physics, chemistry and other quantitative sciences, with expected industrial applications generally beyond our expectation at this writing.

As an illustration, we outline in this section the industrial applications expected from the technology underlying the synthesis of the neutron from the hydrogen which technology has been possible thanks to the “completion” of quantum into hadronic mechanics.

Besides the synthesis of neutroids (Figure 10) and neutrons (11), the

Directional Neutron Source (DNS) [39] (Figure 9) permits the study of the synthesis of a *negatively charged, strongly interacting particles* [48] (patent pending) with rest energy of about 940 MeV, spin 0, charge radius $R = 1 fm$, and mean-life of about 3 s which is called the *protoid*, and denoted $\hat{p}_1$ (Figure 13).

Recall that contact non-Hamiltonian interactions solely depend on wave-overlapping, thus being charge independent. The protoid is predicted to be given by a singlet coupling of an electron essentially at contact with a neutron, thus yielding total angular momentum 0, with the following structure model:

$$\hat{p}_1 = (e_{\downarrow}^-, n_{\uparrow})_{hm}. \quad (79)$$

Hadronic mechanics predicts a second negatively charged proton called *pseudo-proton*, and denoted $\hat{p}_2$, with essentially the same rest energy, charge radius and mean-life of the neutroid but with spin 1 (Figure 14).

Recall that an electric arc submerged within a hydrogen gas creates a plasma in its surrounding comprising: protons, electrons, neutrons and valence electron pairs in singlet couplings known as *isoelectronia* with rest energy of about 1 MeV, null spin and magnetic moment, and a mean-life expected to be of the order of 1 s or fractions thereof (Section 2.8.3).

Following the synthesis of the neutron via the "compression" of an electron inside the proton (Section 2.6.2), the pseudo-proton is predicted to be generated by a "compression" of an electron, this time, inside the neutron, or equivalently, by the "compression" of an isoelectronium inside the proton, with structure model

$$\hat{p}_2 = (\tilde{e}, \tilde{n})_{hm} \approx [\tilde{p}, (\tilde{e}_{\uparrow}^-, \tilde{e}_{\downarrow}^-)]_{hm}. \quad (80)$$

The spin $\S_{\hat{p}_2} = 1$ is due to the sum of the proton spin $S_p = 1/2$ plus the constrained orbital motion of the electron pair along the proton spin $L_{1,2} = 1/2$.

Recall that protons are *repelled* by nuclei. Hence, the industrially significant feature of negatively charged hadrons is that they are *attracted* by nuclei, thus initiating a basically new approach toward the controlled nuclear fusions that resolves the extremely large Coulomb repulsion that has opposed nuclear fusions to date.

In view of the above features, the technologies that can be developed with the DNS are the following:

1. The detection of fissionable nuclear material that can be concealed in suitcases, containers or underground, since the neutron irradiation of fissionable material under controlled directionality, energy and flux, triggers the decay of some of the nuclei with a shower of clearly detectable

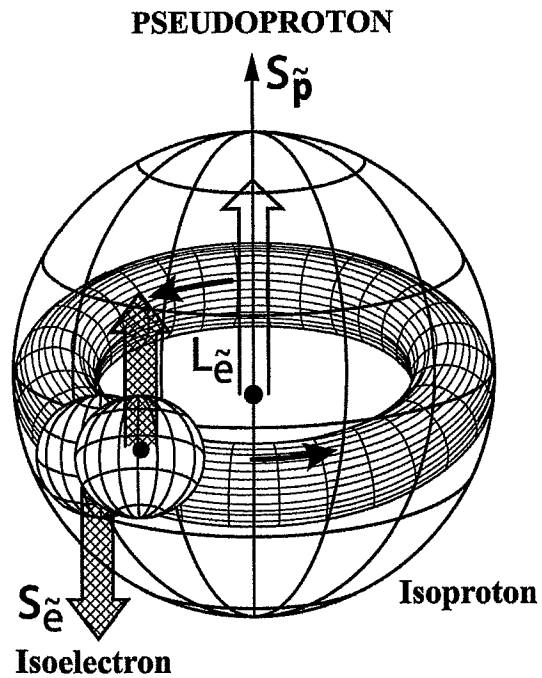


Figure 14: A conceptual rendering of the pseudo-proton created by the “compression” of an electron, this time, inside a neutron, or equivalently, of a valence electron pair in singlet coupling, resulting in a negatively charged hadron with spin 1.

radiations. By comparison, most nuclear material are detected as ordinary metals under currently available X-ray, microwave and other scans.

2. The detection of the presence and concentration of precious metals in mining operations, since the neutron irradiation, for instance, of the walls of mining tunnel triggers known nuclear transmutations with the emission of sharp, clearly detectable photons, while statistical data identify the concentration.

3. The control of large metal welds in civilian and military naval construction, which control is rendered particularly effective by the easy mobility of the DNS.

4. Cancer treatments via pseudo-proton irradiation which is expected to be less invasive and more localized than currently available proton irradiation since, in the former case, pseudo-protons are *attracted* by cancerous nuclei, while, in the latter case, protons are *repelled*.

5. Initiation of the much overdue research and development toward the recycling of nuclear waste via its stimulated decay while delivering energy, which recycling is possible in a number of ways, including the irradiation of nuclear waste pellets with a sufficiently intense pseudo-proton flux resulting in a deficiency of the nuclear charge Z for a given atomic number A and ensuing decay.

Note that the above applications are possible thanks to the *mean-life of DNS synthesized particle of the order of seconds or fractions thereof*, which are quite large for particle standard.

By comparison, negatively charged strongly interacting particles synthesized in contemporary physics laboratory with mean-lives of the order of 10^{-30} s or so have no known industrial or medical applications.

2.7. Einstein's determinism in nuclear structures.

2.7.1. Historical notes. As it is well known, nuclei were first assumed to be bound states of protons and electrons under their mutual Coulomb attraction which force, being inversely proportional to the square of the distance, is extremely big at nuclear distances.

This assumption was soon dismissed because it was considered inconsistent with quantum mechanics and unable to represent experimental data.

Consequently, nuclei were assumed to be bound states of protons and neutrons under a strong nuclear force that remains unknown to this day.

In reality, nuclei provide additional, rather clear evidence on the "lack of completeness" of quantum mechanics beyond scientific doubt, due to

the well known inability by quantum mechanics to achieve an exact representation of nuclear experimental data.

As an illustration, in about one century of efforts and the use of large public funds, quantum mechanics has been unable to achieve a representation of the characteristics of the simplest nucleus, the *deuteron*, with embarrassing deviations between the prediction of the theory and the experimental data for heavier nuclei such as the Zirconium [25] (see also K. R. Popper [15], J. Dunning-Davies [16], J. Horgan [17] and quoted literature):

1. Quantum mechanics has been unable to represent the spin $J = 1$ of the deuteron in its ground state $L = 0$. This is due to the fact that the sole stable quantum mechanical bound state of two particles with spin $1/2$ is the singlet state, in which case the spin of the deuteron should be $J = 0$. For the intent of preserving the validity of quantum mechanics in nuclear physics, the spin $J = 1$ of the deuteron is represented via the assumption of a combination of excited orbital spaces, $L = 0, 1, 2$ contrary to the evidence that isolated deuterons are in their ground state, with the consequential inability to represent the deuteron positive parity $P = (-1)^L = 0$, and other insufficiencies.

2. Quantum mechanics has been unable to achieve an exact representation of the deuteron magnetic moment, because the representation of about 1% of the experimental value is still missing despite all possible relativistic (as well as quark-inspired) corrections, with large deviations occurring for heavier nuclei.

3. Quantum mechanics has been unable to represent the stability of the deuteron, since neutrons are well known to be naturally unstable, thus mandating a quantitative representation of the mechanism turning the neutron into a permanently stable particle when bonded to a proton.

4. Quantum mechanics has been unable to represent the proton-neutron exchange in the deuteron structure.

5. Quantum mechanics has been unable to achieve an explicit and concrete representation of the attractive strong nuclear force bonding together the proton and the neutron. Due to the intent of preserving the exact validity of quantum mechanics in nuclear physics, with the consequential sole representation of strong interactions via a potential in a Hamiltonian, the representation of strong nuclear forces has been attempted by adding potentials in the Hamiltonian, up to fifty additive potentials (sic) without the achievement of the needed representation of experimental data.

In a variety of works outlined in these papers (see monograph [25] and

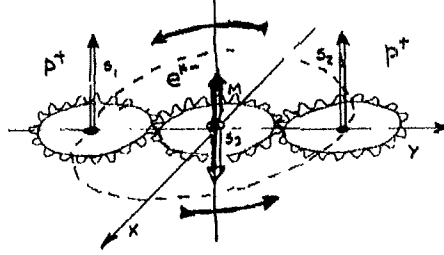


Figure 15: The "gear model" used by Santilli [25], page 180, to illustrate the prediction from the spin $J_D = 1$ that the deuteron is a three-body hadronic bound state of one isoelectron and two isoprotons, with the individual couplings $p - e$ and $e - p$ being in singlet as necessary for stability. The spin $J_D = 1$ is then achieved from the "constrained angular momentum" of the bound state studied in Sections II-3.5.3 and II-3.5.4.

papers quoted therein), Santilli has expressed the view that the "completion" of quantum into hadronic mechanics permits the apparent resolution of the above insufficiencies or sheer inconsistencies via a return to the original conception of nuclei as bound state of electrons and protons, although under non-linear, non-local and non-potential interactions caused by deep mutual overlapping.

2.7.2. Deuteron three-body model. Following the achievement of mathematical, theoretical, experimental and industrial advances in structure model (66) of the neutron as a hadronic bound state of one isoelectron and one isoproton, Santilli proposed in Section IV of Ref. [25], page 143 on (see also independent reviews [31] and [49]), the structure model of the deuteron D as a *restricted three-body hadronic bound state of two isoprotons and one isoelectron* (Figures 13, 14) which can be reduced in first approximation to a two-body hadronic bound state of one isoproton and one isoneutron

$$D = (\tilde{p}_\uparrow^+, \tilde{e}_\downarrow, \tilde{p}_\uparrow^+)_{hm} \approx (\tilde{p}^\uparrow, \tilde{n}^\uparrow)_{hm}. \quad (81)$$

Ref. [25] then presented various arguments showing that model (81) apparently resolves quantum mechanical insufficiencies (1 - 5) of the preceding section.

The analysis of model (81) was conducted in the 1999 monograph [25] following the publication in 1998 of the first proof of the EPR argument [9], as well as its detailed study in the 1995 monograph [33], Chapter 4, particularly Appendix 4C, page 166. Therefore, Santilli was aware that, unlike the case for atomic structures, deuteron model (81) does admit a classical

counterpart (Section II-3.7) due to the inapplicability of Bell's inequality (9) [3] in favor of isoidentity (10).

However, at the time of writing the 1999 monograph [25], Santilli was unaware of the additional apparent proof of the progressive validity of Einstein's determinism in the interior of hadrons, nuclei and stars [10], and its apparent full achievement in the interior of gravitational collapse.

It is, therefore, important to review the main aspects of model (81) to indicate, apparently for the first time, its verification of Einstein's determinism we have called isodeterministic according to Lemma II-3.7 and Corollary II-3.7.1, and verify the apparent resolution of historical problems (1 - 5) of the preceding section.

1. NONRELATIVISTIC REPRESENTATION OF THE SPIN $J_D = 1$ IN THE GROUND STATE $L_D = 0$. With reference to Figure 24, model (81) includes *five* angular momenta: 1) The parallel spins S_1 and S_2 of the isoprotons; 2) The angular momentum L_{1-2} of the two isoprotons; 3) The spin of the isoelectron S_3 ; and 4) The angular momentum L_3 of the isoelectron inside one of the two isoprotons.

Recall that, according to experimental data on nuclear dimensions, the isoproton and the isoneutron are expected to have a diameter of 1.73 fm , while the diameter of the deuteron is 4.26 fm .

Consequently, the two isoprotons are separated by about 0.886 fm . In model (81), this space is occupied by the isoelectron acting like a "gluon" of the two isoprotons, while permitting their triplet alignment necessary for the spin $J_D = 1$.

In fact, the isoelectron is in the singlet coupling with each of the two isoprotons necessary for stability according to hadronic mechanics (Figure 14).

Recall also that, in semiclassical approximation, the wavepacket of the electron is of the order of 2.2 fm .

The above data provide not only a considerable synchronicity in model (81), but also a rather strong bond caused by the nuclear forces, the Coulomb attraction between the isoelectron and the isoprotons, as well as the strong attractive force originating from deep wave-overlapping in singlet coupling.

Recall finally that the diameter of the horizon for the full applicability of hadronic mechanics has been selected in Paper I to be $d = 2 \times 10^{-13} \text{ cm} = 2 \text{ fm}$, while the deuteron is about double that size.

The above data on dimensions suggest the use of the following isosymmetries for the characterization of the angular momentum and spin of the deuteron:

Use the regular isosymmetries $\hat{O}(3)$ and $\hat{S}\hat{U}(2)$ for the characterization of the angular momentum and spin outside the hadronic horizon. We are here referring to isosymmetries which can be constructed via a non-unitary transform of the corresponding Lie symmetries, thus preserving conventional eigenvalues for angular momentum and spin (Section I-3.8 and I-3.9).

Use the irreregular isosymmetries $\hat{O}(3)$ and $\hat{S}\hat{U}(2)$ for the characterization of the angular momentum and spin inside the hadronic horizon. We are here referring to isosymmetries which cannot be constructed via non-unitary transforms of the corresponding Lie symmetries, by therefore having anomalous values of the angular momentum and spin (Section II-3.4).

It is then easy to see that the diameter $D = 2.59 \text{ fm}$ of the orbital motion of the two isoprotons is close to, but bigger than the hadronic horizon $d = 2 \text{ fm}$. We can therefore see the regular isorotational symmetry $\hat{O}(3)$ admitting conventional angular momentum eigenvalues with ground state $L_{1-2} = 0$.

By contrast we have to use the irregular $\hat{O}(3)$ isosymmetry for the rotation of the isoelectron in the interior of the isoproton, resulting in the constrained value $L_3 = 1/2$ for the neutron (Sections 2.6.2 and 2.6.3).

In this way, Ref. [25] achieved the first known representation of the spin of the deuteron in its true ground state according to values:

$$\begin{aligned} J_D &= S_1 + S_2 + S_3 + L_{1-2} + L_3 = \\ &= \frac{1}{2} + \frac{1}{2} - \frac{1}{2} + 0 + \frac{1}{2} = 1. \end{aligned} \tag{82}$$

By comparing the hadronic structure model of the neutron, Figure 10, and that of the deuteron, Figure 15, it is evident that the isoelectron is entirely compressed inside the isoproton in the former case, but only partially compressed in the latter case.

This occurrence is expected due to the presence in model (81) of the second isoproton with ensuing strongly attractive Coulomb force responsible for the partial extraction of the isoelectron from the isoproton, although this possibility has not been investigated to date.

Similarly, structure (81) appears to be particularly suited for the representation of the proton-neutron exchange in the deuteron structure, although this possibility has also not been subjected to a quantitative study to our best knowledge.

2. RELATIVISTIC REPRESENTATION OF THE SPIN $J_D = 1$ IN THE GROUND STATE $L_D = 0$. Ref. [25] was written following the 1995 publication of *Elements*

of *Hadronic Mechanics*, Vols. I [32] and II [33]. Therefore, Ref. [25] studied model (81) with the entire sue of said Volumes whose knowledge, particularly that reviewed in Section II-3, is herein tacitly assumed.

The study of model (81) was done in Ref. [25], Section IV-2.3.4, page 161, via the use of the irregular iso-Dirac equation (II-201) defined on iso-semi-product (II-200) of an iso-Minkowskian space $\hat{M}(\hat{x}, \hat{\eta}, \hat{I}_{orb})$ for the relativistic description of the hadronic orbital motion, multiplied by a three-dimensional complex-valued isounitary isospace for the characterization of the hadronic spin $\hat{E}(\hat{z}, \hat{\eta}, \hat{I}_{spin})$.

It should be recalled that, in principle, *all* conventional intrinsic characteristics of the proton and the electron, including their charges, are expected to be mutated when said particles are in conditions of total mutual penetration. These mutations are quantitatively represented by the Lorentz-Poincaré-Santilli isosymmetry of the irregular iso-Dirac equation (Section II-2.5.11) and its irregular Pauli-Santilli isomatrices (Section II-3.4).

However, Ref. [25], Section 2.3.5, page 162, has shown that, for the case of the deuteron (but not necessarily so for heavier nuclei), the spins and changes of the isoprotons and of the isoelectron can be assumed in first approximation to have conventional values. In this case, the sole mutations achieving the representation of experimental data of the deuteron are those for shapes and constrained angular momenta.

The first known relativistic representation of the spin $J_D = 1$ of the deuteron in its ground state $L_D = 0$ was achieved in Ref. [25], Section 3.5.6, thanks to constrained spin-orbit coupling inside the isoneutron of model (81).

The latter coupling has been reviewed at the non-relativistic level in Section II-3.5.3 and at the relativistic level in Section II-3.5.4, see in particular Eqs. (II-118), (II-119), with a detailed description available in Ref. [33], Chapter 6.

The resulting relativistic characterization of hadronic angular momenta and spins is given by

$$\begin{aligned} S_{k\alpha} &= \epsilon_{kij} \hat{\gamma}_{i\alpha} \star \hat{\gamma}_{j\alpha}, \quad \alpha = \tilde{p}1, \tilde{p}2, \tilde{e}, \\ L_{k,p1-p2} &= \epsilon_{kij} r_{i,p1-p2} \star p_{j,p1-p2}, \end{aligned} \tag{83}$$

where $\hat{\gamma}_k$ are the irregular Dirac-Santilli isomatrices.

We then have the irregular isocommutation rules Ref. [36], Eqs. (6.4),

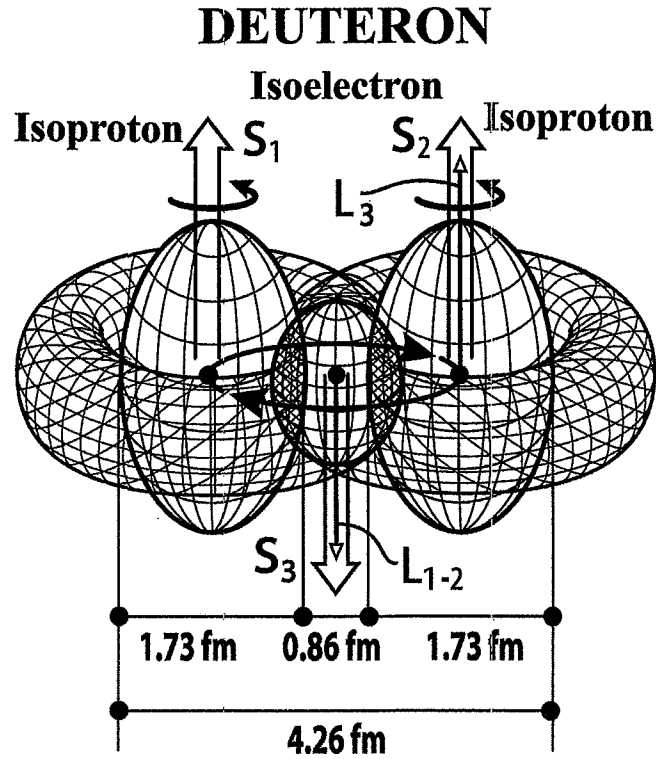


Figure 16: A conceptual rendering of the deuteron model (91) illustrating the representation for the first time of spin $J_D = 1$ with angular momentum $L_D = 0$, where: the two isoprotons have the charge diameter $D_{\bar{p}} = 1.73$ fm; the deuteron has the charge diameter $D_D = 4.26$ fm; the interspace between the isoprotons is then of 0.86 fm; the isoelectron has a point-like charge, the hadronic diameter $D_e^{hm} = 1$ fm, and a semi-classical wavepacket with $D_e^{cl} = 2.2$ fm. The above data illustrate the stability of model (81) with the central isoelectron allowing the isoprotons in the triplet coupling needed to achieve the spin $J_D = 1$ in the true ground state (Figure 14). The doughnuts around the two isoprotons are used to represent the proton-neutron exchange forces.

page 190,

$$[S_{i,\alpha}, \hat{S}_{j\alpha}] = 2\epsilon_{kij} m_{k\alpha}^2 \hat{S}_{k\alpha}, \quad (84)$$

$$[L_{i,1-2}, \hat{L}_{j,1-2}] = \epsilon_{ijk} m_{k,1-2}^2 L_{k,1-2},$$

and isoeigenvalues, Eqs, [36], Eqs. (6.4d), page 190,

$$\hat{S}_{3\alpha} \star |\hat{\psi}\rangle = \pm \frac{1}{m_{1\alpha} m_{2\alpha}} |\hat{\psi}\rangle,$$

$$\hat{S}_{\alpha}^2 \star |\hat{\psi}\rangle = (m_{1\alpha}^{-2} m_{2\alpha}^{-2} + m_{2\alpha}^{-2} m_{3\alpha}^{-2} + m_{3\alpha}^{-2} m_{1\alpha}^{-2}) |\hat{\psi}\rangle,$$

$$\hat{L}_{3,1-2} \star |\hat{\psi}\rangle = \pm m_{1,1-2} m_{2,1-2} |\hat{\psi}\rangle,$$

$$\hat{L}_{\beta}^2 1 - 2 \star |\hat{\psi}\rangle = (m_{1,1-2}^2 m_{2,1-2}^2 + m_{2,1-2}^2 m_{3,1-2}^2 + m_{3,1-2}^2 m_{1,1-2}^2) |\hat{\psi}\rangle. \quad (85)$$

Under the assumption that the hadronic medium in the interior of the proton is homogeneous, we can assume the values [36]

$$m_{1\alpha} = m_{1,1-2} = m_{2\alpha} = m_{2,1-2} = m_{3\alpha} = m_{3,1-2} = \frac{1}{\sqrt{2}} = 0.842, \quad (86)$$

under which the isoeigenvalues become

$$\hat{S}_{3\alpha} \star |\hat{\psi}\rangle = \pm \frac{1}{2} |\hat{\psi}\rangle,$$

$$\hat{S}_{\alpha}^2 \star |\hat{\psi}\rangle = \frac{3}{4} |\hat{\psi}\rangle,$$

$$\hat{L}_{3,1-2} \star |\hat{\psi}\rangle = \pm \frac{1}{2} |\hat{\psi}\rangle,$$

$$\hat{L}_{1-2}^2 \star |\hat{\psi}\rangle = \frac{3}{4} |\hat{\psi}\rangle.$$

(87)

It then follows that the total angular momentum of the isoneutron is given by

$$J_{\bar{n}} = S_2 + S_{\bar{e}}^{spin} + L_{\bar{e}}^{orb} = \frac{1}{2} + \frac{1}{2} - \frac{1}{2} = \frac{1}{2}. \quad (88)$$

The total value of the deuteron spin $J_D = 1$ in its ground state $L_D = 0$ then follows from the fact indicated above that the orbital diameter is bigger than the hadronic horizon.

3. REPRESENTATION OF THE DEUTERON REST ENERGY, STABILITY AND SIZE.

The hadronic representation of the rest energy, stability and charge radius of the deuteron were first achieved in Ref. [25], Eqs. (5.2.16), page 179,

via the use of structural equations (29) (see also independent reviews [31], [49])

$$\left[\frac{1}{r^2} \left(\frac{d}{dr} r^2 \frac{d}{dr} \right) + \bar{m} (E \pm W_1 e^{\frac{2}{r}} + W_2 \frac{e^{-br}}{1-e^{-br}}) \right] = 0,$$

$$E_{tot} = E_1 + E_2 - E = 1875.7 \text{ MeV}, \quad \bar{m} = \frac{m}{T^2} \quad (89)$$

$$\tau^{-1} = 2\pi\lambda^2 |\hat{\psi}(0)|^2 \frac{\alpha^2 E_1}{\hbar} = \infty,$$

$$R = b^{-1} = 2.13 \times 10^{-13} \text{ cm} = 2.13 \text{ fm},$$

with solutions for the k -parameters

$$k_1 = 1 \quad k_2 = 2.5. \quad (90)$$

The total mass/rest energy of the deuteron is then given by

$$M_D = M_p + M_n - E = 1875.7 \text{ MeV}, \quad (91)$$

where the binding energy is given by $E = 2.2 \text{ MeV}$.

Value (90) should be compared to the corresponding values for the neutron, Eqs. (69). As one can see, values (90) verify conditions (35) on the validity of the isodeterminism of Lemma II-3.7 as well as with the increase of the rapid convergence of isoperturbative series of Corollary II-3.7.1.

Note that, according to our assumption, values (81) verify the crucial condition (35) for the lack of existence of excited states, because excited states would imply the existing of the deuteron constituents out of the hadronic horizon with its consequential disintegration.

Note also that the stability of the deuteron is intrinsic in model (81) since the deuteron is reduced to the only known, massive, permanently stable particles existing in the universe, the proton and the electron.

Needless to say, the above representation should be considered as a first approximation, with a number of improvements being possible such as the inclusion of Coulomb interactions with possible excited states within the hadronic horizon.

4. REPRESENTATION OF THE DEUTERON MAGNETIC MOMENT. As it is well known, the experimental value of the deuteron magnetic moment is given by [13] [14]

$$\mu_D^{exp} = 0.857 \mu_N. \quad (92)$$

For the quantum mechanical ground state with $L_D = 0$, in case consistent, one would obtains

$$\mu_D^{qm} = \frac{1}{2} (g_p^S + g_n^S) = 2.792 - 1.913 = 0.879 \mu_N, \quad (93)$$

(where the g 's are tabulated g -factors), by therefore missing about 3% of experimental value (256), which deviation cannot be entirely resolved via relativistic or other corrections.

The first, exact, non-relativistic representation of the deuteron magnetic moment was achieved by Santilli for the approximate two-body version of model (81) in paper [50] of 1994 (written at the Joint Institute for Nuclear Research, Dubna, Russia) under the sole assumption that the charge distributions of protons and neutrons are deformed under strong nuclear forces, resulting in a corresponding mutation of their intrinsic magnetic moment as predicted by E. Fermi [13], J.M. Blatt and V. F. Weisskopf [14], and other founders of nuclear physics (see their recollection in Ref. [25], page 158-159, Section IV-2.3.1, under the title *The Historical Hypothesis*).

Santilli achieved the first, exact, relativistic representation of magnetic moment of the two-body deuteron model in paper [51] of 1996 via the use of the iso-Dirac equation.

The 1998 Ref. [25], Section IV-2.3.6, page 163, presents the first known exact representation of the deuteron magnetic moment for the full three-body model (81).

It is important to review and upgrade the latter representation to show its compatibility with Einstein's determinism according to Lemma II-3.7 and rapid convergence according to Corollary II-3.7.1.

Assume, in first approximation, that the isoparticles of model (81) have the same spheroidal shape with semiaxes n_1^2 , n_2^2 , n_3^2 under the condition of preserving the volume of the original particles assumed for simplicity to be normalized to 1

$$n_1^2 \times n_2^2 \times n_3^2 = 1. \quad (94)$$

Represents with n_4^2 the density of the two isoprotons (denoted with the subindices 1 and 2), resulting in the isotopic element for the two isoprotons

$$\hat{T}_{1,2} = \text{Diag.}(n_1^2, n_2^2, n_3^2, n_4^2). \quad (95)$$

Then, the Lorentz-Santilli isotransforms (II-42) imply the following mutation of the intrinsic magnetic moment of the proton (see Ref. [36], page 190 for a detailed derivation with the notation $b_\mu = 1/n_\mu$, $\mu = 1, 2, 3, 4$)

$$\mu_{1,2}^{qm} \rightarrow \tilde{\mu}_{1,2}^{hm} = \frac{n_4}{n_3} \mu_{1,2}^{qm} = \tilde{g}^S \mu_N, \quad (96)$$

$$\tilde{g} = \frac{n_4}{n_3} g.$$

Recall that the isoprotons have Spin $S_{1,2} = 1/2$ and $L_D = 0$.

In order to compute the magnetic moment of the three-body model (81), we first compute *the intrinsic and orbital contributions of the isoprotons*

via a simple isotopy of the corresponding quantum mechanical expression [45]

$$\begin{aligned}\tilde{\mu}_k^{hm} &= \\ \frac{1}{J+1} < \hat{Y}_{L,S} | \frac{1}{2} \tilde{g}_k^S S + \frac{1}{2} \tilde{g}_k^L L | \hat{Y}(L, S) > = \\ &= \frac{1}{2(J+1)} \tilde{g}_k^S [J(J+1) - L(L+1) + S(S+1)], \quad k = \tilde{p}1, \tilde{p}2,\end{aligned}\quad (97)$$

where $\hat{Y}_{L,S}$ are the isospherical isoharmonics (Ref. [33], Section 6.4, page 240) and $J_k = S_k + L_k$, $k = 1, 2$ is the total angular momentum.

Simple calculations show that, for $L_k = 0$ and $\mu_p^{exp} = 2.79 \mu_N$, expression (261) yields the value

$$\begin{aligned}\tilde{\mu}_k^{hm} &= \\ &= \frac{1}{2} \tilde{g}_k^S = \frac{n_4}{2n_3} g_k^S = \frac{n_4}{n_3} 2.79 \mu_N.\end{aligned}\quad (98)$$

To compute the *intrinsic and orbital contribution from the isoelectron*, we recall from Eq. (69) that its intrinsic value is ignorable in first approximation, the sole expected contribution being that from its constrained orbital motion inside the proton.

By recalling that the electron has a point-like charge which cannot be mutated by isotopies, then the density of the electron remains the conventional one for the vacuum with $n_{4,\bar{e}} = 1$.

Under the approximation that the deformation of the isoelectron is the same as those of the two isoprotons, we have the mutation of the value (70) for the neutron,

$$\mu_{\bar{e},orb}^{hm} = -\frac{1}{n_3} \mu_{\bar{e}}^{orb} = -\frac{1}{n_3} 4.6 \mu_N. \quad (99)$$

The quantum mechanical magnetic moment of the three-body model (81) of the deuteron, in case consistent, would be given by

$$\mu_D^{gm} = 2\mu_p + \mu_e = 2 \times 2.79 - 4.6 \mu_N = 0.98 \mu_N, \quad (100)$$

thus lacking an exact representation of value (92) because in *excess* for about 8%.

By comparison, the magnetic moment for model (81) according to hadronic mechanics is given by

$$\mu_D^{hm} = 2 \frac{n_4}{n_3} \mu_{\bar{p}} + \frac{1}{n_3} \mu_{\bar{e}} = \frac{n_4}{n_3} 5.78 - \frac{1}{n_3} 4.6 \mu_N = 0.857 \mu_N. \quad (101)$$

By assuming the value $n_4 = 0.62$ from the density of the proton-antiproton fireball in the Bose-Einstein correlation [46] [47], we can write the *hadronic representation of the deuteron magnetic moment*

$$\mu_D = \frac{0.62 \times 5.78}{n_3} - \frac{4.6}{n_3} = 0.857 \mu_N, \quad (102)$$

which is verified for

$$n_3 = 1.20. \quad (103)$$

The use of the volume-preserving condition (94) then yields the remaining ellipsoidal values

$$n_1 = n_2 = 0.91. \quad (104)$$

The above values imply that the shape deformation of the deuteron constituents caused by the internal strong nuclear force turns charge distributions and/or wavepackets from a spherical to a *prolate* spheroidal ellipsoid with semiaxes

$$n_1^2 = n_2^2 = 0.83, \quad n_3^2 = 1.44, \quad (105)$$

which prolate deformation was predicted by all preceding works by Santilli in the field.

Note that *the magnetic moment of the deuteron is represented exactly not only in its value, but also in its sign*, that of being parallel to the spins of the isoprotons.

It then follows that *representation (101) can be construed as evidence on the presence of two protons with parallel spins inside the deuteron*, which presence mandates three-body model (81) with a central exchanged electron for stability (Figures 14, 15).

Note also that the *prolate* deformation of shape is necessary because the quantum mechanical expression (93) is in *excess* of experimental value (92). By comparison, an *oblate* deformation would *increase* said excess value due to the increase of the rotating charge distribution at the equator.

Almost needless to say, representation (101) is intended to illustrate the *capability* by hadronic mechanics to achieve an exact representation of the deuteron magnetic moment.

A number of improvement of representation (101) are possible, among which we mention a more accurate value of the density n_4 of the proton, a more accurate value of the orbital contribution of the isoelectron due to its geometric difference with that for the neutron indicated above, and other improvements.

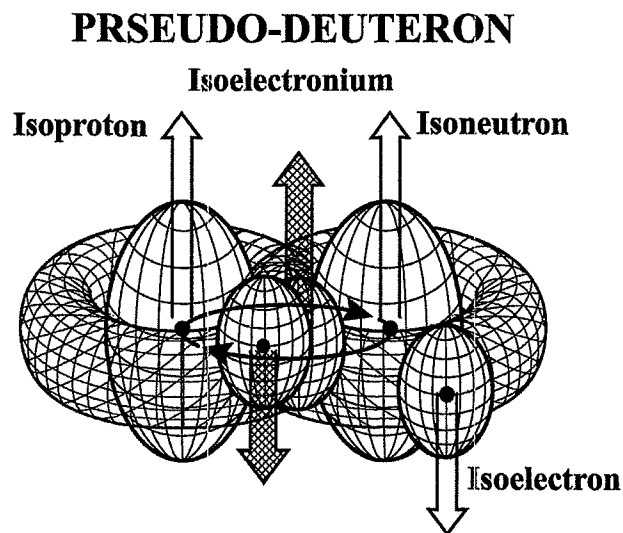


Figure 17: *An illustration of the negatively charged pseudo-deuteron predicted by hadronic mechanics via the use of a deuterium gas in the Directional Neutron Source of Figure 8. Due to its negative charge, the pseudo-deuteron can potentially initiate a basically new type of nuclear fusions, here called hyperfusions, based on the natural attraction of the pseudo-deuteron by natural deuteron resulting in the helium without production of harmful radiations or release of radioactive waste.*

5. **REPRESENTATION OF NUCLEAR FORCES.** Hadronic structure model (81) achieves the first known explicit concrete representation of strong nuclear forces thanks to the structural lifting of the mathematics underlying quantum mechanics into the covering isomathematics.

Such a generalization resulted to be necessary due to the non-Hamiltonian character of strong interactions, with particular reference to the generalization of the associative product into isoproduct (1) and the representation of strong nuclear forces via isotopic element (4), with ensuing lifting of the Schrödinger equation, from its historical form $H\psi = E\psi$, into the isotopic form

$$H \star \psi = H(r, p) \hat{T}(\psi, \dots) \hat{\psi} = E \hat{\psi}. \quad (106)$$

6. **REDUCTION OF STABLE NUCLEI TO ISOELECTRONS AND ISOPROTONS.** The reduction of neutrons to isoprotons and isoelectrons clearly implies the possible reduction of all stable matter in the universe to the only known, massive and stable particles, protons and electrons [52].

This important possibility has been confirmed by the reduction of the deuteron to isoprotons and isoelectrons [25], and it is under study by A. A. Bhalekar and R. M. Santilli [54].

2.7.3. Industrial applications. In Section I-1.4 we have indicated the societal duty of seeking basically *new* forms of clean energies, while continuing research along conventional lines.

This is due to the inability of identifying industrially viable nuclear fusions in about three quarters of a century and the investment of billions of dollars of public funds.

Recall that the primary obstacle opposing the achievement of the controlled nuclear fusion of two deuterons $D(1, 2, 1)$ into the Helium $He(2.4.0)$ is the *extreme repulsive Coulomb force between nuclei* that, the distance of $1 \text{ fm} = 10^{-15} \text{ m}$ needed to activate strong nuclear forces is given by

$$\begin{aligned} F &= k \frac{e^2}{r^2} = \\ &= (8.99 \times 10^9) \frac{(1.60 \times 10^{-19})^2}{(10^{-15})^2} = 230 \text{ N}, \end{aligned} \quad (107)$$

which force is extremely large for nuclear standards.

As it is well known, the efforts done to date for overcoming such a large repulsive force have been the use of high energies resulting, as expected, in uncontrollable instabilities at the initiation of the fusion itself.

Another problem that has prevented the achievement of the controlled

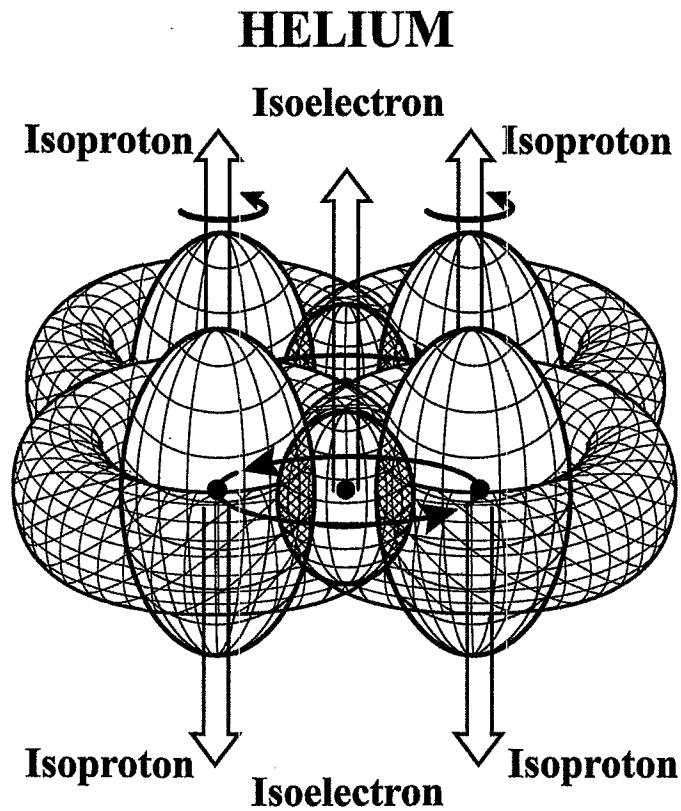


Figure 18: An illustration of the helium nucleus as predicted by hadronic mechanics and consisting of four isoprotons and two isoelectrons. Note that the illustration depicts the conventional conception of the Helium as being composed by two protons and two neutrons, with the sole replacement of the neutrons with their physical constituents permitted by hadronic mechanics, the proton and the electron. Note finally that the uncertainties of the constituents of the helium follow Einstein's determinism per Lemma II-3.7 and Corollary II-3.7.1.

fusion of deuterons into helium is the need for their *singlet coupling*

$$D(1, 2, 1_{\downarrow}) + D(1, 2, 1_{\uparrow}) \rightarrow He(2, 4, 0), \quad (108)$$

which is of extremely difficult engineering realization at low energies, and virtually impossible at high energies, resulting in nuclear fusions that can at best be at random.

For additional technical requirements needed to achieve the fusion of deuterons into helium, interested readers may consult the *hadronic laws for nuclear fusions*, which have been systematically studied in Ref. [25], Chapter 4 and Section 4.2, page 188, in particular.

The Directional Neutron Source (DNS, Figure 8) is produced and sold by Thunder Energies Corporation, now called Hadronic Technologies Corporation, for the synthesis of neutrons (Section 2.6.3) and pseudo-protons (Section 2.6.5) from a commercially available hydrogen gas.

However, the same DNS is produced for the use, without any modification, of a commercially available *deuterium gas* as a basic feedstock, in which case, hadronic mechanics predicts the synthesis of a *negatively charged deuteron* called *pseudo-deuteron* [48] (patent pending) which is predicted to have a structure of the type (Figure 17)

$$\hat{D} = [\tilde{p}_{\uparrow}, (\tilde{e}_{\uparrow}^-, \tilde{e}_{\downarrow}^-), \tilde{n}_{\uparrow}]_{hm}. \quad (109)$$

The mechanism for the synthesis of the pseudo-deuteron (which we write with nuclear symbols $D(-1, 2, 1)$) from a deuterium gas stems from the main feature of the DNS, that of actually “compressing” electrons in the interior hadrons (Figure 9) thanks to a specially conceived electric arc and other engineering features. Consequently, pseudo-deuteria are expected to be synthesized in the DNF operating on deuterium gas, following molecular separation with the ensuing presence of valence electron pairs called *isoelectronia* (Section 2.8.2 and Figure 20), of course, in a way dependent on power, pressure, flow, and other factors.

Preliminary calculations via hadronic mechanics indicate that the pseudo-deuteron should have a mean-life of the order of a second, thus being sufficient for industrial applications.

The importance of the research under consideration is that, contrary to current trends, *pseudo-deuterons are attracted, rather than being repelled by natural deuterons*.

This basic feature provides plausible means to search for basically *new* nuclear fusions called *hyper-fusions*, here referred to fusions of natural, positively charged nuclei and synthesized negatively charged nuclei, such as

the hyper-fusion of a pseudo-deuteron and a deuteron into the helium

$$\hat{D}(-1, 2, 1_{\uparrow}) + D(1, 2, 1_{\downarrow}) \rightarrow He(2, 4, 0) + 2e^{-}, \quad (110)$$

where the singlet coupling is naturally achieved because the pseudo-deuteron has not only the charge, but also the magnetic moment opposite that of the natural deuteron.

The energy expected to be released by each hyperfusion (110) is given by

$$\Delta E = E_{He} - (E_{\hat{D}} + E_D) = 23.8 \text{ MeV} = 3.81 \times 10^{-12} \text{ J}, \quad (111)$$

and occurs *without any emission of harmful radiation (since electrons can be stopped with a metal shield), and without the release of radioactive waste.*

As an illustration, the possible achievement of the rather realistic number of 10^{18} controlled fusions (110) per hour would yield the significant release of about 10^6 J of clean energy per hour without harmful radiations or waste.

In regard to the possible achievement of controlled hyper-fusions, we should mention that the DNS is considered in this section for the mere intent of achieving experimental measurements on the sole *existence* of pseudo-deuterons. Their production in the needed number and energy, and the hadronic reactor needed for the utilization of the produced heat, evidently require specialized equipment under proper funding.

By recalling that conventional nuclear fusions have requested billions of dollars of investments of public funds without any industrially viable result to date, readers should be warned against any expectation prior to the investment of equally large funds

2.8. Einstein's determinism in molecular structures.

2.8.1. Insufficiencies of quantum chemistry. There is no doubt that quantum chemistry has permitted truly historical advances in the 20th century. However, as it is the case for nuclear physics, serious scholars are expected to admit that quantum chemistry cannot be *exactly valid* for chemical structures and processes because of a number of insufficiencies, such as [30] (for dissident views, see also Refs. [15] [16] [17]):

1. Even though possessing excellent practical values, *the quantum chemical notion of valence is a 'nomenclature' without quantitative treatment because, due to their equal charge, the Schrödinger equation predicts that valence electron pairs 'repel' (rather than attract) each other, due to the necessary sign + of the Coulomb potential in the equation for the electron pair*

$$i \frac{\partial}{\partial t} \psi(t, r) = \left[-\frac{\hbar^2}{m} \Delta_r + \frac{e^2}{r} \right] \psi(t, r). \quad (112)$$

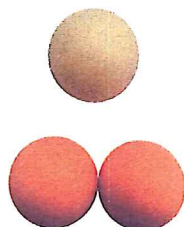


Figure 19: Lectures in hadronic chemistry are generally initiated by showing a ball in representation of a hydrogen atom with the recollection that quantum mechanics achieved an exact representation of all its experimental data. The speaker then shows two joined balls representing a hydrogen molecule, with the recollection that, this time, quantum mechanics and chemistry have not achieved an exact representation of molecular data. The reason indicated for this dichotomy is that mutual distances in the atomic structure, that are of the order of 10^{-8} cm, are such to allow an effective point-like approximation of the constituents. By contrast, mutual distances for valence electron pairs in molecular structures, which are of the order of 10^{-13} cm, are smaller than the size of the electron wavepackets, which is of about 2.2×10^{-13} cm, thus prohibiting an effective point-like approximation of particles in favor of the representation of their actual size and as well as of the ensuing contact non-potential interactions due to deep wave overlapping.

In particular, the repulsive force between the two identical valence electrons has the extremely big value of 230 N at mutual distances of 1 fm, Eq. (107), with repulsive values remaining big at atomic distances. Consequently, quantum chemistry misses a quantitative model of molecular structures.

2. According to quantum chemistry (see, e.g., Ref. [55]), valence electron bonds between two atoms are created by the overlap of atomic orbitals. Even though approximately valid, such a model is “incomplete” because, due to the point-like approximation of the electrons, said orbitals remain mostly independent, thus allowing their polarizations under a sufficiently strong electric or magnetic field, with the consequential prediction that substances are generally ferromagnetic.

3. Quantum mechanics has achieved an exact representation of all experimental data of the hydrogen atom H , but in the transition to the hydrogen molecule $H_2 = H - H$ (where $-$ represents valence bond), quantum mechanics and chemistry still miss 1 % of the H_2 binding energy which corresponds to the rather significant value of 950 BTU.

4. Quantum chemistry still misses to this day the quantitative identi-

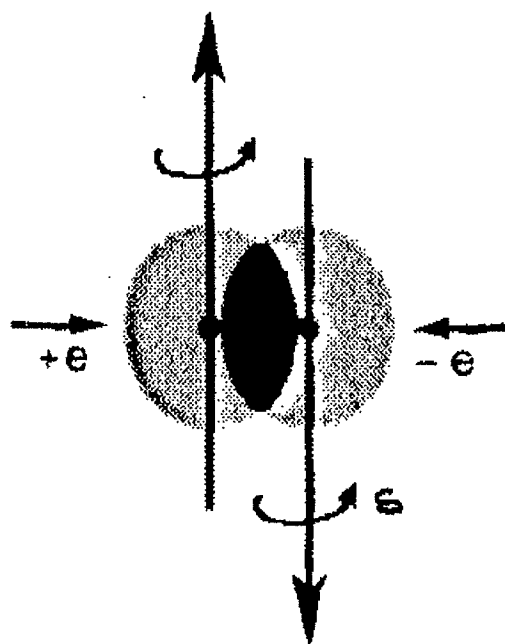


Figure 20: According to the basic principles of quantum mechanics and chemistry, electron valence pairs in molecular structures should “repel” (rather than “attract”) each other with a force of the order of 230 N, Eq. (107), which is extremely big for particle standards. Hadronic mathematics, mechanics and chemistry have achieved an attractive force between valence electron pairs so strong to turn them into a quasi-stable particle called the isoelectronium [30], that allowed the first known achievement of exact representation of experimental data on the hydrogen [52] and water [53] molecules that are not possible with quantum chemistry. In Santilli’s view, the impossibility in quantum chemistry to achieve an attractive force between valence electron pairs as occurring in nature, is additional evidence supporting the need for the “completion” of quantum principles into covering vistas according to Einstein’s legacy.

fication of the *attractive force* bonding together neutral, dielectric and diamagnetic molecules in their liquid state, with ensuing lack of quantitative representation of the liquid state for fuels.

5. The basic axioms of quantum chemistry are reversible over time. Consequently, quantum chemistry is structurally unable to provide an exact representation of combustion, as well as all energy-releasing processes with ensuing implications for the solution of problems of large societal values, such as the improvement of fossil fuel combustion.

We cannot possibly review in this paper the ongoing process toward the solution of the above insufficiencies of quantum chemistry via the covering hadronic chemistry [30]. Nevertheless, we believe it is important to indicate the main evidence supporting the validity for molecular structures of the "completion" of quantum chemistry into a suitable covering theory according to Einstein-Podolsky-Rosen argument [1].

Insufficiencies (1 - 5) above are known to chemists, as attested by the widespread distribution of works in the field by Santilli and other scholars. For this reason, said insufficiencies are referred to as *the best kept secrets in the best Ph. D. courses in chemistry around the world*.

2.8.2. Attractive force in valence electron bonds. Recall that in the 1978 paper [19], Santilli: 1) Identified the non-potential, strongly attractive force created by the deep overlapping of the wavepackets of particles in singlet coupling; 2) Constructed the foundations of hadronic mechanics with particular reference to the iso-Schrödinger equation for quantitative treatments of non-potential interactions; and 3) Applied the emerging new methods to achieve an exact representation of all characteristics of mesons as bound states of electrons and positrons (Sections 2.5.3 to 2.5.5).

In the 1995 paper [95], A. O. E. Animalu and R. M. Santilli showed that the indicated non-potential force is essentially charge independent since it remains strongly attractive also for electron-electron pairs in superconductivity thanks to the "absorption" of the Coulomb potential by the Hulthén potential irrespective of its sign, Eq. (5.1.15), page 836, Ref. [19] (reviewed in Eq. (31) above).

Thanks to the "completion" of quantum into hadronic chemistry (hc), in the 2001 monograph [30] Santilli worked out a new notion of valence bonds, today known as *strong valence bond*, which consists of valence isoelectron pairs, known as *isoelectronium* and denoted $\mathcal{I}$, with a fully identified *attractive force* with the structure (see the independent reviews [31]

[96] [97])

$$\mathcal{I} = (\tilde{e}_\uparrow, \tilde{e}_\downarrow)_{hc}. \quad (113)$$

The achievement of the attractive force between the identical valence electrons, apparently done for the first time in Ref. [30], Chapter 4, is described in detail in Section 2.3, and it is essentially based on the "completion" of all products into isoproducts (1) with consequential "completion" of the local Newton's differential calculus into the non-local isodifferential calculus (7) (8), resulting in the iso-Schrödinger equation

$$\begin{aligned} i \frac{\partial}{\partial t} \psi(t, r) &= \left[-\frac{\hbar^2}{m} \Delta_r + \frac{e^2}{r} \right] \psi(t, r) \rightarrow \\ \rightarrow i \frac{\hat{\partial}}{\hat{\partial} t} \psi(t, r) &= \left[-\frac{\hbar^2}{m} \hat{\Delta}_r + \frac{e^2}{r} \right] \hat{T}(\psi, \hat{\psi}, \dots) \hat{\psi}(t, r) \approx \\ \approx i \frac{\hat{\partial}}{\hat{\partial} t} \hat{\psi}(t, r) &= \left[-\frac{\hbar^2}{m} \hat{\Delta}_r - W \frac{e^{-br}}{1-e^{-br}} \right] \hat{T}(\psi, \hat{\psi}, \dots) \hat{\psi}(t, r), \end{aligned} \quad (114)$$

with a strong attraction characterized by the Hulthen potential.

In Santilli's view, *the achievement of an attractive force in valence electron pairs via the "completion" of the quantum wavefunction $\psi(t, r)$ into the hadronic isowavefunction $\hat{\psi}(t, r)$ is a strong confirmation of the final statement by Einstein, Podolski and Rosen [1] according to which quantum wavefunctions do not represent the entire physical or chemical reality.*

The strength of the internal bond allows the reduction of four-body molecules, such as the hydrogen $H_2 = H - H$, down to a *restricted three-body system* that, as such, admit full analytic solutions with major simplification of otherwise notoriously complex elaborations.

It should be noted that, besides the existence of molecules, the biggest evidence on the existence of the isoelectronium as a particle is provided by experimental data on the photoionization of the hydrogen and helium molecules (see, e.g., Ref. [56]) in which bonded electron pairs in singlet have been systematically detected to survive molecular separation.

For comparison with preceding hadronic structures, let us recall that Ref. [30], Eqs. (4.5) to (4.23), pages 169 to 173, provided the following representation via model (29) of the rest energy $E = 1 \text{ MeV}$ and change radius $R = 6.84 \times 10^{-11} \text{ cm}$ of the isoelectronium for the case of full stability

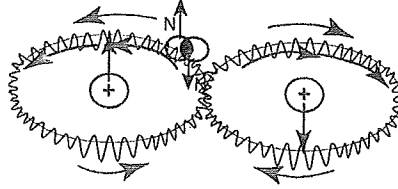


Figure 21: An illustration of the isochemical model of the hydrogen molecule achieved by R. M. Santilli and D. D. Shillady [52] with the following main features: 1) Valence electron pairs are bonded into isoelectronia according to the principles of hadronic chemistry; 2) Due to their strong internal forces, isoelectronia are forced to have oo orbits around corresponding nuclei resulting in opposite angular momenta that represent the diamagnetic character of the hydrogen and other molecules; 3) The conventional four-body equations for the hydrogen molecule are reduced to a restricted three-body form admitting a full analytic solution.

$$\begin{aligned} \tau = \infty \quad & \left[\frac{1}{r^2} \left(\frac{d}{dr} r^2 \frac{d}{dr} \right) + \tilde{m} (E \pm W_1 e_r^2 + W_2 \frac{e^{-br}}{1-e^{-br}}) \right] = 0, \\ E_{tot} = E_1 + E_2 - E = 1 \text{ MeV}, \quad \tilde{m} = \frac{m}{I^2} \end{aligned} \quad (115)$$

$$\tau^{-1} = 2\pi\lambda^2 |\hat{\psi}(0)|^2 \frac{\alpha^2 E_1}{\hbar} = \infty,$$

$$R = b^{-1} = 6.84 \times 10^{-11} \text{ cm},$$

with solutions for the k -parameters

$$k_1 = 0.19 \quad k_2 = 1, \quad (116)$$

for which the binding energy is null (in first approximation) as expected from the sole use of contact non-potential interactions, resulting in the total energy of the isoelectronium $E = E_1 + E_2 = 1.022 \text{ MeV}$.

Note that, as expected from the charge independence of contact interactions, values (114) for the isoelectronium $(\tilde{e}_1, \tilde{e}_2)_{hc}$ are rather close to the corresponding values (50) for the π^0 meson $(\tilde{e}^-, \tilde{e}^+)_{hm}$.

Interested readers should meditate a moment on the capability of contact interactions to achieve a strongly attractive force by overcoming the extremely big repulsive Coulomb force of 230 N.

This occurrence emphasizes the need in hadronic structure models such as (112) of the notion of *isoparticles* in which *all* intrinsic characteristics are mutated, thus including the charge.

This occurrence also suggests the *study of structure models of the elementary charge*, such as that of Ref. [44], capable of such a mutation in condition of deep mutual overlapping to represent the physical evidence of a strong attraction in singlet valence couplings.

In 1999, D. D. Shillady and R. M. Santilli proved that the isotopic branch of hadronic chemistry achieves the first known, numerically exact representation of the binding energy and other characteristics of the hydrogen [52] (Figure 23) and water [53] (Figure 24) molecules via isoperturbative isoseries (namely isoseries based on isoproduct $A\hat{T}B$ with $\hat{T} \ll 1$) whose convergence is at least *one thousand times faster* than the convergence of conventional quantum chemical series, thus providing an important experimental confirmation in molecular structures of both Einstein's determinism per Lemma II-3.7 as well as Corollary II-3.7.1.

In regard to the rather complex case in which the isoelectronium is partially unstable, the numerically exact representation of H_2 and H_2O data, and other aspects, we refer the interested reader to the above quoted literature.

The strong valence bond of isoelectronia evidently resolves Insufficiencies 1) and 2) of the preceding section.

Insufficiency 3) is resolved by the fact that, due to the strength of its bond, isoelectronium (112) is forced to have an oo-orbit around the respective nuclei (Figure 22) with ensuing opposite angular momenta and opposite magnetic polarities that permit a quantitative representation of the diamagnetic character of the hydrogen, water and other molecules.

The apparent resolution of Insufficiency 4) was provided by the new chemical species of *magnecules* [30] (see the U. S. patent [57], the latest experimental verification [58] and Figure 25) that allowed the identification of an actual attractive force between water molecules in their liquid state [98].

The resolution of Insufficiency 5) can be studied via the *Lie-admissible branch of hadronic chemistry*, also called *genochemistry* [30], although it does not appear that, at this writing, a consistent representation of energy releasing processes is of interest to contemporary chemists at large.

2.8.3. Industrial applications. It is a truism to state that the combustion of fossil fuels occurring in the ongoing disproportionate number of civilian, industrial and military vehicles is the same as it was at the dawn of our civilization some 50,000 years ago because, in all cases, we strike a spark and lit the fuel.

To be "True Researchers" in Einstein's words, it is our societal duty to

Species	H ₂	H ₂ O	HF
SCF-energy (DH) (a.u.)	-1.132800 ^a	-76.051524	-100.057186
Hartree-Fock ^d (a.u.)			-100.07185 ^d
Iso-energy (a.u.)	-1.174441 ^c	-76.398229 ^c	-100.459500 ^c
Horizon R_o (Å)	0.00671	0.00038	0.00030
QMC energy ^{d,e} (a.u.)	-1.17447	-76.430020 ^e	-100.44296 ^d
Exact non-rel. (a.u.)	-1.174474 ^f		-100.4595 ^d
Correlation (%)	99.9 ^b	91.6 ^b	103.8
SCF-dipole (D)	0.0	1.996828	1.946698
Iso-dipole (D)	0.0	1.847437	1.841378
Exp. dipole (D)	0.0	1.85 ^g	1.82 ^g
Time ^h (mins)	0:15.49	10:08.31	6:28.48

(DH⁺) Dunning-Huzinaga (10S/6P), [6,2,1,1,1/4,1,1]+H₂P₁+3D1.

^aLEAO-6G1S + optimized GLO-2S and GLO-2P.

^bRelative to the basis set used here, not quite HF-limit.

^cIso-energy calibrated to give exact energy for HF.

^dHartree-Fock and QMC energies from Luchow and Anderson [33].

^eQMC energies from Hammond et al. [30].

^fFirst 7 sig. fig. from Kolos and Wolniewicz [34].

^gData from Chemical Rubber Handbook, 61st ed., p. E60.

^hRun times on an O2 Silicon Graphics workstation (100 MFLOPS max.).

Figure 22: A reproduction of Table 4.2 of Ref. [52] presenting the first known exact representation of the experimental data of the hydrogen molecule via isomathematics and isochemistry, by providing an experimental confirmation of the validity in molecular structures of Einstein's determinism according Lemma II-3.7, with a convergence of the isoperturbative series at least one thousand times faster than conventional series, by therefore providing an experimental confirmation of Corollary II-3.7.1.

initiate the laborious process of trial and errors in seeking a clean combustion of fossil fuels that, to be as such, as to be *new*, namely based on new mathematical, physical and chemical principles.

In the hope of initiating the expectedly long and laborious process of trials and errors, Santilli has submitted in Ref. [59] the study of the new form of combustion of carbon and oxygen, known as *hypercombustion*, intended to achieve the full combustion of fossil fuels via a small percentage of nuclear fusions of the novel magnecular forms of $6-C-12$ and $8-O-16$ into the stable $14-Si-28$, thus without the emission of harmful radiations and without the release of radioactive waste [69] [70] (Figure 24).

We should mention in this respect the gaseous fuel *magnegas* [30] which is synthesized by hadronic reactors from a mixture of oil and water, in production and sale world wide by Magnegas Corporation, now called Taronis Corporation, whose combustion in air shows no appreciable carbon monoxide CO and hydrocarbons (HC) (Figure 25).

This environmental result was achieved via a combustion temperature more than double that of commercially available fuels, which temperature is permitted by magnecular bonds that are *weaker* than molecular bonds, and at which temperature CO and HC cannot remain unburned.

HyerCombustion aims at achieving the needed higher combustion temperature of petroleum fuels via the use of the indicated nuclear processes that are solely possible under the "completion" of quantum into hadronic mechanics.

It appears advisable to indicate that, besides the expected environmental advances, the new chemical species of magnecules appears to have significant medical applications, such as the possible killing of the corona virus in lungs (and other organs) of patients via ventilators releasing the new polarized species of *magneoxygen*, rather than conventional oxygen [62] - [65].

2.9. Einstein's determinism in gravitational collapse.

In the recent paper [10], Santilli recalls that iso-space-time metrics contain as particular cases all possible symmetric metrics in $(3 + 1)$ -dimensions, thus including the Riemannian metric.

Ref. [10] then factorizes the space component of the Schwartzchild metric $g_s(r)$ according to isotopic rule introduced in Refs. [66] [67]

$$g_s(r) = \hat{T}(r)\delta, \quad (117)$$

where δ is the Euclidean metric.

In this way, Santilli reaches the following realization of the isotopic

Studies on the EPR argument, I: Basic methods

	OH ⁺	OH ⁻	H ₂ O	HF
SCF-Energy ^a	-74.860377	-75.396624	-76.058000	-100.060379
Hartree-Fock ^b				-100.07185 ^b
Iso-Energy ^c	-75.056678	-75.554299	-76.388340	-100.448029
Horizon R_c (Å)	0.00038	0.00038	0.00038	0.00030
QMC Energy ^{b,d}	-76.430020 ^d			-100.44296 ^b
Exact non-rel.				-100.4595
Iso-Dipole (D)	5.552581	8.638473	1.847437	1.8413778
Exper. Dipole			1.84	1.82

^a Dunning-Huzinaga (10s/6p), (6,2,1,1,1/4,1,1)+H2s1+H2p1+3d1.

^b Iso-Energy calibrated to give maximum correlation for HF.

^c Hartree-Fock and QMC energies from Luchow and Anderson [22].

^d QMC energies from Hammond, Lester and Reynolds [21].

Figure 23: A reproduction of Table 5.1. of Ref. [53] presenting the first exact representation of the experimental data of the water molecule, including its electric and magnetic moments, via isomathematics and isochemistry, by therefore providing a second experimental confirmation of the validity in molecular structures of Einstein's determinism according to Lemma II-3.7, with a convergence of the isoperturbative series at least one thousand time faster than conventional series, thus providing an experimental confirmation of Corollary II-3.7.1.

element

$$\hat{T} = \frac{1}{1 - \frac{2M}{r}} = \frac{r}{r - 2M}, \quad (118)$$

where M is the gravitational mass of the body considered with ensuing isodeterministic isoprinciple, Ref. [10], Eq. (46), page 16,

$$\Delta \hat{r} \Delta \hat{p} \approx \hat{T} = \frac{r}{r - 2M} \Rightarrow_{r \rightarrow 0} = 0, \quad (119)$$

which confirms the possible recovering of full classical determinism in the interior of gravitational collapse essentially as predicted by Einstein (see Ref. [16], Chapter 6 in particular, for a critical analysis of black holes).

It should perhaps be indicated that the 1993 paper [100] identified the universal isosymmetry of all possible (non-singular) Riemannian line elements in $3 + 1$ -dimensions formulated on iso-Minkowskian isospaces [78] over isofields. Papers [66] [67] introduced the factorization of a full Riemannian metric $g(x)$, $x = (r, t)$ in $(3 + 1)$ -dimensions

$$g(x) = \hat{T}_{gr}(x)\eta, \quad (120)$$

where $\hat{T}_{gr}$ is the *gravitational isotopic element*, and η is the Minkowski metric $\eta = \text{Diag.}(1, 1, 1, -1)$.

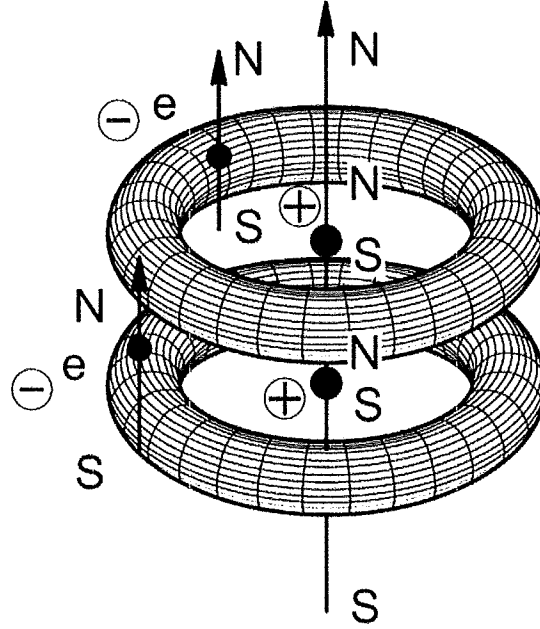


Figure 24: In this figure, we present a conceptual rendering of the new chemical species of Santilli magnecules [30] for the case of the elementary hydrogen magnecules $\tilde{H}_2 = H \times H$ (where $\times$ denotes magnecular bond). The species is obtained via the use of special electric arcs producing electric and magnetic fields over certain minimum values [?], which arcs ionize the hydrogen molecule and polarize the electron orbits into toroids by therefore creating a new magnetic dipole moment which does not exist in the natural state. Polarized atoms bond together in the configuration of the figure which is stable at ambient temperatures by creating the new species of magnehydrogen (see Refs. [62] to [65]). It should be indicated that elementary magnecules between other selected atoms verify all hadronic laws for controlled nuclear fusions ([25], Section 4), for instance, of $\tilde{C}O = 6 - C - 12 \times 8 - O - 16$ into $14 - Si - 28$ without the emission of harmful radiations and without the release of radioactive waste [69] [70].

Refs. [66] [67] then reformulated the Riemannian geometry via the transition from a formulation over the field of real numbers $\mathcal{R}$ to that over the isofield of isoreal isonumbers $\tilde{\mathcal{R}}$ where the gravitational isounit is evidently given by

$$\hat{I}_{gr}(x) = 1/\hat{T}_{gr}(x). \quad (121)$$

The above reformulation turns the Riemannian geometry into a new

geometry called iso-Minkowskian isogeometry [78] which is locally isomorphic to the *Minkowskian* geometry, while maintaining the mathematical machinery of the Riemannian geometry (covariant derivative, connection, geodesics, etc.) although reformulated in terms of the isodifferential isocalculus.

The following advantages should be mentioned for the *identical* iso-Minkowskian reformulation of general relativity, including Einstein's field equations, known as *isogravitation* [83]:

- 1) The achievement of a consistent operator form of gravity via the axiom-preserving embedding of the gravitational isounit $\hat{I}_{gr}(x)$ in the unit of relativistic quantum mechanics [33] [21];
- 2) The achievement of the universal *LPS isosymmetry* of all non-singular Riemannian metrics [100], which symmetry is locally isomorphic to the LP symmetry, (Section II-2), while being notoriously impossible in a conventional Riemannian space over the reals;
- 3) The achievement of clear compatibility, actually a *true isounification*, of general and special relativity since the latter can be identically recovered with the simple limit

$$\hat{I}_{gr} \rightarrow I = \text{Diag.}(1, 1, 1, 1), \quad (122)$$

implying the transition from the universal LPS isosymmetry to the LP symmetry of special relativity with ensuing recovering of conservation and other special relativity laws;

- 4) The possibility of initiating systematic studies on *interior gravitational problems* along the forgotten Schwartzchild's second paper [73];
- 5) The achievement of axiomatic compatibility between gravitation and electroweak interactions thanks to the replacement of curvature into the covering notion of *isoflatness*, while offering realistic hopes to achieve a grand unification [38]; and other intriguing advances.

3. CONCLUDING REMARKS.

At the end of their historical paper [1], Einstein, Podolsky and Rosen state:

While we have thus shown that the wavefunction [of quantum mechanic] does not provide a complete description of the physical reality, we left open the question of whether or not such a description exists. We believe, however, that such a theory is possible.

In the preceding Papers I and II we have provided a review and upgrade of the apparent proof of the existence of classical counterparts for



Figure 25: A picture of the 300 kW mobile magnegas refinery built by Santilli when Chief Scientist of Magnegas Corporation (now Taronis Corporation) to process a mixture of fossil oil and water into magnegas whose combustion with the proper stoichiometric oxygen has no detectable CO or HC [57]. The oxygen content of magnegas appears to have special polarizations [62] - [65] deserving tests for its possible use in ventilators to kill Corona viruses due to the emission of strong UV light in its depolarization in human lungs.

extended particles within physical media [9] (Section II-3.7), and of the apparent progressive verification of Einstein determinism in the interior of hadrons, nuclei and stars [10] (Section II-3.8).

In this third paper, it appears we have additionally proved, apparently for the first time, the above quoted, concluding EPR statement. In fact, the "completion" of the quantum wavefunction $\psi(t, r)$ of the Schrödinger equation into the hadronic isowavefunction $\hat{\psi}(\hat{t}, \hat{r})$ of the Schrödinger-Santilli isoequation allows the achievement, otherwise impossible via quantum mechanics, of an exact representation of all characteristics of: the neutron in its synthesis from the hydrogen atom (Section 2.6); the deuteron (Section 2.7); the attractive force between the identical electrons in valence couplings (Section 2.8); and the progressive achievement of Einstein's determinism in interior dynamical conditions (Section 2.9).

Acknowledgments. The writing of these papers has been possible thanks to pioneering contributions by numerous scientists we regret not having been able to quote in this paper (see Vol. I, Refs. [?] for comprehensive literature), including:

The mathematicians: H. C. Myung, for the initiation of iso-Hilbert spaces; S. Okubo, for pioneering work in non-associative algebras; N. Kamiya, for basic advances on isofields; M. Tomber, for an important bibliography on non-associative algebras; R. Ohemke, for advances in Lie-admissible algebras; J. V. Kadeisvili, for the initiation of isofunctional isoanalysis; Chun-Xuan Jiang, for advances in isofield theory; Gr. Tsagas, for advances in the Lie-Santilli theory; A. U. Klimyk, for the initiation of Lie-admissible deformations; Raul M. Falcon Ganfornina and Juan Nunez Valdes, for advances in isomanifolds and isotopology; Svetlin Georgiev, for advances in the isodifferential isocalculus; A. S. Muktibodh, for advances in the isorepresentations of Lie-Santilli isoequations; Thomas Vougiouklis, for advances on Lie-admissible hypermathematics; and numerous other mathematicians.

The physicists and chemist: A. Jannussis, for the initiation of isocreation and isoannihilation operators; A. O. E. Arimalu, for advances in the Lie-admissible formulation of hadronic mechanics; J. Fronteau, for advances in the connection between Lie-admissible mechanics and statistical mechanics; A. K. Aringazin for basic physical and chemical advances; R. Mignani for the initiation of the isoscattering theory; P. Caldirola, for advances in isotime; T. Gill, for advances in the isorelativity; J. Dunning Davies, for the initiation of the connection between irreversible Lie-admissible mechanics and thermodynamics; A. Kalnay, for the connection be-

tween Lie-admissible mechanics and Nambu mechanics; Yu. Arestov, for the experimental verification of the iso-Minkowskian structure of hadrons; A. Ahmar, for experimental verification of the iso-Minkowskian character of Earth's atmosphere; L. Ying, for the experimental verification of nuclear fusions of light elements without harmful radiation or waste; Y. Yang, for the experimental confirmation of magnecules; D. D. Shillady, for the isorepresentation of molecular binding energies; R. L. Norman, for advances in the laboratory synthesis of the neutron from the hydrogen; I. Gandzha for important advances on isorelativities; A. A. Bhalekar, for the exact representation of nuclear spins; Simone Beghella Bartoli important contributions in experimental verifications of hadronic mechanics; and numerous other physicists.

I would like to express my deepest appreciation for all the above courageous contributions written at a point in time of the history of science dominated by organized oppositions against the pursuit of new scientific knowledge due to the trillions of dollars of public funds released by governmental agencies for research on pre-existing theories.

Special thanks are due to A. A. Bhalekar, J. Dunning-Davies, S. Georgiev, R. Norman, T. Vougiouklis and other colleague for deep critical comments.

Additional special thanks are due to Simone Beghella Bartoli for a detailed critical inspection of this Paper I.

Special thanks and appreciation are due to Arun Muktibodh for a very detailed inspection and control of all three papers, although I am solely responsible for their content due to numerous revisions implemented in their final version.

Thanks are also due to Mrs. Sherri Stone for an accurate linguistic control of the manuscript.

Finally, I would like to express my deepest appreciation and gratitude to my wife Carla Gandiglio Santilli for decades of continued support of my research amidst a documented international chorus of voices opposing the study of the most important vision by Albert Einstein.

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